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Estimating threshold age between early deaths and late deaths for a developing country

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Abstract

The increase in life expectancy, from under 40 years in all areas of the world two centuries ago to over 80 years today in many developed countries, has fundamentally improved the human condition. Though life expectancy is a well known measure for longevity of a population, period life expectancy is affected by age-specific mortality intensely. Early aged mortality creates disparity in life expectancies, which is very common for many under developed and developing countries. Adequate link exists between changes in age-specific mortality and lifespan inequality, and key to this relationship is a young–old threshold age, below and above which mortality decline respectively decreases and increases lifespan inequality. Besides socio-economic importance, this threshold's location modifies the correlation between changes in life expectancy and lifespan inequality. Zhang-Vaupel's theorem of estimating threshold age is applied to estimate the threshold age for a developing country Bangladesh, using the data of Matlab Health and Demographic Surveillance System. The graphical estimate of threshold age was found to be 70 to 72 years for male and 72 to 75 years for female of Matlab HDSS-2010. These findings imply the imminent longevity for the population of Matlab HDSS.

Keywords: Threshold age, Longevity, Zhang-Vaupel's theorem, Matlab HDSS, Bangladesh.

Introduction

Period life expectancy at birth is defined as the average life span in year that a newborn may live given a set of mortality rates seen in a calendar year.[1] The first of these averages (denoted by *e(0)*), is known as life expectancy at birth, which is also a summary indicator of population structure of a country; along with public health related major concern. Life expectancy for different ages is computed from life table. As it is computed from the life table function, the mortality pattern affects the value of life expectancy at different ages. Whatever the differentials are, life expectancy by age became a monotonic decreasing function with increasing age for the developed countries since the second half of the twentieth century.[2] It is not historical, considerable gaps between the life expectancy at birth and older ages can be still found in many countries.[3] Significant effect of infant mortality is obtained in life expectancy at birth and age 1, death create subsequent decrease in life expectancy at birth.[2]

Dissimilarities have been observed for most of the under developed and developing countries. Besides the main continents, research is also done for historical data of the industrialized countries, as well as the minority groups and various special sub populations.[4] Changes in mortality in the earlier life significantly affect life expectancy at birth, and it has been recommended that the time series of *e(0)* alone is not well suited for studying the length of life in aging populations.[5] Furthermore, it turns out that the derivatives of some other measures with respect to the change in mortality have a similar form and all imply the existence of an age separating early and late deaths, which may be considered a threshold age before early age mortality to longevity.[6]

The necessity of estimating threshold age is important for a country, where age-specific mortality patterns are changing with a monotonic trend from a long period of time.[7] Lifespan inequality is the defin-

Practice Points

- Period life expectancy is affected by age-specific mortality intensely, although it is a widely used indicator of longevity.
- Young–old threshold age indicates the specific age which separate the decreases and increases of lifespan inequality.
- For most of the developing countries threshold age lies between 75 to 80 years.
- Current study revealed threshold ages for Matlab HDSS lie between 70 to 72 years for male and 72 to 75 years for female.
- The results suggest forthcoming population aging for Matlab HDSS.

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ing measure of social and health disparity alongside life expectancy at a particular age is a key indicator of population health.[8,9] Changes in life expectancy in particular age/age groups are well understood in terms of the underlying changes in mortality pattern: mortality reduction at any age increases life expectancy accordingly.[6,9,10] However, the precise link between lifespan inequality and age-specific mortality is less clear. Several ways to quantify lifespan inequality and understand its change through time are suggested in previous studies.[7,9,11] Although highly correlated, these measures behave differently in response to change in the age structure of mortality, and results from all of these studies suggest that, only mortality decline at ages below a young–old threshold can decrease lifespan inequality.[6,11] The impact of this threshold age is thus huge for developing countries, too. It can be used to have a clear profile of age-specific mortality pattern, aging parameter, public health policies, to define appropriate retirement age and so on. Trend analysis of threshold ages enables the population researcher to suggest future public health policies for demographic transition.

Earlier studies attempted to estimate threshold age from age-specific hazard of death, $\mu(x)$; under the assumption that $\mu(x)$ follows the Gompertz curve.[7] Numerous other measures of lifespan disparity in a life table have been proposed.[8] Variation of the age at death, the standard deviation, the standard deviation above age 10, and the entropy of the life table can be named for example.[9,10] These measures are highly correlated with each other, so not particularly appropriate for presenting threshold age for a developing country, where early mortality is present in a notable amount.[6] Also, most of the illustrations were made for industrialized countries and developed countries; developing countries have less focus due to lack of vital registration system.[2,11]

Fortunately, there is other option to estimate differentiating age between early and late mortality. Zhang-Vaupel's relationship between earlier and later death defines a separating age between early and late demise (in terms of their effect on disparity) and investigate the factors determining this threshold age. Zhang-Vaupel's theorem is preferred for estimating threshold age because of its desirable mathematical properties and it can be readily explained and interpreted.[6] In current study, Zhang-Vaupel's theorem is applied to estimate threshold age graphically for a developing country, Bangladesh.

Materials and methods

Like many other developing countries, Bangladesh does not have any vital registration system. For current study the vital registration and maternal and child health data gathered from Matlab Health and Demographic Surveillance System 2010, Bangladesh is utilized.[12] Since 1966, the Health and Demographic Surveillance System (HDSS) has maintained the registration of births, deaths, and migrations; in addition to carrying out periodical censuses in Matlab. Matlab HDSS is recognized worldwide as one of the long-term demographic surveillance sites for a developing country. Life tables for both sexes have been considered for the current study.[12] It should be noted that, current analysis is based on period life tables only; cohort life tables may produce different results than this. Statistical package *R* (version 2.15.2) is used for statistical analysis.

General mathematical relationships of abridged life tables are discussed in the following sections:

Number of survivors in a particular age x is $l(x)$ and number of deaths in a particular age interval $x+n$ is, $d(x,n) = l(x) - l(x,n)$. At age 0, the value of $l(x)$ is known as radix and it is considered as 1.0 in standard.

Probability of surviving in particular age x to $x+n$ is, $p(x,n)$ and death is $q(x,n)$, where $p(x,n) = 1 - q(x,n)$ and $q(x,n) = d(x,n)/l(x)$. Thus, in a time interval $x+n$, $l(x,n) = l(x)p(x,n)$ and $d(x,n) = l(x).q(x,n)$.

Number of person-year lived by the cohort is:

$$L(x,n) = \int_0^n l(x,t)dt$$

Which is equivalent to:.

$$L(x,n) = \frac{n\,l(x) + l(x,n)}{2}; (x \geq 2)$$

For $x<2$; $L(0) = 0.201\ l(0) + 0.8\ l(1)$ and $L(1) = 0.410\ l(1) + 0.590\ l(2)$.

Central death rate is defined as: $m(x,n) = d(x,n)/L(x,n)$. Also,

$$m(x) = \frac{2q(x)}{2 - q(x)}$$

Number of person-year lived by the life table population is:

$$T(x) = \int_0^\infty L(x,t)dt$$

Which is equivalent to:

$$T(x) = \sum_{t=0}^{\infty} L(x,t)$$

Expectancy of life at age x is: $e(x) = T(x) / l(x)$

Force of mortality is defined as age-specific hazard of death at x, which is a function of number of survivors (l_x) at age x; symbolically,

$$\mu(x) = \lim_{t\to 0} \frac{l(x,t) - l(x)}{t.l(x)} = -\frac{1}{l(x)}.\frac{d}{dx}l(x) = -\frac{d}{dx}\ln(l(x))$$

$\mu(x)$ can be estimated by considering a function of $l(x)$ and age x. For current study, a third degree polynomial is fitted to estimate the values of $\mu(x)$.[13] It appears from the scattered plot of the number of persons surviving at an exact age x for both sexes of Matlab HDSS-2010 by age groups that $l(x)$ may be distributed by polynomial model for different ages rather than a linear model. Hence n^{th} degree polynomial is fitted to the values of l

(x) instead of linear regression model. The structure of the model is:

$$y = a_0 + \sum_{i=1}^{n} a_i . x^i + u \quad \text{.......... (a)}$$

Where, x is the exact age, y is the number of survivors at age x, a_i is the coefficients of the model (for i=1,2,3; a_0 is the constant term of the fitted model) and u is random error.

Model validation technique for force of mortality

Cross validity prediction power (*CVPP*); ρ^2_{CV}; is applied here to test the stability of the fitted model. Symbolically,

$$\rho^2_{CV} = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}(1-R^2)$$

Where 'n' is the number of observations, 'k' is the number of predictors in the model, 'R' is the correlation between observed and predicted values of the dependent variable. The shrinkage of the model is the absolute difference of *CVPP* and R^2. Moreover, the stability of R^2 of the model is defined as difference between 1 and shrinkage.[13]

Again, from (a),

$$y = a_0 + a_1 x + a_2 x^2 + \ldots\ldots + u$$

$$y' = a_1 + 2a_2 x + 3a_3 x^2 + \ldots\ldots + na_n x^{n-1} \quad \text{........(b)}$$

Using (a) and (b) in definition of force of mortality, we have:

$$\mu(x) = -\frac{d}{dx}\ln(l(x)) = -\frac{1}{l(x)} \cdot \frac{d}{dx} l(x)$$

$$= -\frac{(a_1 + 2a_2 x + 3a_3 x^2 + \ldots\ldots + na_n x^{n-1})}{(a_0 + a_1 x + a_2 x^2 + \ldots\ldots + u)} \quad \text{................(c)}$$

Estimation of threshold age

For a particular age *a*, *l(a)* is the survivors, *e(a)* denotes life expectancy and *d(a)* denotes the distribution of death.

Then, by definition, *d(a)=l(a).μ(a)* and

$$e(a) = \int_a^\infty \frac{l(x)}{d(x)} dx$$

Let, cumulative hazard of death at age a is denoted by *H (a)*. Then:

$$H(a) = \int_0^a \mu(x) dx$$

With *H(0)=0* as:

$$l(a) = \exp(-\int_0^a \mu(x) dx$$

Let, life disparity by life expectancy lost due to death is:

$$e^\dagger = \int_0^\infty e(a) d(a) da$$

For age a:

$$e^\dagger(a) = \int_0^\infty \frac{e(x)d(x)}{l(a)} dx$$

is the life expectancy lost due to death among people surviving to age a.

Then Zhang-Vaupel's relationship between early and late mortality states that, a^* should be the threshold age if the following equation is satisfied,

$$e^\dagger(a) = e(a)(1-H(a)) \qquad (1)$$

That is, the age for which the intersect of the lines $e^\dagger(a)$ and *e(a)(1-H(a))* occurs, will be the threshold age.[6]

Results

The *l(x)* values from Matlab HDSS-2010 are used to calculate the parameters of the models for male and female respectively.[12] Estimation of force of mortality for male and female of Matlab HDSS-2010 are taken from previous study.[13] For both male and female, the third degree polynomial fits the data well. The findings of the models are presented in Table 1, model (i) and (ii) represent separate models for male and female respectively.

Both of the models are highly cross-validated and their shrinkages are only 0.001372 and 0.0041908 respectively for male and female. Furthermore, both of the

Table 1: Information on Model Fittings

Model	n	k	R^2	F-statistic	p-value	ρ^2_{CV}	Shrinkage	Parameter	t-statistic	*p*-value
								a_0	147.235	0.0000
								a_1	-5.225	0.0001
								a_2	7.825	0.0000
(i)	22	3	0.99704	2020.72	0.0000	0.995668	0.001372	a_3	-14.597	0.0000
								a_0	95.743	0.0000
								a_1	-6.621	0.0000
								a_2	9.422	0.0000
(ii)	22	3	0.99096	658.03	0.0000	0.986769	0.0041908	a_3	-13.767	0.0000

fitted models will be more than 99% stable. Besides, the parameters of the fitted models are highly significant; *CVPP* indicates 99% and 98% of variance are explained respectively for male and female. From t-statistics, it is found that all the parameters of the model are also highly significant. In both models, the stability of R^2 is more than 99%. The calculated values of the F-test of the models are 2020.72322 and 658.02939 respectively for males and females with degrees of freedom (3, 18), whereas the corresponding tabulated value is only 5.09 at a 1% level of significance. Therefore, the overall measure of the fitted models and its R^2 are highly significant. The fitted model for male and female are presented below correspondingly.

$$y = 97996.03-478.24x+21.66x^2-0.31x^3 \quad (2)$$
$$y = 99811.38-949.172x+40.85x^2-0.482x^3 \quad (3)$$

The *l(x),* e(x) and μ(x) values for corresponding age are presented in Table 2 for Matlab HDSS (both sexes). The values of μ(x) are estimated using equation (2) and (3) respectively for male and female.[13]

J-shaped life expectancy curves are observed in Matlab HDSS like other developing countries.[2] It can be also observed from the distribution of *l(x)* values for both sexes in Table 1 for Matlab HDSS. This nature of life expectancy create irregular trend in case of $e^\dagger(a)$. Table 3 presents the $e^\dagger(a)$, *H(a)* and *1-H(a)* for both sexes of Matlab HDSS-2010. It had been discussed already in methodology that, *H(0) = 0*.

Graphical illustrations of the estimates of threshold age a^* for both sexes in Matlab HDSS are presented in figure 1. For both sexes, intersect of the lines present the threshold age for death between earlier and later age. Irregular trend may be observed in the lines of life expectancy disparity for both sexes. Life disparity by life expectancy lost due to death has its highest peak at age zero, soon it declines to a lower value at age 1 and so on.

From figure 1, the threshold age lies somewhere between 70 to 72 years for male; while for female the threshold age lies somewhere between 72 to 75 years.

Discussion

For a developing country with presence of high infant mortality, life expectancy at birth does not provide clear depiction of the overall mortality pattern, or concise idea about longevity.[5,14,15] Also, the distribution of deaths contain deaths before the age of 30, which means the researcher is unable to distinguish between early and aged mortality for a population of a developing country.[16,17] The main objective of the current study was to determine threshold age for a developing country, Bangladesh. Zhang-Vaupel's theorem of estimating threshold age is applied to estimate the threshold age for a developing country Bangladesh, using the data of Matlab Health and Demographic Surveillance System.[6,12]

To estimate the threshold age for Matlab HDSS, first the force of mortality is estimated.[13] The pattern of force of

Table 2: *l(x), e(x)* and estimated μ(x) values for both sexes in Matlab (Matlab HDSS-2010)

Age *x*	**Male**			**Female**		
	l(x)	*e(x)*	$\mu(x)$	*l(x)*	*e(x)*	$\mu(x)$
0	1.0	69.3	0.004880276	1.0	73.2	0.0095095053
1	0.96710	70.6	0.004468924	0.97279	74.2	0.0087853056
2	0.96399	69.8	0.004071917	0.96755	73.6	0.0080703811
3	0.96141	69.0	0.003690025	0.96565	72.8	0.0073669233
4	0.95985	68.1	0.003323958	0.96447	71.9	0.0066770571
5	0.95873	67.2	0.00974374	0.96327	71.0	0.0060028264
10	0.95620	62.4	0.001492690	0.96067	66.2	0.0029304269
15	0.95379	57.5	0.000489962	0.95905	61.3	0.0004894886
20	0.94992	52.8	7.992256e-06	0.95549	56.5	7.902256e-06
25	0.94229	48.2	5.963364e-06	0.95245	51.7	5.863364e-06
30	0.93821	43.4	5.328055e-04	0.95035	46.8	4.928055e-04
35	0.92852	38.8	0.001580488	0.94633	41.9	0.0015689562
40	0.92213	34.0	0.003177760	0.93999	37.2	0.0003062654
45	0.90485	29.6	0.005390802	0.93535	32.4	0.0017637566
50	0.88521	25.2	0.008347626	0.92304	27.8	0.0046494459
55	0.85552	21.0	0.01228084	0.91493	23.0	0.0085341663
60	0.79580	17.4	0.01761312	0.88087	18.8	0.0137834645
65	0.69702	14.5	0.02514985	0.82756	14.8	0.0211153564
70	0.59041	11.6	0.03657859	0.73060	11.4	0.0320340924
75	0.45144	9.4	0.05605122	0.59341	8.5	0.0501635778
80	0.31637	7.3	0.09712505	0.38477	6.7	0.0867835349
85	0.17906	6.1	0.2436403	0.19988	5.7	0.2030147583

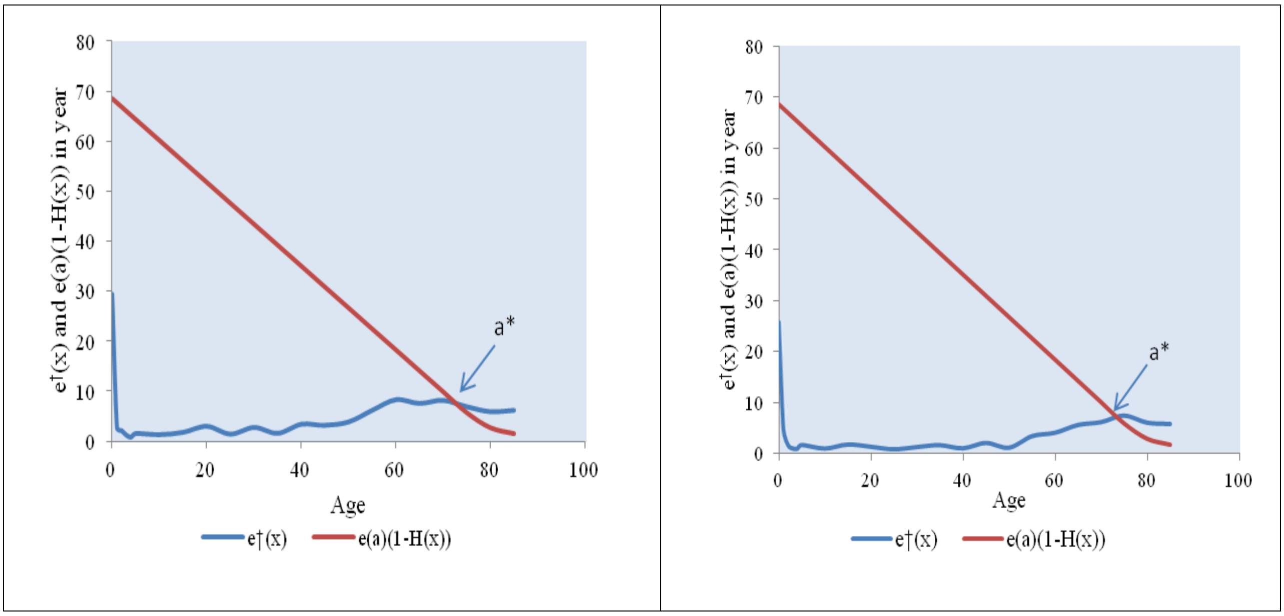
Table 3: Estimated age-specific cumulative hazards for both sexes in Matlab (Matlab HDSS 2010)

Age	Male			Female		
	H(a)	*1-H(a)*	$e^{\dagger}(a)$	*H(a)*	*1-H(a)*	$e^{\dagger}(a)$
0	0.004880	1.0	29.386281	0.0	1.0	25.778754
1	0.009349	0.990651	2.6494975	0.474056	0.5259441	4.7089382
2	0.013421	0.986579	2.0161143	0.455761	0.5442390	1.5709782
3	0.017111	0.982889	1.1090586	0.447691	0.5523093	0.8876425
4	0.020435	0.979565	0.7170287	0.440324	0.5596763	0.8132135
5	0.030179	0.969821	1.4418992	0.433647	0.5663533	1.5700894
10	0.031672	0.968328	1.2077724	0.427644	0.5723561	0.8612052
15	0.032161	0.967839	1.6911647	0.424713	0.5752866	1.6499870
20	0.032169	0.967831	2.8859893	0.424224	0.5757761	1.2191943
25	0.032175	0.967825	1.3271074	0.424216	0.5757840	0.7203213
30	0.032708	0.967292	2.6677684	0.424017	0.5759826	1.1632556
35	0.034289	0.965711	1.4789245	0.423717	0.5762826	1.5288409
40	0.037467	0.962533	3.2999772	0.422148	0.5778516	0.9196183
45	0.042857	0.957143	3.0843167	0.421842	0.5781578	1.9622826
50	0.051205	0.948795	3.7732572	0.420078	0.5799216	1.0253478
55	0.063486	0.936514	6.0940200	0.415429	0.5845710	3.3094707
60	0.081099	0.918901	8.2295979	0.406895	0.5931052	3.9882491
65	0.106249	0.893751	7.4793105	0.393111	0.6068887	5.5184107
70	0.142827	0.857173	8.0970309	0.371996	0.6280040	6.0652026
75	0.198879	0.801121	6.8217172	0.339962	0.6600381	7.3483359
80	0.296004	0.703996	5.8158296	0.289798	0.7102017	5.9584583
85	0.539644	0.460356	6.1025083	0.203015	0.7969852	5.7100458

mortality is decreasing up to 35 years in Matlab HDSS, but it has an increasing trend after 40 years; rapidly increasing pattern is observed after the age interval 55 and above.[13] Force of mortality is higher for female in earlier age compare to male. The crossover of force of mortality occurs approximately at age 15 for both sexes, and remained almost equal at the adolescent age. After the age of 20 and onwards, the force of mortality is lower in the case of females compare to males.[13] This pattern of force of mortality also has an impact on the threshold age.[6] Life expectancy at birth is lower than life expectancy at age 1 and 2 for the people of Matlab. Presence of infant mortality and early childhood mortality is responsible behind this pattern; many other developing countries also faced this situation before certain decline in infant mortality.[2] It can be also observed from the distribution of *l(x)* values in Table. This nature of life expectancy create irregular trend in case of $e^{\dagger}(a)$.[5,9]

Figure 1 presents the graphical illustrations of the

Figure 1: Graphical illustration of the estimate of the threshold age *a** for (a) Male (*left*), (b) Female (*right*) of Matlab (Matlab HDSS-2010). The upper (lower) curve of each figure is $e^{\dagger}(x)$ and *e(a)(1-H(x))*, respectively.



estimates of the threshold age a^* for both sexes in Matlab HDSS. For both sexes, intersect of the lines present the threshold age for death between earlier and later age.[6] The lines of life expectancy disparity follow an irregular trend for both sexes, though the second line follow almost linear trend. Life disparity by life expectancy lost due to death has its highest peak at age zero, soon it declines to a lower value at age 1 and so on. The reason behind such rapid decline in age zero and one is higher life expectancy at age one compared to birth. Previous studies suggest this pick as imbalance in life table due to the presence of a high infant mortality rate, which is also present in Matlab HDSS.[2,12] The trend of lost life expectancy due to death is lowest up to age 50 for both sexes. After rapid decline at age 1, almost similar pattern can be observed in early childhood. Adult mortality is minimal for Matlab HDSS, which is also reflected in pattern of lost life expectancy due to death.[9,14]

Obtained results suggest that, threshold age lies somewhere between 70 to 72 years for males; while for females the threshold age lies somewhere between 72 to 75 years. The obtained result is interesting, because the life expectancy at birth is near to these intervals for Matlab HDSS, and the highest observed life expectancy is found at age 1 for both sexes (for male 70.6 and female 74.2 years). For most of the developed countries the threshold age lies between 75 to 80 years, for US females, this age was close to 80years in 2005.[6,9,15] Since the last age of current life tables are 85 years and the existing age-specific mortality pattern along with life expectancies; the results clearly indicates about the imminent population aging of Matlab HDSS.

Conclusion

Almost all over the world, lifespan inequality has varied greatly within and among countries in the past six decades; even while life expectancy has continued to increase. Estimation of young–old threshold age; below and above which mortality decline respectively decreases and increases lifespan inequality; is necessary because threshold's location modifies the correlation between changes in life expectancy and lifespan inequality. For several developed countries like Canada, USA, Japan the threshold age centered on retirement age. Current study tried to fill the gap for developing countries, threshold age is estimated for a developing country Bangladesh. Using the data of Matlab Health and Demographic Surveillance System (Matlab HDSS-2010) threshold age was found to be 70 to 72 years for male and 72 to 75 years for female, which clearly indicates forthcoming aging of Matlab HDSS.

The impact of this threshold age is very precise. If mortality decline continues to stagnate at young ages, but to progress steadily at old ages, the implication is that lifespan inequality increases will become more common, that is, the primary profile of an upcoming aging population. From the results of the current study, the retirement age can be redefined by Bangladesh Government. Also, new public health policies are required along with existing polices for upcoming aging. Several future researches can be done on the basis of findings and limitations of the current study. This study is done only for period life tables; cohort life tables may produce different result than this. Also, a point estimate of threshold age was unavailable due to lack of any well-defined model of life expectancy and age-specific hazard of death. A trend analysis of the threshold age between early and aged mortality from previous study will help the researchers and planners for better policy implication about longevity in Matlab HDSS. Regional variation in threshold age can be verified in prospective studies for Bangladesh. These sorts of comparison can be helpful to define separate clusters or sub population with different age-specific mortality pattern in Bangladesh.

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