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On the Use of Inverse Exponentiation to Improve the Efficiency of Calibration Estimators in Stratified Double Sampling

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Abstract

This study introduces the concept of inverse exponentiation in formulating calibration weights in stratified double sampling and proposes a more improved calibration estimator based on Koyuncu and Kadilar (2014) calibration estimator. The variance of the proposed logarithmic calibration estimator has been derived under large sample approximation. Calibration asymptotic optimum estimator ($CAOE$) and its approximate variance estimator are derived for the proposed logarithmic calibration estimator. Results of empirical study showed that the proposed logarithmic calibration estimator ($\bar{Y}^*_{new}$) performs better than the Koyuncu and Kadilar (2014) calibration estimator ($\bar{Y}^*_{kk}$) with appreciable gains in efficiency. Also, simulation study for the comparison of the proposed logarithmic estimator with a Global estimator [Generalized Regression (GREG) estimator ($\bar{Y}^*_{GREG}$)] proved the robustness of the proposed logarithmic calibration estimator and by extension the efficacy of inverse exponentiation in calibration weightings. Analysis and evaluation are presented.

Keywords and Phrases: Calibration constraint, large sample approximation, logarithmic estimator, optimality conditions, percentage relative efficiency.

AMS Classification: 62D05, 62G05, 62H12.

1. Introduction

The integration of supplementary information holds significant importance in constructing efficient estimators for population parameter estimation and enhancing efficiency in diverse sampling designs. Exploring the knowledge of the supplementary variables, several authors have developed different estimation techniques for estimating the finite population mean of the study variable; [Cochran (1977),Singh and Tailor (2003), Gupta and Shabbir (2008), Sharma and Tailor (2010), Diana *et al*. (2011), Singh and Audu (2013), Shittu and Adepoju (2014), Lone and Tailor (2015); Clement and Enang (2015), Clement (2016, 2017), Clement *et al* (2021), Inyang and Clement (2023)] among others, have worked on the estimation of population parameters using supplementary information.

Calibration estimation extensively explores the use of supplementary information to adjust the original design weights to improve the precision of survey estimates of population or subpopulation parameters. The calibration weights are chosen to minimize a given distance measure (or loss function) and these weights satisfy the constraints related supplementary variable information. The concept of calibration estimation was introduced by Deville and Sarndal (1992) and a wealth of research, featuring scholars like Wu and Sitter (2001), Arnab and Singh (2005), Kim *et al.* (2007), Sarndal (2007), Kim and Park (2010), Rao *et al.* (2012), Clement *et al.* (2014), Koyuncu and Kadilar (2016), Clement and Enang (2017), Clement (2021, 2022), Clement and Inyang (2020, 2021), Enang and Clement (2020), has delved into calibration estimation.

Tracy *et al.* (2003) introduced the concept of calibration estimation to stratified double sampling using multi-parametric calibration weightings. Multi-parametric calibration weightings is the formulation of calibration constraints with respect to a given distance measure to obtain expression of calibration weights using information from two or more parameters of the same supplementary variable. Work in this aspect include among others, Tracy *et al.* (2003), Koyuncu and Kadilar (2016), Clement (2018), Clement (2020) and Clement and Etukudoh (2023).

In the progression to improve calibration estimation, this paper based on Koyuncu and Kadilar (2014) calibration estimator, introduces a new improved calibration estimator for population mean in stratified double sampling with equal probability using the concept of inverse exponentiation. The choice is obvious, because inverse exponentiation reduces both the non-response bias and the sampling error, thereby increasing the efficiency of the proposed calibration estimator.

2. Sample Design and Procedure

In double sampling for stratification the population is stratified into H strata such that the h-th stratum consists of N_h units and $\sum_{h=1}^{H} N_h = N$, $\sum_{h=1}^{H} n_h = n$. From the N_h units a preliminary large sample of $n_h^{'}$ units is drawn by the simple random sampling without replacement (*SRSWOR*) and the supplementary character x_{hi} is measured only. A subsample of n_h is then selected from the given preliminary large sample of $n_h^{'}$ units by *SRSWOR* and both the study variable y_{hi} and the supplementary variable x_{hi} are measured.

Let $\bar{x}_h^{'} = \frac{1}{n_h^{'}}\sum_{i=1}^{n_h^{'}} x_{hi}$, $S_{hx}^{'2} = \frac{1}{n_h^{'}-1}\sum_{i=1}^{n_h^{'}}\left(x_{hi} - \bar{x}_h^{'}\right)$, denote the first phase sample mean and variance respectively for the supplementary variable.

Similarly, let $\bar{x}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} x_{hi}$, $S_{hx}^2 = \frac{1}{n_h-1}\sum_{i=1}^{n_h}(x_{hi} - \bar{x}_h)^2$, $\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi}$, and

$S_{hy}^2 = \frac{1}{n_h-1}\sum_{i=1}^{n_h}(y_{hi} - \bar{y}_h)^2$ denote the second phase sample means and variances for the supplementary variable and study variable respectively.

Let the relative errors be defined as follows:

$e_{hy} = \left(\frac{\bar{y}_h - \bar{Y}_h}{\bar{Y}_h}\right)$ so that $\bar{y}_h = \bar{Y}_h\left(1 + e_{hy}\right)$

$e_{hx} = \left(\frac{\bar{x}_h - \bar{X}_h}{\bar{X}_h}\right)$ so that $\bar{x}_h = \bar{X}_h(1 + e_{hx1})$

$e'_{hx} = \left(\frac{\bar{x}'_h - \bar{X}_h}{\bar{X}_h}\right)$ so that $\bar{x}'_h = \bar{X}_h(1 + e'_{hx1})$

$e_{hs} = \left(\frac{s_{hx}^2 - S_{hx}^2}{S_{hx}^2}\right)$, so that $s_{hx}^2 = S_{hx}^2(1 + e_{hs})$

$e'_{hs} = \left(\frac{s'^2_{hx} - S^2_{hx}}{S^2_{hx}}\right)$, so that $s'^2_{hx} = S^2_{hx}(1 + e'_{hs})$

Let the expected values of the relative errors be defined as follows:

$E(e_{hy}) = E(e_{hx}) = (e_{hs}) = E(e'_{hx}) = E(e'_{hs}) = 0$

$E(e^2_{hy}) = \gamma_h C^2_{hy}, E(e^2_{hx}) = \gamma_h C^2_{hx}, E(e^2_{hs}) = \gamma_h C^2_{hs},$

$E(e'^2_{hx}) = \gamma'_h C^2_{hx}, E(e'^2_{hs}) = \gamma'_h C^2_{hs},$

$E(e_{hy} e_{hx}) = \gamma_h \rho_{hyx} C_{hy} C_{hx}$, $E(e_{hy} e_{hs}) = \gamma_h \rho_{hys} C_{hy} C_{hs}$,

$E(e_{hy} e'_{hx}) = \gamma'_h \rho_{hyx} C_{hy} C_{hx}$, $E(e_{hy} e'_{hs}) = \gamma'_h \rho_{hys} C_{hy} C_{hs}$,

$E(e'_{hx} e_{hx}) = \gamma'_h C^2_{hx}, E(e'_{hs} e_{hs}) = \gamma'_h C^2_{hs}$

$E(e'_{hx} e_{hxs}) = \gamma'_h \rho_{hxs} C_{hx} C_{hs}$ $E(e'_{hs} e_{hx}) = \gamma'_h \rho_{hxs} C_{hx} C_{hs}$,

$E(e'_{hx} e'_{hs}) = \gamma'_h \rho_{hxs} C_{hx} C_{hs}$, ,

$E(e_{hx} e_{hs}) = \gamma_h \rho_{hxs} C_{hx} C_{hs},$

where $\gamma'_h = \left(\frac{1}{n'_h} - \frac{1}{N_h}\right)$, $\gamma_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right)$ and $\gamma^*_h = \gamma_h - \gamma'_h = \left(\frac{1}{n_h} - \frac{1}{n'_h}\right)$

and the parameters are defined wherever they appear as the following:

$\bar{y}_h$ is the second phase sample stratum mean of the study variable
$\bar{Y}_h$ is the second phase population stratum mean of the study variable
$\bar{x}'_h$ is the first phase sample stratum mean of the supplementary variable
$\bar{x}_h$ is the second phase sample stratum mean of the supplementary variable
$\bar{X}_h$ is the second phase population stratum mean of the supplementary variable
s'^2_{hx} is the first phase sample stratum variance of the supplementary variable
s^2_{hx} is the second phase sample stratum variance of the supplementary variable
S^2_{hx} is the second phase population stratum variance of the supplementary variable
C^2_{hx} is the coefficient of variation of the supplementary variable
C^2_{hy} is the coefficient of variation of the supplementary variable
ρ_{hxy} is the correlation coefficient between the supplementary variable and the study variable.
ρ_{hxs} is the correlation coefficient between the mean and variance of the supplementary variable.
ρ_{hys} is the correlation coefficient between the mean of the study variable and variance of the supplementary variable.

3. The Koyuncu and Kadilar (2014) calibration estimator

Motivated by Tracy et al. (2003), Koyuncu and Kadilar (2014) proposed the following calibration estimator in stratified double sampling:

$$\bar{Y}^*_{kk} = \sum_{h=1}^{H} \psi_h \bar{y}_h \tag{1}$$

using the chi-square loss functions of the form:

$$L(, \psi_h, w_h) = \sum_{h=1}^{H} \frac{(\psi_h - w_h)^2}{w_h q_h} \tag{2}$$

and subject to the calibration constraints defined by:

$$\sum_{h=1}^{H} \psi_h \bar{x}_h = \sum_{h=1}^{H} w_h \bar{x}_h' \quad (3)$$
$$\sum_{h=1}^{H} \psi_h s_{hx}^2 = \sum_{h=1}^{H} w_h s_{hx}'^2 \quad (4)$$
$$\sum_{h=1}^{H} \psi_h = \sum_{h=1}^{H} w_h \quad (5)$$

obtained the calibration weights

$$\psi_h = w_h + w_h q_h(\lambda_{11}\bar{x}_h + \lambda_{22}s_{hx}^2 + \lambda_{33}) \quad (6)$$

Substituting (6) in [(3), (4), (5)] respectively and solving the resulting system of equations gives the values of the $\lambda_{ii}s$.

On substituting the $\lambda_{ii}s$ in (6) and the resulting equation in (1); Koyuncu and Kadilar (2014) obtained their calibration regression estimator as:

$$\bar{Y}_{kk}^* = \bar{y}_{st} + \mathrm{B}_{h,11}\sum_{h=1}^{H} w_h(\bar{x}_h' - \bar{x}_h) + \mathrm{B}_{h,22}\sum_{h=1}^{H} w_h\left(s_{hx}'^2 - s_{hx}^2\right) \quad (7)$$

where $\bar{y}_{st} = \sum_{h=1}^{H} w_h \bar{y}_h$ is the Horvitz-Thompson-type estimator; $\mathrm{B}_{h,11}$ and $\mathrm{B}_{h,22}$ are coefficients of regression and are given by

$$\mathrm{B}_{h,11} = \frac{v_{22}v_{13} - v_{12}v_{23}}{v_{11}v_{22} - v_{12}^2}, \qquad \mathrm{B}_{h,22} = \frac{v_{11}v_{23} - v_{12}v_{13}}{v_{22} - v_{12}^2}$$

where

$v_{11} = \sum_{h=1}^{H} w_h \bar{x}_h^2$, $v_{12} = \sum_{h=1}^{H} w_h \bar{x}_h s_{hx}^2$, $v_{13} = \sum_{h=1}^{H} w_h \bar{x}_h \bar{y}_h$, $v_{22} = \sum_{h=1}^{H} w_h s_{hx}^4$
$v_{23} = \sum_{h=1}^{H} w_h s_{hx}^2 \bar{y}_h$, [See Koyuncu and Kadilar (2014) for detail]

3.1 Theoretical Variance Estimation

This section derives the estimator of variance for the Koyuncu and Kadilar (2014) calibration estimator. Thus, expressing (7) in the relative error terms gives

$$[\bar{Y}_{kk}^* - \bar{Y}] = \sum_{h=1}^{H} w_h\big[\bar{Y}_h e_{hy} + \mathrm{B}_{h,11}\bar{X}_h e_{hx}' + \mathrm{B}_{h,22}S_{hx}^2 e_{hs}'$$
$$-\mathrm{B}_{h,11}\bar{X}_h e_{hx} - \mathrm{B}_{h,22}S_{hx}^2 e_{hs}\big] \quad (8)$$

Squaring both sides of (8) gives

$$[\bar{Y}_{kk}^* - \bar{Y}]^2 = \sum_{h=1}^{H} w_h^2\,\big[\bar{Y}_h^2 e_{hy}^2 + \mathrm{B}_{h,11}^2\bar{X}_h^2\left(e_{hx}'^2 + e_{hx}^2\right)$$
$$+\mathrm{B}_{h,22}^2 S_{hx}^4\left(e_{hs}'^2 + e_{hs}^2\right) + 2\bar{Y}_h\mathrm{B}_{h,11}\bar{X}_h e_{hy}e_{hx}' + 2\bar{Y}_h\mathrm{B}_{h,22}S_{hx}^2 e_{hy}e_{hs}'$$
$$+2\mathrm{B}_{h,11}\mathrm{B}_{h,22}\bar{X}_h S_{hx}^2\left(e_{hx}e_{hs} + e_{hx}'e_{hs}'\right) - 2\mathrm{B}_{h,11}^2\bar{X}_h^2 e_{hx}'e_{hx}$$
$$-2\mathrm{B}_{h,22}^2 S_{hx}^4 e_{hs}'e_{hs} - 2\bar{Y}_h\mathrm{B}_{h,11}\bar{X}_h e_{hy}e_{hx} - 2\bar{Y}_h\mathrm{B}_{h,22}S_{hx}^2 e_{hy}e_{hs}$$
$$-2\mathrm{B}_{h,11}\mathrm{B}_{h,22}\bar{X}_h S_{hx}^2\left(e_{hx}e_{hs}' + e_{hx}'e_{hs}\right)\big] \quad (9)$$

Taking expectation of both sides of (9) gives

$$\hat{V}[\bar{Y}_{kk}^*] = \sum_{h=1}^{H} w_h^2\,\bar{Y}_h^2\gamma_h C_{hy}^2 + \left(\gamma_h - \gamma_h'\right)\sum_{h=1}^{H} w_h^2\,[\mathrm{B}_{h,11}^2\bar{X}_h^2 C_{hx}^2 - 2\bar{Y}_h\mathrm{B}_{h,11}\bar{X}_h\rho_{hyx}C_{hy}C_{hx}$$
$$+\mathrm{B}_{h,22}^2 S_{2h}^4 C_{hs}^2 + 2\mathrm{B}_{h,22}\bar{Y}_h S_{hx}^2\rho_{hsy}C_{hs}C_{hy} - 2B_{h,11}\mathrm{B}_{h,22}\bar{Y}_h S_{hx}^2\rho_{hxs}C_{hx}C_{hs}] \quad (10)$$

3.2 Optimality conditions

This section deduces the optimality conditions that would guarantee optimum performance of the Koyuncu and Kadilar (2014) calibration estimator.

Setting $\frac{\partial \hat{V}[\bar{Y}_{kk}^*]}{\partial \mathrm{B}_{h,11}} = 0$ and $\frac{\partial \hat{V}[\bar{Y}_{kk}^*]}{\partial \mathrm{B}_{h,22}} = 0$ respectively gives:

$$\mathrm{B}_{h,11} = \frac{\bar{Y}_h \rho_{hyx} C_{hy} C_{hx} - \mathrm{B}_{h,22} S_{hx}^2 \rho_{hxs} C_{hx} C_{hs}}{\bar{X}_h C_{hx}^2} \tag{11}$$

$$\mathrm{B}_{h,22} = \frac{\bar{Y}_h \rho_{hys} C_{hy} C_{hs} - \mathrm{B}_{h,11} \bar{X}_h \rho_{hxs} C_{hx} C_{hs}}{S_{hx}^2 C_{hs}^2} \tag{12}$$

Substituting (11) in (12) or vice verse, gives the optimum values of $\mathrm{B}_{h,11}(opt)$ and $\mathrm{B}_{h,22}(opt)$ respectively as:

$$\mathrm{B}_{h,11}(opt) \quad = \quad \frac{\bar{Y}_h C_{hy}\left(\rho_{hyx} - \rho_{hys}\rho_{hxs}\right)}{\bar{X}_h C_{hx}(1-\rho_{hxs}^2)} \tag{13}$$

$$\mathrm{B}_{h,22}(opt) \quad = \quad \frac{\bar{Y}_h C_{hy}\left(\rho_{hys} - \rho_{hyx}\rho_{hxs}\right)}{S_{hx}^2 C_{hs}(1-\rho_{hxs}^2)} \tag{14}$$

Substituting the value of $\mathrm{B}_{h,11}(opt)$ in (13) and $\mathrm{B}_{h,22}(opt)$ in (14) for $\mathrm{B}_{h.11}$ and $\mathrm{B}_{h,22}$ in (7), gives the Koyuncu and Kadilar (2014) calibration asymptotically optimum estimator (*CAOE*) for population mean in stratified double sampling as:

$$\bar{Y}_{kk,Opt}^* = \sum_{h=1}^{H} w_h \bar{y}_h + \frac{\bar{Y}_h C_{hy}\left(\rho_{hyx} - \rho_{hys}\rho_{hxs}\right)}{\bar{X}_h C_{hx}(1-\rho_{hxs}^2)} \sum_{h=1}^{H} w_h(\bar{x}_h' - \bar{x}_h)$$
$$+ \frac{\bar{Y}_h C_{hy}\left(\rho_{hys} - \rho_{hyx}\rho_{hxs}\right)}{S_{hx}^2 C_{hs}(1-\rho_{hxs}^2)} \sum_{h=1}^{H} w_h\left(s_{hx}'^2 - s_{hx}^2\right) \tag{15}$$

Similarly, substituting the value of $\mathrm{B}_{h,11}(opt)$ in (13) and $\mathrm{B}_{h,22}(opt)$ in (14) for $\mathrm{B}_{h,11}$ and $\mathrm{B}_{h,22}$ in (10), gives the variance of Koyuncu and Kadilar (2014) calibration asymptotically optimum estimator $(CAOE)$ $\bar{Y}_{kk,opt}^*$ [or minimum variance of $\bar{Y}_{kk}^*$] as:

$$\hat{V}_{opt}[\bar{Y}_{kk}^*] = \sum_{h=1}^{H} w_h^2 \gamma_h \bar{Y}_h^2 C_{hy}^2 + \sum_{h=1}^{H} w_h^2 \bar{Y}_h^2 C_{hy}^2 \,(1-\rho_{hxs}^2)^{-2} \gamma_h^* \times$$
$$\Big[\left(\rho_{hyx} - \rho_{hys}\rho_{hxs}\right)^2 + \left(\rho_{hsy} - \rho_{hxs}\rho_{hxy}\right)^2 - 2(1-\rho_{xs}^2) \times$$
$$\left[\rho_{hxy}\left(\rho_{hxy} - \rho_{hxs}\rho_{hsy}\right) + \rho_{hsy}\left(\rho_{hsy} - \rho_{hxs}\rho_{hxy}\right)\right] +$$
$$2\rho_{hxs}\left(\rho_{hxy} - \rho_{hxs}\rho_{hsy}\right)\left(\rho_{hsy} - \rho_{hxs}\rho_{hxy}\right)\Big] \tag{16}$$

4. The Suggested estimator

The objective of this study is to introduce the concept of inverse exponentiation in formulating calibration constraints. Therefore, motivated by Koyuncu and Kadillar (2014), a new calibration estimator of population mean in stratified double sampling is suggested as:

$$\bar{Y}_{new}^* = \sum_{h=1}^{H} \varphi_h log\bar{y}_h \tag{17}$$

where φ_h are calibration weights, using the chi-square loss functions

$$L(\varphi_h, w_h) = \sum_{h=1}^{H} \frac{(\varphi_h - w_h)^2}{w_h q_h} \tag{18}$$

and subject to the following calibration constraints

$$\sum_{h=1}^{H} \varphi_h log\bar{x}_h = \sum_{h=1}^{H} w_h \, log\bar{x}_h^{'} \tag{19}$$
$$\sum_{h=1}^{H} \varphi_h \, logs_{hx}^2 = \sum_{h=1}^{H} w_h \, logs_{hx}^{'2} \tag{20}$$
$$\sum_{h=1}^{H} \varphi_h = \sum_{h=1}^{H} w_h \tag{21}$$

The Lagrange function is given by

$$\Delta = \sum_{h=1}^{H} \frac{(\varphi_h - w_h)^2}{w_h q_h} - 2\lambda_{11}^* \left(\sum_{h=1}^{H} \varphi_h log\bar{x}_{hx} - \sum_{h=1}^{H} w_h log\, \bar{x}_{hx}' \right)$$
$$-2\lambda_{22}^* \left(\sum_{h=1}^{H} \varphi_h log\, s_{hx}^2 - \sum_{h=1}^{H} w_h \, logs_{hx}^{'2} \right) - 2\lambda_{33}^* \left(\sum_{h=1}^{H} \varphi_h - \sum_{h=1}^{H} w_h \right) \tag{22}$$

Minimizing the chi-square loss functions (18) subject to the calibration constraints [(19), (20), (21)] gives the calibration weights for stratified double sampling as follows:

$$\varphi_h = w_h + w_h q_h (\lambda_{11}^* \bar{x}_{hx} + \lambda_{22}^* \, s_{hx}^2 + \lambda_{33}^*) \tag{23}$$

Substituting (23) into [(19), (20), (21)] respectively gives the following system of equations:

$$\begin{bmatrix} \omega_{16} & \omega_{15} & \omega_{14} \\ \omega_{15} & \omega_{12} & \omega_{13} \\ \omega_{14} & \omega_{13} & \omega_{11} \end{bmatrix} \begin{bmatrix} \lambda_{11}^* \\ \lambda_{22}^* \\ \lambda_{33}^* \end{bmatrix} = \begin{bmatrix} \mathrm{M}_{11} \\ \mathrm{M}_{22} \\ \mathrm{M}_{33} \end{bmatrix} \tag{24}$$

Solving the system of equations in (24) for $\lambda_{ii}^* s$ gives

$$\lambda_{11}^* = \frac{\mathrm{M}_{11}(\omega_{11}\omega_{12} - \omega_{13}^2) + \mathrm{M}_{22}(\omega_{14}\omega_{15} - \omega_{11}\omega_{16})}{(\omega_{11}\omega_{12}\omega_{16} - \omega_{12}\omega_{14}^2 - \omega_{11}\omega_{15}^2 - \omega_{13}^2\omega_{16} + 2\omega_{13}\omega_{14}\omega_{15})}$$
$$\lambda_{22}^* = \frac{\mathrm{M}_{22}(\omega_{11}\omega_{16} - \omega_{14}^2) - \mathrm{M}_{11}(\omega_{11}\omega_{15} - \omega_{13}\omega_{14})}{(\omega_{11}\omega_{12}\omega_{16} - \omega_{12}\omega_{14}^2 - \omega_{11}\omega_{15}^2 - \omega_{13}^2\omega_{16} + 2\omega_{13}\omega_{14}\omega_{15})}$$
$$\lambda_{33}^* = \frac{\mathrm{M}_{11}(\omega_{13}\omega_{15} - \omega_{12}\omega_{14}) + \mathrm{M}_{22}(\omega_{14}\omega_{15} - \omega_{13}\omega_{16})}{(\omega_{11}\omega_{12}\omega_{16} - \omega_{12}\omega_{14}^2 - \omega_{11}\omega_{15}^2 - \omega_{13}^2\omega_{16} + 2\omega_{13}\omega_{14}\omega_{15})}$$

Where $\omega_{11} = \sum_{h=1}^{H} w_h q_h$ $\omega_{12} = \sum_{h=1}^{H} w_h q_h \, (logs_{hx}^2)^2$ $\omega_{13} = \sum_{h=1}^{H} w_h q_h logs_{hx}^2$ $\omega_{14} = \sum_{h=1}^{H} w_h q_h log\bar{x}_h$, $\omega_{15} = \sum_{h=1}^{H} w_h q_h (log\bar{x}_h)(logs_h^2)$
$\omega_{16} = \sum_{h=1}^{H} w_h q_h \, (log\bar{x}_h)^2$ $\omega_{16} = \sum_{h=1}^{H} w_h q_h \, (log\bar{x}_h)^2$
$\mathrm{M}_{11} = \sum_{h=1}^{H} w_h (log\bar{x}_h' - log\bar{x}_h)$ $\mathrm{M}_{22} = \sum_{h=1}^{H} w_h \left(logs_{hx}^{'2} - logs_{hx}^2\right)$, $\mathrm{M}_{33} = 0$

Substituting the $\lambda_{ii}^* s$ in (23) and the resulting equation in (17) while setting $q_h = 1$, gives the proposed logarithmic calibration regression estimator for population mean in stratified double sampling as follows:

$$\bar{Y}_{new}^* = \sum_{h=1}^{H} w_h (log\bar{y}_h) + B_{h,11}^* \sum_{h=1}^{H} w_h (log\bar{x}_h' - log\bar{x}_h) +$$
$$B_{h,22}^* \sum_{h=1}^{H} w_h \left(logs_{hx}^{'2} - logs_{hx}^2\right) \tag{25}$$

where $B_{h,11}^*$ and $B_{h,22}^*$ are the coefficients of regression and are given by:

$$B_{h,11}^* = \frac{\mathrm{A}_{44}(\alpha_{11}\alpha_{12} - \alpha_{13}^2) - \mathrm{A}_{55}(\alpha_{11}\alpha_{15} - \alpha_{13}\alpha_{14}) + \mathrm{A}_{66}(\alpha_{13}\alpha_{15} - \alpha_{12}\alpha_{14})}{(\alpha_{11}\alpha_{12}\alpha_{16} - \alpha_{12}\alpha_{14}^2 - \alpha_{11}\alpha_{15}^2 - \alpha_{13}^2\alpha_{16} + 2\alpha_{13}\alpha_{14}\alpha_{15})}$$
$$B_{h,22}^* = \frac{\mathrm{A}_{44}(\alpha_{13}\alpha_{14} - \alpha_{11}\alpha_{15}) - \mathrm{A}_{55}(\alpha_{11}\alpha_{16} - \alpha_{14}^2) + \mathrm{A}_{66}(\alpha_{14}\alpha_{15} - \alpha_{13}\omega_{16})}{(\alpha_{11}\alpha_{12}\alpha_{16} - \alpha_{12}\alpha_{14}^2 - \alpha_{11}\alpha_{15}^2 - \alpha_{13}^2\alpha_{16} + 2\alpha_{13}\alpha_{14}\alpha_{15})}$$

Where $\alpha_{11} = \sum_{h=1}^{H} w_h$ $\alpha_{12} = \sum_{h=1}^{H} w_h \, (logs_{hx}^2)^2$ $\alpha_{13} = \sum_{h=1}^{H} w_h logs_{hx}^2$
$\alpha_{14} = \sum_{h=1}^{H} w_h log\bar{x}_h$, $\alpha_{15} = \sum_{h=1}^{H} w_h (log\bar{x}_h)(logs_h^2)$ $\alpha_{16} = \sum_{h=1}^{H} w_h \, (log\bar{x}_h)^2$
$\mathrm{A}_{44} = \sum_{h=1}^{H} w_h (log\bar{x}_h)(log\bar{y}_h)$. $\mathrm{A}_{55} = \sum_{h=1}^{H} w_h (logs_h^2)(log\bar{y}_h)$, $\mathrm{A}_{66} = \sum_{h=1}^{H} w_h log\bar{y}_h$

4.1 Theoretical Variance Estimation

This section derives the estimator of variance for the proposed logarithmic calibration estimator using the large sample approximation (*LASAP*) method.

$$\bar{Y}^*_{new} = \sum_{h=1}^{H} w_h log\bar{y}_h + B^*_{h,11}\sum_{h=1}^{H} w_h(log\bar{x}'_h - log\bar{x}_h) + B^*_{h,22}\sum_{h=1}^{H} w_h\left(logs'^2_{hx} - logs^2_{hx}\right)$$

$$\bar{Y}^*_{new} = \sum_{h=1}^{H} w_h log\bar{y}_h + B^*_{h,11}\sum_{h=1}^{H} w_h\left(log\frac{\bar{x}'_h}{\bar{x}_h}\right) + B^*_{h,22}\sum_{h=1}^{H} w_h\left(log\frac{s'^2_{hx}}{s^2_{hx}}\right)$$

$$\bar{Y}^*_{new} = \sum_{h=1}^{H} w_h log\bar{Y}_h\left(1+e_{hy}\right) + B^*_{h,11}\sum_{h=1}^{H} w_h log\left(\frac{1+e'_{hx}}{1+e_{hx}}\right)$$

$$+ B^*_{h,22}\sum_{h=1}^{H} w_h\left(\frac{1+e'_{hs}}{1+e_{hs}}\right)$$

$$\bar{Y}^*_{new} = \sum_{h=1}^{H} w_h log\bar{Y}_h + \sum_{h=1}^{H} w_h log\left(1+e_{hy}\right) + B^*_{h,11}\sum_{h=1}^{H} w_h log\left(1+e'_{hx}\right)(1+e_{hx})^{-1}$$

$$+B^*_{h,22}\sum_{h=1}^{H} w_h log\left(1+e'_{hs}\right)(1+e_{hs})'$$

$$\bar{Y}^*_{new} = \sum_{h=1}^{H} w_h log\bar{Y}_h + \sum_{h=1}^{H} w_h log\left(1+e_{hy}\right) + B^*_{h,11}\sum_{h=1}^{H} w_h log\left(1+e'_{hx}\right)$$

$$-B^*_{h,11}\sum_{h=1}^{H} w_h log(1+e_{hx}) + B^*_{h,22}\sum_{h=1}^{H} w_h log\left(1+e'_{hs}\right) - B^*_{h,22}\sum_{h=1}^{H} w_h log(1+e_{hs})$$

$$(\bar{Y}^*_{new} - \bar{Y}) = \left[\sum_{h=1}^{H} w_h\left(e_{hy} - \frac{e^2_{hy}}{2!} + \frac{e^3_{hy}}{3!} - \cdots\right) + B^*_{h,11}\sum_{h=1}^{H} w_h\left(e'_{hx} - \frac{e'^2_{hy}}{2!} + \frac{e'^3_{hy}}{3!} - \cdots\right)\right.$$

$$-B^*_{h,11}\sum_{h=1}^{H} w_h\left(e_{hx} - \frac{e^2_{hx}}{2!} + \frac{e^3_{hx}}{3!} - \cdots\right) + B^*_{h,22}\sum_{h=1}^{H} w_h\left(e'_{hs} - \frac{e'^2_{hs}}{2!} + \frac{e'^3_{hs}}{3!} - \cdots\right)$$

$$\left.-B^*_{h,22}\sum_{h=1}^{H} w_h\left(e'_{hx} - \frac{e^2_{hs}}{2!} + \frac{e^3_{hs}}{3!} - \cdots\right)\right] \quad (26)$$

Squaring both sides of (26) and retaining terms to the first degree of approximation gives:

$$[\bar{Y}^*_{new} - \bar{Y}]^2 = \sum_{h=1}^{H} w_h^2\left[e^2_{hy} + \mathrm{B}^{*2}_{h,11}\left(e'_{hx} - e_{hx}\right)^2 + \mathrm{B}^{*2}_{h,22}\left(e'_{hs} - e_{hs}\right)^2\right.$$

$$+2B^*_{h,11}e_{hy}\left(e'_{hx} - e_{hx}\right) + 2B^*_{h,22}e_{hy}\left(e'_{hs} - e_{hs}\right)$$

$$\left.+2\mathrm{B}_{h,11}\mathrm{B}_{h,22}B^*_{h,11}B^*_{h,11}\left(e'_{hx} - e_{hx}\right)\left(e'_{hs} - e_{hs}\right)\right] \quad (27)$$

Taking expectation of both sides of (27) gives

$$\hat{V}[\bar{Y}^*_{new}] = \sum_{h=1}^{H} w_h^2\left[\gamma_h C^2_{hy} + \left(\gamma_h - \gamma'_h\right)[\mathrm{B}^{*2}_{h,11}C^2_{hx} + \mathrm{B}^{*2}_{h,22)}C^2_{hs}\right.$$

$$\left.-2B^*_{h,11}\rho_{hyx}C_{hy}C_{hx} - 2B^*_{h,22}\rho_{hys}C_{hy}C_{hs} + 2B^*_{h,11}B^*_{h,22}\rho_{hxs}C_{hx}C_{hs}]\right] \quad (28)$$

4.2 Optimality conditions

This section deduced the optimality conditions that would guarantee optimum performance of the proposed logarithmic calibration estimator on satisfaction.

Setting $\frac{\partial\hat{V}[\bar{Y}^*_{new}]}{\partial\mathrm{B}_{h,11}} = 0$ and $\frac{\partial\hat{V}[\bar{Y}^*_{new}]}{\partial B^*_{h,22}} = 0$ in (28) respectively gives:

$$B^*_{h,11} = \frac{\rho_{hyx}C_{hy} - B^*_{h,22}\rho_{hxs}C_{hs}}{C_{hx}} \quad (29)$$

$$B^*_{h,22} = \frac{\rho_{hyx}C_{hy} - B^*_{h,11}\rho_{hxs}C_{hx}}{C_{hs}} \quad (30)$$

Substituting (29) in (30) or vice verse, gives the optimum values of $B^*_{h,11}(opt)$ and $B^*_{h,22}(opt)$ respectively as:

$$B^*_{h,11}(opt) = \frac{C_{hy}\left(\rho_{xy} - \rho_{xs}\rho_{sy}\right)}{C_{hx}(1-\rho^2_{hxs})} \tag{31}$$

$$B^*_{h,22}(opt) = \frac{C_{hy}\left(\rho_{sy} - \rho_{xs}\rho_{xy}\right)}{C_{hs}(1-\rho^2_{hxs})} \tag{32}$$

Substituting the value of $B^*_{h,11}(opt)$ in (31) and $B^*_{h,22}(opt)$ in (32) for $B^*_{h,11}$ and $B^*_{h,22}$ in (25), gives the proposed logarithmic calibration asymptotically optimum estimator ($CAOE$) for population mean in stratified double sampling as:

$$\bar{Y}^*_{new,opt} = \sum_{h=1}^{H} w_h log\bar{y}_h + \frac{C_{hy}(\rho_{xy}-\rho_{xs}\rho_{sy})}{C_{hx}(1-\rho^2_{hxs})}\sum_{h=1}^{H} w_h (log\bar{x}'_h - log\bar{x}_h)$$
$$+ \frac{C_{hy}(\rho_{sy}-\rho_{xs}\rho_{xy})}{C_{hs}(1-\rho^2_{hxs})}\sum_{h=1}^{H} w_h\left(logs'^2_{hx} - logs^2_{hx}\right) \tag{33}$$

Similarly, substituting the value of $B^*_{h,11}(opt)$ in (31) and $B^*_{h,22}(opt)$ in (32) for $B^*_{h,22}$ and $B^*_{h,11}$ in (28), gives the variance of the proposed logarithmic calibration asymptotically optimum estimator $(CAOE)$ $\bar{Y}^*_{new,opt}$ [or minimum variance of $\bar{Y}^*_{new}$] as:

$$\hat{V}_{opt}[\bar{Y}^*_{new}] = \sum_{h=1}^{H} w_h^2 C^2_{hy}(1-\rho^2_{xs})^{-2}\{\gamma_h(1-\rho^2_{xs})^2 + \gamma^*_h\left[\left(\rho_{hxy} - \rho_{hxs}\rho_{hsy}\right)^2\right.$$
$$+\left(\rho_{hsy} - \rho_{hxs}\rho_{hxy}\right)^2 - (1-\rho^2_{xs})[2\rho_{hxy}\left(\rho_{hxy} - \rho_{hxs}\rho_{hsy}\right)$$
$$+2\rho_{hsy}\left(\rho_{hsy} - \rho_{hxs}\rho_{hxy}\right)] + 2\rho_{hxs}\left(\rho_{hxy} - \rho_{hxs}\rho_{hsy}\right)\left(\rho_{hsy} - \rho_{hxs}\rho_{hxy}\right)\} \tag{34}$$

5. Empirical study

The relative performances of the proposed logarithmic calibration estimator over members of its class in stratified double sampling was determined using the data set in Table 1 adapted from Clement (2018). Two measuring criteria; variance and percent relative efficiency (PRE) were used to compare the performance of each estimator.

The percent relative efficiency (PRE) of an estimator ϕ with respect to the conventional regression estimator in stratified double sampling $\bar{Y}^*_{lr}$ is defined by:

$$PRE[\phi, \bar{Y}^*_{lr}] = \frac{V(\bar{Y}^*_{lr})}{V(\phi)} \times 100$$

The variance of the conventional regression estimator of population mean for double sampling for stratification defined by Cochran (1977) is given by:

$$V(\bar{Y}^*_{lr}) = \sum_{h=1}^{H}\left\{\frac{S^2_{hy}(1-\rho^2_{hxy})}{n_h} + \frac{\rho^2_{hxy}S^2_{hy}}{n'_h} - \frac{S^2_{hy}}{N_h}\right\} = 4137.2834$$

$$V_{Opt}(\bar{Y}^*_{kk}) = 3530.17655$$

$$V_{Opt}(\bar{Y}^*_{new}) = 2642.2146$$

The percent relative efficiency of the conventional regression estimator in stratified double sampling $\bar{Y}^*_{lr}$, Koyuncu and Kadilar (2014) calibration regression estimator in stratified double sampling $(\bar{Y}^*_{kk})$ and the proposed logarithmic calibration regression estimator in stratified double sampling $(\bar{Y}^*_{new})$ with respect to $(\bar{Y}^*_{lr})$ were calculated and presented in Table 2.

Table 1: Data Statistics [Clement (2018)]

Parameter	*Stratum* 1	*Stratum* 2	*Stratum* 3
N_h	52	76	82
n'_h	15	20	28
n_h	4	5	7
$\bar{X}_h$	6.813	10.12	7.967
$\bar{Y}_h$	417.33	503.375	340.00
S^2_{hx}	15.9712	132.66	38.438
S^2_{hy}	74775.467	259113.70	65885.6
S^2_{hxy}	1007.6547	5709.1629	1404.71
γ'_h	0.0474	0.0368	0.0235
γ_h	0.2308	0.1868	0.1307
ρ_{hyx}	0.703	0.738	0.805
ρ_{hys}	0.802	0.761	0.826
$\rho_{hy\beta}$	0.86	0.764	0.726
ρ_{hxs}	0.714	0.812	0.742
$\rho_{hx\beta}$	0.82	0.803	0.782
$\rho_{hs\beta}$	0.836	0.846	0.812

Table 2: Performance of estimators from empirical study

Estimator	*Variance*	*PRE* $(\phi, \bar{Y}^*_{lr})$
$\bar{Y}^*_{lr}$	4137.2834	100
$\bar{Y}^*_{kk}$	3530.1765	117.1976
$\bar{Y}^*_{new}$	2642.2146	158.4763

6. Simulation Study

This section compares the performance of the Koyuncu and Kadilar (2014) calibration estimator $(\bar{Y}^*_{kk})$ and the proposed logarithmic calibration estimator $(\bar{Y}^*_{new})$ with a global estimator [The Generalized Regression (GREG) estimator $(\bar{Y}^*_{GREG})$]

For a given estimator (say) $\hat{\hat{Y}}^*_i$, let $\hat{\hat{Y}}_i^{*(m)}$ be the estimate of $\hat{\hat{Y}}^*_i$ in the m-th simulation run; m =1, 2… M (=4,500). Four performance criteria namely; Relative Root Mean Square Error (RRMSE), Percent Relative Efficiency (PRE), Average Length of Confidence Interval (AL) and Coverage Probability (CP) were used to compare the performance of the Koyuncu and Kadilar (2014) calibration estimator $(\bar{Y}^*_{kk})$ and the proposed logarithmic calibration estimator $(\bar{Y}^*_{new})$ with the GREG-estimator $(\bar{Y}^*_{GREG})$. Each measuring criterion is calculated using the following mathematical expressions:

(i) $$RRMSE\left(\hat{\hat{Y}}^*_i\right) = \sqrt{\frac{1}{M}\sum_{i=1}^{M}\left(\frac{\hat{\hat{Y}}_i^{*(m)} - \overline{\hat{\hat{Y}}^*_i}}{\hat{\hat{Y}}^*_i}\right)^2}$$

where $\overline{\hat{\hat{Y}}^*_i} = \frac{1}{M}\sum_{m=1}^{M}\hat{\hat{Y}}_i^{*(m)}$ and $\hat{Y}_i^{*(m)}$ is the estimated total based on sample m and M is the total number of samples drawn for the simulation.

(ii) The percent relative efficiency (PRE) of an estimator $\hat{\hat{Y}}^*_i$ with respect to the Generalized Regression (GREG) estimator $(\bar{Y}^*_{GREG})$ is defined by:

$$PRE\ [\bar{Y}^*_i, \bar{Y}^*_{GREG}] = \frac{RRMSE(\bar{Y}^*_{GREG})}{RRMSE(\hat{\bar{Y}}^*_i)} \times 100$$

(iii) $CP\left(\hat{\bar{Y}}_i^*\right) = \frac{1}{M}\sum_{m=1}^{M}\left(\hat{\bar{Y}}_L^{*(m)} < \hat{\bar{Y}}_i^{*(m)} < \hat{\bar{Y}}_U^{*(m)}\right)$

where $\hat{\bar{Y}}_L^{*(m)}$ is the lower confidence limit and $\hat{\bar{Y}}_U^{*(m)}$ is the upper confidence limit. For each estimator of $\hat{\bar{Y}}_i^*$, a 95% Confidence Interval $\left(\hat{\bar{Y}}_L^{*(m)}, \hat{\bar{Y}}_U^{*(m)}\right)$ is constructed, where,

$$\hat{\bar{Y}}_L^{*(m)} = \hat{\bar{Y}}_i^{*(m)} - 1.96\sqrt{V\left(\hat{\bar{Y}}_i^{*(m)}\right)}, \quad \hat{\bar{Y}}_U^{*(m)} = \hat{\bar{Y}}_i^{*(m)} + 1.96\sqrt{V\left(\hat{\bar{Y}}_i^{*(m)}\right)}$$

and $V\left(\hat{\bar{Y}}_i^{*(m)}\right) = \frac{1}{M-1}\sum_{m=1}^{M}\left(\hat{\bar{Y}}_i^{*(m)} - \overline{\hat{\bar{Y}}_i^*}\right)^2$.

(iv) $ALCI\left(\hat{\bar{Y}}_i^*\right) = \frac{1}{M}\sum_{m=1}^{M}\left(\hat{\bar{Y}}_U^{*(m)} - \hat{\bar{Y}}_L^{*(m)}\right)$.

Table 3: Performance of estimators from simulation study

Estimators	RRMSE	$PRE\ [\bar{Y}_i^*, \bar{Y}_{GREG}^*]$	ALCI	CP
$\bar{Y}_{GREG}^*$	182.7423	100.0000	1668.72	0.7324
$\bar{Y}_{kk}^*$	168.6332	108.3667	1464.68	0.6446
$\bar{Y}_{new}^*$	123.4642	148.0124	1034.42	0.5278

7. Results and Discussion

Numerical results for the percent relative efficiency (*PREs*) in Table 2 reveals that the proposed logarithmic calibration estimator ($\bar{Y}_{new}^*$) has 58 percent gains in efficiency while the Koyuncu and Kadilar (2014) calibration estimator ($\bar{Y}_{kk}^*$) has 17 percent gains in efficiency; this shows that the proposed logarithmic calibration estimator ($\bar{Y}_{new}^*$) is 41 percent more efficient than the Koyuncu and Kadilar (2014) calibration estimator($\bar{Y}_{kk}^*$). This means that in using the proposed logarithmic calibration estimator ($\bar{Y}_{new}^*$) one will have 41 percent efficiency gain over the Koyuncu and Kadilar (2014) calibration estimator ($\bar{Y}_{kk}^*$).

Similarly, the simulation study for the comparison of performance of estimators reveals that the proposed logarithmic calibration estimator ($\bar{Y}_{new}^*$) has 48 percent gains in efficiency while the Koyuncu and Kadilar (2014) calibration estimator ($\bar{Y}_{kk}^*$) has 8 percent gains in efficiency; this shows that the proposed logarithmic calibration estimator ($\bar{Y}_{new}^*$) is 40 percent more efficient than the Koyuncu and Kadilar (2014) calibration estimator ($\bar{Y}_{kk}^*$) with respect to the Generalized Regression (GREG) estimator ($\bar{Y}_{GREG}^*$) as shown in the percent relative efficiency (*PREs*) in Table 3 This means that in using the proposed logarithmic calibration estimator ($\bar{Y}_{new}^*$), one will have 40 percent efficiency gains over the Koyuncu and Kadilar (2014) calibration estimator ($\bar{Y}_{kk}^*$). The simulation study also showed that the Average Length of Confidence Interval (ALCI) and Coverage Probability (CP) for the proposed logarithmic calibration estimator are significantly smaller than that of Koyuncu and Kadilar (2014) calibration estimator and GREG-estimator. These results prove the robustness of the proposed logarithmic calibration estimator and by extension inverse exponentiation.

8. Conclusion

This study introduces the concept of inverse exponentiation to formulate new calibration weights in stratified double sampling and proposes a more improved calibration estimator based on Koyuncu and Kadilar (2014) calibration estimator. The variance of the proposed calibration estimator has been derived under large sample approximation. Calibration asymptotic optimum estimator $(CAOE)$ and its approximate variance estimator are derived for the proposed calibration estimator and existing calibration estimators in stratified double sampling. Results of empirical and simulation studies conducted showed that the proposed logarithmic calibration estimator $(\bar{Y}^*_{new})$ is more efficient than both the Koyuncu and Kadilar (2014) calibration estimator $(\bar{Y}^*_{kk})$ and the Generalized Regression (GREG) estimator $(\bar{Y}^*_{GREG})$.

It is observed that the proposed logarithmic calibration estimator $(\bar{Y}^*_{new})$ is very attractive and should be preferred in practice as it provides consistent and more precise parameter estimates than existing calibration estimators in stratified double sampling.

9. Conflicts of Interest

There is no conflict of interest associated with the publication.

References

[1] Arnab, R. and Singh, S. (2005): A note on variance estimation for the generalized regression predictor. Australia and New Zealand Journal of Statistics 47 (2): 231-234.

[2] Clement, E. P., Udofia, G. A and Enang, E. I. (2014): Sample design for domain alibration estimators. International Journal of Probability and Statistics, 3(1), 8-14.

[3] Clement, E. P. and Enang, E. I. (2015): Multivariate calibration estimation for domain in stratified random sampling. International Journal of Modern Mathematical Sciences, 13 (2), 187-197.

[4] Clement, E. P. (2016): An Improved Ratio Estimator for Population Mean in Stratified Random Sampling. European Journal of Statistics and Probability4 (4): 12-17

[5] Clement, E. P. (2017): Efficient Exponential Estimators of Population Mean in Survey Sampling. International Journal of Mathematics and Computation28 (3): 94-106.

[6] Clement, E. P. and Enang, E. I. (2017): On the Efficiency of Ratio Estimator over the Regression Estimator, Communication in Statistics: Theory and Methods, 46 (11): 5357-5367.

[7] Clement, E. P. (2018): A Note on Calibration Weightings in Stratified Double Sampling with Equal Probability, Communication in Statistics: Theory and Methods, Vol. 47 No.12 pp: 2835-2847.

[8] Clement, E. P. (2020): On the Improvement of Multivariate Ratio Method of Estimation in Sample Surveys by Calibration Weightings. Asian Journal of Probability and Statistics, 10 (1) 1-12. DOI: 10.9734/AJP AS/2020/v10i130236.

[9] Clement, E. P. and Inyang, E. J. (2020): A Class of Modified Calibration Ratio Estimators of Population Mean with Known Coefficient of Kurtosis in Stratified Double Sampling Journal of modern Mathematics and Statistics 14 (4):55-60.

[10] Clement, E. P. (2021): Ratio-Product Estimator in Stratified Double Sampling based on the coefficient of skewness of the Auxiliary Variable. International Journal of Statistics and Applied Mathematics, 6 (1): 24-28.

[11] Clement, E. P. and Inyang, E. J. (2021): Improving the Efficiency of Ratio Estimators by Calibration Weightings. International Journal of Statistics and Mathematics; 8(1): 164 – 172.

[12] Clement, E. P., Ekpoese, E. D. and Inyang, E. J. (2021): Improving the Efficiency of Estimators of Population Mean in Systematic Sampling Using Inverse Exponentiation International Journal of Statistics and Applied Mathematics; 6 (3): 87- 97.

[13] Clement, E. P. (2022): On the Precision of Estimators in Sampling Surveys by Multi-Parametric Calibration weightings, Thailand Statistician 2022 (3) 655-668.

[14] Clement, E. P. and Etukudoh, I. O. (2023): Calibration Ratio Estimator for Estimating Population Mean in Stratified Random Sampling Design Using Properties of Two Supplementary Variables, International Journal of Scientific Research in Mathematical and Statistical Sciences 10 (5),14 -21

[15] Cochran, W. G. (1977): Sampling techniques. New York: Wiley and Sons.

[16] Deville, J. C. and Sarndal, C. E. (1992): Calibration estimators in survey sampling Journal of the American Statistical Association, 87, 376-382.

[17] Diana, G., Giordan, M. and Perri P. F. (2011): An improved class of estimators for the population mean, Statistical methods and Applications. 20:123-140,

[18] Enang, E. I. and Clement, E. P. (2020): An efficient class of calibration Ratio Estimators of Domain Mean in Survey Sampling, Communication in Mathematics and Statistics Vol. 8: pp 279-293 DOI: 10.1007/s40304-018-00174-z

[19] Gupta, S. and Shabbir, J. (2008): On the improvement in estimating the population mean in simple random sampling, Journal of Applied Statistics 35 (5): 559-566,

[20] Inyang, M. O. and Clement, E. P. (2023): On the Estimation of Current Population Median in Two-Occasion Rotation Sampling, International Journal of Scientific Research in Mathematical and Statistical Sciences 10 (5), 26 -33

[21] Kim, J. M., Sungur, E. A. and Heo, T. Y. (2007): Calibration approach estimators in stratified sampling. Statistics and Probability Letters, 77(1), 99-103.

[22] Kim, J. K. and Park, M. (2010): Calibration estimation in survey sampling. International Statistical Review, 78(1):21-29.

[23] Koyuncu, N., and Kadillar, C. (2014): A new calibration estimator in stratified double sampling. Hacettepe Journal of Mathematics and Statistics, 43 (2): 337-346.

[24] Koyuncu, N., and Kadilar, C. (2016): Calibration weighting in stratified random sampling. Communications in Statistics: Simulation and Computation 45(7): 2267-2275.

[25] Lone, H. A. and Tailor, R., (2015): Dual to separate product type exponential estimator in sample surveys, Journal of Statistics Applications & Probability Letters. 2(2)89–96.

[26] Rao, D., Khan, M. G. M. and Khan, S. (2012): Mathematical programming on multivariate calibration estimation in stratified sampling. World Academy of Science, Engineering and Technology, 72, 12-27.

[27] Sarndal, C-E (2007): The calibration approach in survey theory and practice. Survey Methodology, 33, 99-119.

[28] Sharma, B. and Tailor, R. (2010): A new ratio-cum-dual to ratio estimator of finite population mean in simple random sampling. Global Journal of Science 10 (1):27-31.

[29] Shittu, O. I. and Adepoju, K. A. (2014): On the efficiency of modified ratio estimator based on linear combination of kurtosis, median and quartile deviation. International Journal of Modern Mathematics Sciences.11 (2), 103-107.

[30] Singh, H. P. and Tailor, R. (2003): Use of known correlation coefficient in estimating the finite population mean. Statistics in Transition, 6(4), 555-560.

[31] Singh, R. V. K. and Audu, A. (2013): Efficiency of ratio estimators in stratified random sampling using information on auxiliary attribute. International Journal of Engineering and Innovative Technology, 2(1), 166-172.

[32] Tracy, D. S., Singh, S. and Arnab, R. (2003): Note on calibration in stratified and double sampling. Survey Methodology, 29: 99 – 104.

[33] Wu, C. and Sitter, R. R. (2001): A Model-calibration approach to using complete auxiliary information from survey data, Journal of the American Statistical Association, 96,185-193.