

INTELLIGENT ESTIMATION OF FAILURE PROCESSES IN CRITICAL SYSTEMS USING HYBRID ANN-ABC UNDER NON-HOMOGENEOUS POISSON MODELING

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SUMMARY

This study introduces an intelligent hybrid estimation framework for modeling the time-dependent failure behavior of repairable systems. The proposed approach embeds the Inverse Weibull Process (IWP) within a Non-Homogeneous Poisson Process (NHPP) structure and compares the performance of the traditional Maximum Likelihood Estimation (MLE) method with a hybrid algorithm that integrates Artificial Neural Networks and the Artificial Bee Colony optimization technique (ANN-ABC). Both simulation experiments and an application to real clinical data from 299 patients with heart failure demonstrate that the ANN-ABC estimator achieves lower Root Mean Squared Error (RMSE) and Bayesian Information Criterion (BIC) values, particularly for moderate to large datasets. These findings highlight the potential of hybrid intelligent methods as robust alternatives to conventional estimation techniques, offer improved precision in modeling failure intensities, and support predictive maintenance as well as intelligent healthcare monitoring systems.

Keywords and phrases: Inverse Weibull Process; Non-Homogeneous Poisson Process; Artificial Neural Network; Artificial Bee Colony; Reliability Modeling; Predictive Maintenance; Failure Data Analytics

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1 Introduction

Predictive maintenance optimization and close analysis of failure in repairable systems and smart monitoring, particularly in safety-bound fields like healthcare, power systems, and infrastructure involving the IoT-based system, require reliability modeling frameworks. The NHPP is a widely used modeling framework, as it allows one to model time-varying failure rates. In this regard, the IWP provides a flexible framework for modeling aged systems in which hazards decrease over time and capturing early-life failures (Lawson, 2001, Muralidharan, 2018).

Traditional methods, such as the MLE, may be inadequate in nonlinear and real-world contexts. To address these issues, a hybrid intelligent method that integrates the ANN-ABC optimization algorithm has been proposed by this study. This method aims to enhance the accuracy of estimates and computational efficiency, particularly in small-to-medium data settings. This development helps in establishing intelligent estimation frameworks in complex and time-varying failure modeling that offers promising potential in smart healthcare, anomaly detection, and secure Internet tools and systems (Lawson, 2001, Li et al., 2023, Hussein et al., 2024).

The current study aims to estimate parameters of IWP to reflect variation in failure rate over time (Mohsin et al., 2025, Hou et al., 2022). Two estimation schemes are used: MLE and ANN-ABC. They both are based on frequentist and probabilistic models. In reliability growth analysis, it is essential to understand whether there is an improvement trend or rather a trend that is getting worse. The inter-failure interval is increasing; an increasing inter-failure interval indicates improvement, whereas a decreasing interval indicates degradation. Because of the time-varying intensity function, the NHPP is well-suited to model these dynamics. Renewal processes, in contrast, assume that the random variables governing the times between failures are identically distributed and independent, but otherwise are very general: the case of exponentially distributed inter-arrival times gives a Homogeneous Poisson Process (Sulaiman et al., 2025, Brito et al., 2022, Hussain et al., 2025d).

The current work contributes to modeling reliability, as it compares empirical and machine learning estimators based on IWP to diversify the approach to predictive maintenance and failure prediction. Many fuzzy reliability estimation procedures have been studied along with their counterparts. This paper introduced one approach that brought the idea of frailty-based modeling of repairable systems with dependent failure times, assuming perfect repairs. The estimation of the parameters and a validation exercise of the real data from sugarcane harvesters and dump trucks were carried out to capture unobserved heterogeneity and failure dependence, which helps in making the maintenance decision.

The Bayes adjustments of the change-point in a Weibull-intensity NHPP provided MCMC and model-based choice through the Bayes factor and DIC (Van-Dyck and Verdonck, 2014, Hussain et al., 2025c). The survival model based on Burr Type XII distribution, MLE, and SVM was used as a hybrid model in predicting the survival of cancer (Hussain et al., 2025a). Telecommunications and healthcare are some other fields where the application of NHPP models has been undertaken. A different study used fuzzy inverse Gompertz model-based survival to compare the classical and artificial intelligence estimators based on simulated and real data (Shinohara et al., 2021, De-Oliveira et al., 2012, Hou et al., 2022).

2 Inverse Weibull Process

The IWP distribution is flexible and analytically tractable, making it appropriate for use in life-testing and reliability analysis. The IWP can be used to model time-dependent failure rates under non-homogeneous Poisson processes. As the IWP distribution contains a unimodal probability density, the distribution can be used in predicting complex system failures, which can be applied in manufacturing, engineering, and health care fields (Hussain et al., 2025b).

Suppose

$$f(t) = \lambda(t) e^{-m(t)}, 0 < t < \infty, \quad (2.1)$$

is the probability density function of the time t until the first failure/event occurs, $\lambda(t)$ is the intensity function of the NHPP at time t , indicating the instantaneous rate at which events (e.g., failures) are expected to occur, and $m(t)$ is the Mean Value Function (MVF) evaluated at time t , representing the expected cumulative number of events from time 0 up to time t . Suppose (Ramesh et al., 2022, Pozza et al., 2024)

$$\lambda(t) = k\beta^k t^{-(k+1)}, 0 < t < \infty, \beta, k > 0, \quad (2.2)$$

where parameters β and k are positive constants.

The mean value function is expressed as

$$\begin{aligned} m(t) &= \int_0^t \lambda(u) du, 0 < t < \infty \\ &= \int_0^t k\beta^k u^{-(k+1)} du \\ &= (\beta/t)^k, 0 \leq t \leq t_0. \end{aligned} \quad (2.3)$$

Substituting Eq. (2.2) and Eq. (2.3) into Eq. (2.1). the IWP is obtained through the probability density function (Hussain et al., 2025d,c,e)

$$f(t) = k\beta^k t^{-(k+1)} e^{-\left(\frac{\beta}{t}\right)^k}, t > 0, \quad (2.4)$$

where β and k control the shape of the inverse Weibull intensity, acting as a scale parameter. It is important to correctly estimate β and k , since it shapes how well the model works and how it describes the data. For a Non-Homogeneous Poisson Process (NHPP), the MVF and the intensity function are linked as follows: (Schwarz, 1978, Brown et al., 2002, Cook and Lawless, 2002, Cox and Isham, 1980)

$$m(t) = E[N(t)] = \int_0^t \lambda(u) du, \text{ and } \lambda(t) = m'(t)$$

assuming $m(t)$ is differentiable. The distribution of the number of events by time t is given by

$$P\{N(t) = n\} = \frac{[m(t)]^n}{n!} e^{-m(t)}, n = 0, 1, 2, \dots$$

Furthermore, the probability of zero events is $P\{N(t) = 0\} = e^{-m(t)}$, which corresponds to the survival function of the first event time T_1 . Differentiating it gives the pdf of the first failure as

follows:

$$f_{T_1}(t) = -\frac{d}{dt}e^{-m(t)} = \lambda(t)e^{-m(t)}, \quad (2.5)$$

where $f_{T_1}(t)$ is the pdf of the first failure, and $\lambda(t)$ is the intensity function, the latter equals $\lambda(t)$ multiplied by $e^{-m(t)}$, and is not itself the intensity. For the IWP considered here, the explicit form of $\lambda(t)$ is obtained by differentiating the closed-form $m(t)$ given in Eq. (2.3).

3 Methods of Estimation

Here we study both the MLE and FFNN-ABC methods for estimating the parameters in IWP.

3.1 Maximum Likelihood Method

The MLE is a cornerstone of parametric inference, which has generally been found useful because, apart from being consistent, asymptotically unbiased under standard regularity conditions, and efficient with standard conditions, it is also simple to compute. The MLE finds the parameter values that maximize the likelihood of the observed data. The joint probability density of the time points $(t_1, t_2, \dots, t_n)$ at which events are observed, is given by Khair et al. (2017) and Hussain et al. (2025f).

$$f(t_1, t_2, \dots, t_n) = \prod_{i=1}^n \lambda(t_i) e^{-m(t_i)}. \quad (3.1)$$

Substituting Eq. (2.4), into Eq. (3.1), we obtain

$$f(t_1, t_2, \dots, t_n) = \prod_{i=1}^n k \beta^k t_i^{-(k+1)} e^{-\left(\frac{\beta}{t_i}\right)^k}.$$

The likelihood function is given by

$$L = \prod_{i=1}^n k \beta^k t_i^{-(k+1)} e^{-\left(\frac{\beta}{t_i}\right)^k}.$$

The log-likelihood function is expressed as

$$\ln L(\beta, k) = n \ln k + nk \ln \beta - (k+1) \sum_{i=1}^n \ln t_i - \sum_{i=1}^n \beta^k t_i^{-k}. \quad (3.2)$$

Differentiating Eq. (3.2) with respect to parameter β , we get

$$\frac{\partial \ln L}{\partial \beta} = \frac{nk}{\beta} - k \beta^{k-1} \sum_{i=1}^n t_i^{-k}.$$

Setting $\partial L / \partial \beta = 0$ yields the closed-form relation (for fixed k)

$$\hat{\beta}(k) = \left(\frac{n}{\sum_{i=1}^n t_i^{-k}} \right)^{1/k},$$

and differentiating Eq. (3.2) with respect to parameter k , we get

$$\frac{\partial \ln L}{\partial k} = \frac{n}{k} + n \ln \beta - \sum_{i=1}^n \ln t_i - \sum_{i=1}^n \beta^k t_i^{-k} \ln(\beta/t_i).$$

Substituting $\beta = \hat{\beta}(k)$ and using $\hat{\beta}(k)^k = n / \sum_j t_j^{-k}$ simplifies the k -equation substantially. Let

$$A(k) = \sum_{i=1}^n t_i^{-k}, \quad B(k) = \sum_{i=1}^n t_i^{-k} \ln t_i.$$

After substitution and algebra, one obtains the single nonlinear equation for k :

$$\frac{1}{k} - \frac{1}{n} \sum_{i=1}^n \ln t_i + \frac{B(k)}{A(k)} = 0. \quad (3.3)$$

Because the likelihood equations are highly nonlinear, the conventional analytical methods cannot be used to estimate the parameters β and k . Rather, numerical methods, like Newton-Raphson, or the EM algorithm, must be applied to solve the system iteratively and to provide maximum likelihood estimates.

3.2 Modified Artificial Neural Network

An Artificial Neural Network (ANN) consists of interconnected neurons, each with related weights and bias. Once the network structure is determined, the next step is to train optimal weights and biases to improve prediction accuracy. This is done through various techniques, where the ABC algorithm is utilized in this study to improve the training performance. A thorough description of the ABC algorithm is provided in Hussain et al. (2025f), and a detailed implementation applicable in this work is in Shirawia et al. (2024). Implementation in MATLAB is also available.

Algorithmic Workflow of the ANN-ABC Estimation Procedure

Input: event times vector T (sorted or unsorted).

Step 1: Preprocessing Sort T , rescale to $[0, 1]$, compute empirical intensity $\hat{\lambda}_{\text{emp}}(t)$ by smoothing the derivative of the empirical mean value function.

Step 2: Initialize ANN One hidden layer with 10 neurons, tanh activation. Flatten weights and biases into vector W .

Step 3: Initialize ABC Set $NP = 40$, limit = $0.6 \times NP$, maxIter = 500. Randomly initialize population $\{W_j\}$.

Step 4: Fitness Evaluation For each W_j , compute fitness as RMSE between $\hat{\lambda}_{ANN}(t; W_j)$ and $\hat{\lambda}_{\text{emp}}(t)$.

Step 5: Bee Phases Update employed, onlooker, and scout bees according to ABC rules until stopping criterion is met.

Step 6: Extract Best ANN Select W^* with minimal RMSE. Compute $\hat{\lambda}_{ANN}(t) = ANN(t; W^*)$.

Step 7: Parametric Fitting Fit $\lambda(t; k, \beta) = k\beta^k t^{-(k+1)}$ to $\hat{\lambda}_{ANN}(t)$ using nonlinear least squares.

Return: Estimated $(\hat{k}, \hat{\beta})$ with diagnostics.

3.3 Artificial Neural Network Training Approach Using ABC Algorithm

One of the most extensively used tools of artificial intelligence is the Artificial Neural Networks (ANNs), whose use is proven in classification, prediction, and pattern recognition problems. ANNs are classically trained by different optimization techniques like gradient descent but these tend to be topology-sensitive and can easily end up at a local minimum thus preventing optimal performance. In order to overcome this shortcoming, nature-inspired metaheuristic optimization methods have been suggested to replace it. In this study, we utilize the Artificial Bee Colony (ABC) algorithm to train ANNs to provide a powerful mechanism of escaping local minima and enhancing convergence. The ABC algorithm, conceptually, is similar to the Firefly Algorithm, where individual agents (e.g. bees or fireflies) are directed towards others based on intensity, which in optimization corresponds to fitness (Hussain, 2019). ANN training with ABC typically proceeds through twelve stages, outlined as follows:

Step 1: Initialization ANN

- Input: time variable t (preprocessed and normalized to $[0, 1]$).
- Hidden layers: one hidden layer with 10 neurons; activation function: $\tanh(\cdot)$. (ReLU was also tested with similar performance.)
- Output: estimated intensity $\hat{\lambda}(t)$.
- All weights and biases are flattened into a single parameter vector W .

Step 2: Data Preprocessing and Mapping

- Event times were sorted and rescaled to $[0, 1]$ for stable training.
- The ANN was trained to approximate the empirical intensity function $\hat{\lambda}_{emp}(t)$, obtained by smoothing the derivative of the empirical mean value function.

Step 3: Initialize ABC

- Population size $NP = 40$ bees.
- Proportions: employed bees = 50%, onlooker bees = 40%, scout bees = 10%.
- Abandonment limit parameter: $0.6 \times NP$.
- Maximum iterations: 500.
- Initial weights sampled uniformly from $[-5, 5]$.

Step 4: Training and Validation

- Training was conducted on the full simulated dataset.
- Validation was achieved by monitoring convergence of RMSE.
- Stopping criteria: maximum iterations (500) or no RMSE improvement over 50 iterations.
- Random seed fixed at 123 for reproducibility.

Step 5: Evaluate ANN

- Evaluate the fitness of each bee (solution) using the ANN.
- The fitness function is defined as the negative of the RMSE between the predicted output of the ANN and the target output.

Step 6: Employed Bees Phase Each employed bee searches for a new food source (solution) in the neighborhood of its current position using the following equation:

$$v_{ij} = x_{ij} + \pi_{ij} * (x_{ij} - x_{kj}),$$

where v_{ij} is the new food source (solution), x_{ij} is the current position of the employed bee, π_{ij} is a random number between -1 and 1, and x_{kj} is the position of a randomly selected bee k .

Step 7: Onlooker Bees Phase Each onlooker bee selects a food source (solution) from the employed bees based on the probability.

Step 8: Scout Bees Phase If a food source (solution) is abandoned, a scout bee is sent to search for a new food source using the following equation:

$$x_{ij} = x_{min,j} + \text{rand}(0, 1) * (x_{max,j} - x_{min,j}),$$

where x_{ij} is the new food source (solution), $x_{max,j}$ and $x_{min,j}$ are the minimum and maximum bounds of the j^{th} .

Step 9: Once the Weights and Biases Have Been Updated, Check the MSE If the $MSE_{new} \leq MSE_{old}$, then choose $M_{new} = M_{old}/B$ and go to step 2. Otherwise, choose $M_{new} = M_{old} * B$ and go to step 3.

Step 10: Fitness Function Calculate

$$\text{Fitness}(W) = \sqrt{\frac{1}{n} \sum_{i=1}^n [\hat{\lambda}_{ANN}(t_i; W) - \hat{\lambda}_{emp}(t_i)]^2}$$

Step 11: Post-Processing

- After ANN convergence, the best ANN intensity $\hat{\lambda}_{ANN}(t)$ is extracted.
- The parametric IWP intensity function

$$\lambda(t; k, \beta) = k\beta^k t^{-(k+1)}, \quad (3.4)$$

is then fitted as $\hat{\lambda}_{ANN}(t)$ via nonlinear least squares (MATLAB lsqcurvefit).

- Initial guesses: $k = 1, \beta = \bar{t}/10$.

Step 12: Iteration Repeat steps 5-9 until a stopping criterion is reached.

4 Simulation

Simulation enables systematic evaluation of estimator performance under controlled conditions. Its flexibility enables researchers to study the impacts of changes in parameters and test hypotheses without the logistical, financial, or ethical constraints of physical experimentation. Where empirical studies are either time-consuming, expensive, or impractical, simulation is especially useful to provide an efficient way to research the behavior of a system. Simulation makes it possible to validate models by allowing the systematic variation of inputs and increases the knowledge in a wide range of disciplines, such as engineering, economics, and software systems (Alruhaili et al., 2025, Ibrahim et al., 2024, Veza et al., 2022).

Stage I: Model Initialization and Parameter Specification

The first stage establishes the groundwork, in which one finds the main hypotheses and chooses the parameter values that characterize the further simulation process. It has three sequential elements necessary to configure processes.

Step 1: Default Parameter Values get selected during this first step of the procedure The simulation algorithm starts with default values of the Inverse Weibull process parameters, based on past empirical research and solid validation. Out of several configurations that were tested, two sets of parameters demonstrated superior performance.

Set 1: $\beta = 0.6$, $k = 1.6$

These parameters respectively define the shape, scale, location, and additional distributional characteristics necessary for generating synthetic data that closely resemble the theoretical behavior of the Inverse Weibull Distribution, as shown in Table 1.

Step 2: Determination of Sample Sizes Different sample sizes (small, medium, and large) were used to evaluate the stability and performance of the estimators during the simulation.

$$n = 50, 100, 500.$$

This stratification allows for rigorous analysis of estimator sensitivity and efficiency under varying data volumes.

Stage II: Random Data Generation via Inverse Transformation

This stage involves the generation of pseudo-random data points that follow the probability distribution function of the Inverse Weibull Process, utilizing the Inverse Transform Sampling Method.

Step 1. Generation of Uniform Random Variables Let $u_i \sim U(0, 1)$, $i = 0, 1, 2, \dots, n$. MATLAB provides the built-in rand function to produce independent identical distributed (*i.i.d.*) random variables distributed uniformly from the interval $(0, 1)$ during this stage.

Step 2: Transformation to Inverse Rayleigh Distribution Data The generated uniform variables are transformed into data that follow the Inverse Weibull Process via the Inverse Cumulative Distribution Function (CDF). This transformation leverages the known CDF of the Inverse Weibull Distribution, denoted as Eq. (2.1) in the study, and applies the inverse mapping: $x_i = F^{-1}(y)$. This simplifies to

$$t_i = \sqrt{-u_i/\alpha^2}, i = 0, 1, 2, \dots, n.$$

This procedure ensures that the synthetic dataset accurately represents the statistical characteristics of the Inverse Weibull Process under study.

Stage III: Parameter Estimation

The last step in the simulation model is parameter estimation of the Inverse Weibull distribution in Software Reliability Growth Models (SRGMs). The method used in this phase is to use several estimation methods on the entire observation duration to ensure both statistical accuracy and predictive reliability. The following techniques are used:

- Maximum Likelihood Estimations,
- Artificial Neural Network with Artificial Bee Colony algorithm (ANN-ABC).

Stage IV:

The optimal assessed method was identified using RMSE, evaluated for the estimated probability density function.

Stage V:

The experiment is repeated 1000 times. To ensure statistical stability, all RMSE and MAE values reported in Table 1 were computed by averaging the results over 1000 independent Monte Carlo repetitions. This repetition-based design provides reliable comparisons between MLE and ANN-ABC by minimizing sampling variability and ensuring reproducibility of the simulation outcomes.

Stage VI:

Compute the Root Mean Square Error (RMSE) for each observation λ_i , based on the estimated distribution parameters k and β

$$\text{RMSE}(\hat{\lambda}) = \sqrt{\sum_{i=1}^Q (\hat{\lambda}_i - \lambda_i)^2 / Q}.$$

In addition to RMSE, this study also considers the Mean Absolute Error (MAE):

$$\text{MAE}(\hat{\lambda}) = \frac{1}{n} \sum_{i=1}^n |\lambda_i - \hat{\lambda}_i|.$$

The evaluation criteria indicate a tradeoff between the parsimony and the model fit. In the case of both the MLE and ANN-ABC estimators, the RMSE and MAE were calculated using the simulated data as well as the empirical data. The model with the smallest RMSE and MAE is considered as the most desirable one, because it gives the best trade-off between explanatory power and model complexity.

Table 1 presents empirical properties of the estimates of the model parameters of the IWP obtained from the MLE and ANN-ABC techniques. A comparison of the models using the RMSE and the MAE values shows that the ANN-ABC is performing better than the MLE method.

5 Dataset

The medical records of 299 patients who were hospitalized with heart failure and were admitted to the Faisalabad Institute of Cardiology and Allied Hospital (Punjab, Pakistan) between the months of April and December 2015 are used in this study (Hacilar et al., 2025, Sharma et al., 2025). The data

consisted of 105 women and 194 men between 40 and 95 years. Each patient was reported to have left ventricular systolic dysfunction, and categorized into New York Heart Association (NYHA) class III or IV heart failure at baseline (Shilpa et al., 2025, Prabhas et al., 2025). In the process of recurrent event modeling, the event of interest was hospital readmission due to worsening heart failure. Every patient was tracked since the hospitalization date until the end of the study in December 2015 or censoring. Censoring occurred administratively at the end of follow-up, upon loss to follow-up, or due to death. When we use the IWP in a framework based on a Non-Homogeneous Poisson Process (NHPP), we assume that increments are independent and that they are not subject to unobserved heterogeneity across individuals. These assumptions offer analytic tractability, but in clinical applications where comorbidities, variation in treatments, and unmeasured patient characteristics can create dependence between event times, these assumptions are often difficult to guarantee. These limitations are acknowledged, and in cases of poor model fit, renewal processes or frailty-based models may offer improved flexibility. Data was de-identified before analysis, and the relevant institutional review board provided ethical consent to this research in compliance with the Helsinki Declaration. The patient confidentiality and research governance standards were strictly observed during the research.

5.1 Homogeneity Testing for the Inverse Weibull Process

To assess whether the observed failure stream is consistent with a constant intensity (Homogeneous Poisson Process) or exhibits time-varying behavior (NHPP), we apply the time-rescaling (transformed interarrival) test (Brown et al., 2002). Let $t_1 < \dots < t_n$ be observed event times and let $\hat{\lambda}(t)$ be a candidate intensity (parametric IWP with fitted $\hat{k}, \hat{\beta}$, or the ANN-ABC fitted $\hat{\lambda}_{\text{ANN}}(t)$). Define the integrated intensity

$$\Lambda(t) = \int_{t_0}^t \hat{\lambda}(s) ds,$$

with $t_0 > 0$ chosen to avoid singularities (see below). Under the null hypothesis that the model and parameters are correct, the transformed interarrival increments

$$\Delta z_i = \Lambda(t_i) - \Lambda(t_{i-1}), \quad \Lambda(t_0) = 0,$$

are i.i.d. We test the empirical $\{\Delta z_i\}$ against $\text{Exp}(1)$ using a one-sample Kolmogorov-Smirnov (KS) test. Equivalently, $u_i = 1 - \exp(-\Delta z_i)$ should be i.i.d. $\text{Unif}(0, 1)$ and can be tested by KS . As a complementary check, we perform a Pearson chi-square test on observed counts in m equal time bins. For the IWP intensity

$$\lambda(t) = k\beta^k t^{-(k+1)},$$

the integrated intensity has the closed form

$$\Lambda(t) = \beta^k (t_0^{-k} - t^{-k}),$$

so that Δz_i can be computed analytically. We used $t_0 = \min(t_i)/2$ to avoid the singularity at $t = 0$; we verified robustness to other small positive t_0 .

Table 1: Simulated RMSE and MAE of MLE and ANN-ABC Estimators for the Intensity Function of the IWP.

n	Parameters	RMSE($\lambda(t)$)			MAE($\lambda(t)$)			MLE			ANN – ABC		
		MLE	ANN	ABC	MLE	ANN	ABC	95%CI	Bias		95%CI	Bias	
50	$\beta = 0.6; k = 1.6$	29.287	27.459*		18.187	15.369*		95.7	1.28733		95.8	0.18722	
	$\beta = 1.6; k = 0.6$	29.188	21.453*		18.098	10.263*		96.5	1.00281		96.7	0.41170	
100	$\beta = 0.6; k = 1.6$	18.804	18.211*		27.714	16.111*		95.8	0.25754		95.9	0.14754	
	$\beta = 1.6; k = 0.6$	18.275	15.030*		27.395	13.031*		97.5	0.42766		97.6	0.31755	
500	$\beta = 0.6; k = 1.6$	13.050	12.842*		22.051	10.653*		95.9	0.18064		96.2	0.07064	
	$\beta = 1.6; k = 0.6$	10.734	10.082*		19.734	08.083*		97.4	0.37110		97.5	0.26110	

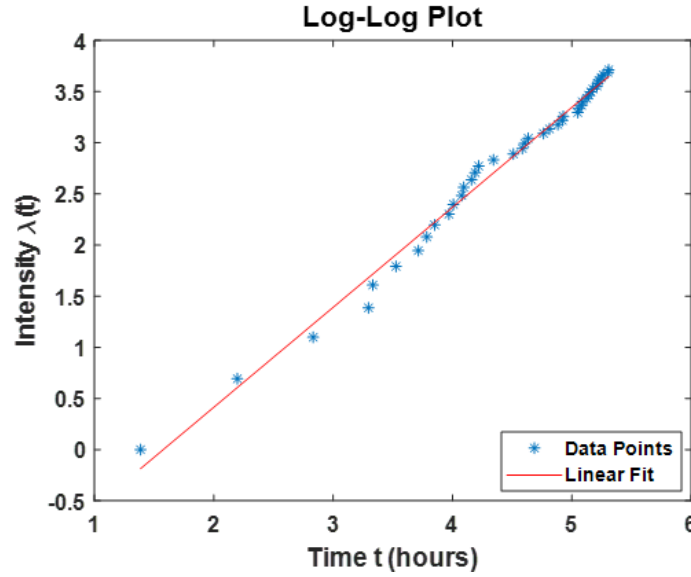


Figure 1: Cumulative number of days of operation between two shutdowns with their occurrence times on a logarithmic scale.

The scatter plot shows an approximately linear trend, which implies that the data are highly suitable for modeling by means of the IWP function. Similar performance of the MLE and ANN-ABC, especially in a sample size of 299, could be referred to as the strengths on which the two methodologies are based. Both methods aim to minimize the gap between observed and predicted values, though they do so through different inferential frameworks. The MLE finds the value of the parameters that maximizes the likelihood of the data given a set of (fixed) parameters.

Table 2: RMSE and MAE values for methods used to estimate the IWP model

Units	Size	Method	$\hat{\beta}$	$\hat{k}$	RMSE($\lambda(t)$)	MAE($\lambda(t)$)
DS1	105	MLE	0.20268	0.20047	25.848	23.626
		ANN-ABC	0.35294	0.24183	21.806*	20.616*
DS2	194	MLE	0.2138	0.1027	28.885	26.675
		ANN-ABC	0.3760	0.2651	27.456*	24.376*

Table 2 shows estimates of the parameters of IWP computed from the proposed estimation schemes of this study. The values of the RMSE and MAE for the ANN-ABC method demonstrated superior performance, as it provided more efficient estimators in describing the underlying structure in data.

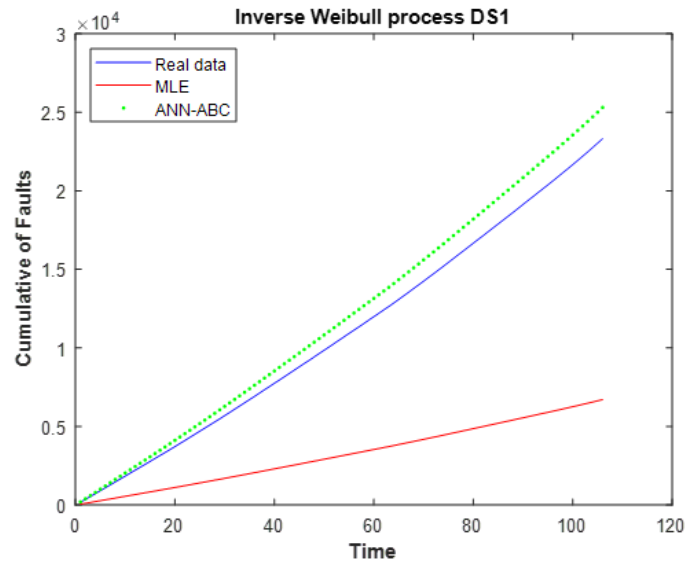


Figure 2: Comparative Estimation of the Cumulative Failure Function for 194 Men Aged 40 and Above Using MLE and ANN-ABC Methods Under the Inverse Weibull Process Model (DS1).

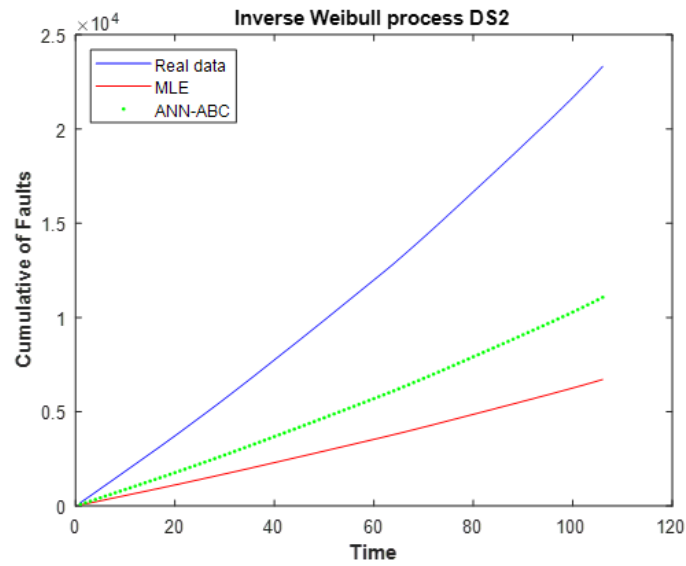


Figure 3: Comparative Estimation of the Cumulative Failure Function for 105 women Aged 40 and Above Using MLE and ANN-ABC Methods Under the Inverse Weibull Process Model (DS2).

The estimates of cumulative failure functions of the male ($n=194$) and female ($n = 105$) heart failure patients under the IWP model are shown in Figures 2 and 3, respectively. In both instances, the ANN-ABC model fits better to the actual cumulative failure data, especially in the case of early and mid-life failures. The MLE also has a similar trend to the general trajectory, but the deviation remains at a slightly higher point than in general, paying more attention to the period over time. These graphical results agree with the quantitative results in Table 2, where ANN-ABC cells showed smaller Mean Absolute Error (MAE) values, which represent an improvement in fit in the model, even though RMSE was only somewhat smaller in small samples. The graphs emphasize the stability of ANN-ABC in the estimation of failure intensity in non-homogeneous settings.

6 Conclusion and Future Work

This research paper presents and tests a hybrid estimation model that integrates the ABC algorithm with ANN to estimate parameters in the IWP, which is modeled under the NHPP. The ANN-ABC approach, based on both simulated data and clinical data of heart failure patients, achieved better performance than the classical MLE method, particularly at moderate and large sample sizes. It demonstrated superior performance in terms of lower Root Mean Squared Error (RMSE) and Mean Absolute Error because both methods indicate their ability to reveal time-varying failure dynamics in a better way in comparison to traditional methods. The results highlight how hybrid intelligent algorithms can be used to increase reliability modeling in complex, data-driven settings, including predictive maintenance in cyber-physical systems, health informatics, and IoT-based monitoring. Future work includes extending the framework to incorporate Bayesian inference and imperfect repair processes, and also including imperfect repair processes to make it more realistic in real-world applications. The other direction of significance is the generalization of the methodology to support streaming or real-time data to provide flexibility in dynamic operational settings. Lastly, there should be efforts to build adversarial robustness and other superior methods of measuring uncertainty to make AI-driven reliability models safe, resilient, and reliable in urgent uses.

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References

- Alruhaili, H., Hussain, A., Ajlouni, A., TÜRK, F., Az-Zo'bi, E., and Tashtoush, M. (2025), "Solving Time-Fractional Nonlinear Variable-Order Delay PDEs Using Feedforward Neural Networks," *Iraqi Journal For Computer Science and Mathematics*, 6(3), 171–184. <https://doi.org/10.52866/2788-7421.1231>.
- Brito, Ê. S., Tomazella, V. L., and Ferreira, P. H. (2022), "Statistical modeling and reliability analysis

- of multiple repairable systems with dependent failure times under perfect repair,” *2025 Reliability Engineering & System Safety*, 222, 108375. <https://doi.org/10.1016/j.ress.2022.108375>.
- Brown, E., Barbieri, R., Ventura, V., Kass, R., and Frank, L. (2002), “The time-rescaling theorem and its application to neural spike train data analysis,” *Neural Computation*, 14(2), 325–346. <https://doi.org/10.1162/08997660252741149>.
- Cook, R. and Lawless, J. (2002), “The statistical analysis of recurrent events,” *New York: Springer*, <https://doi.org/10.1007/978-0-387-69810-6>.
- Cox, D. and Isham, V. (1980), “Point processes,” *New York: Springer*, (Vol. 12). CRC Press.
- De-Oliveira, M. D., Colosimo, E. A., and Gilardoni, G. L. (2012), “Bayesian inference for power law processes with applications in repairable systems,” *Journal of Statistical Planning and Inference*, 142(5), 1151–1160. <http://doi.org/10.1016/j.jspi.2011.11.016>.
- Hacilar, H., Dedetürk, B. K., Ozmen, M., Celik, M. E., and Gungor, V. C. (2025), “Accelerated artificial bee colony optimization for cost-sensitive neural networks in multi-class problems,” *Expert Systems*, 42(5), e70045. <https://doi.org/10.1111/exsy.70045>.
- Hou, Y., Huang, C., and Fang, C. (2022), “The Using the methods of statistical data analysis to improve the trustworthiness of software reliability modeling,” *IEEE Access*, 10, pp.25358–25375. <https://doi.org/10.1109/ACCESS.2022.3154103>.
- Hussain, A. (2019), “Stability of Riccati stochastic differential equations,” *Journal of Duhok University*, 22(1), 11–16.
- Hussain, A., Mahmood, K., Ibrahim, I., Jameel, A., Nawaz, S., and Tashtoush, M. (2025a), “The Parameters estimation of the Gompertz-Makeham process in non-homogeneous Poisson processes: Using modified maximum likelihood estimation and artificial intelligence methods,” *Mathematics and Statistics*, 13(1), 1–11. <http://doi.org/10.13189/ms.2025.130101>.
- Hussain, A., Mashikhin, Z., Yaseen, N., Abu Hammad, M., Az-Zo’bi, E., and Tashtoush, M. (2025b), “Hybrid Deep Learning Framework for Estimating the Occurrence Rate of the Inverse Exponential Process: A Comparative Analysis of LSTM, ANN, and Traditional Methods,” *2025 12th International Conference on Information Technology (ICIT 2025): Innovation Technologies, Amman, Jordan, May 27–30*, pp. 181–187. DOI: 10.1109/ICIT64950.2025.11049273.
- Hussain, A., Mohsin, S., Alsaedi, H., and Tashtoush, M. (2025c), “Parameter Estimation in Linear Rate Process: Modified Maximum Likelihood Estimation Using Particle Swarm Optimization, Least Square Estimation, and Simulation Methods,” *Int. J. Advance Soft Compu. Appl.*, 17(2). <https://doi.org/10.15849/IJASCA.250730.19>.
- Hussain, A., Oraibi, Y., Mashikhin, Z., Jameel, A., Tashtoush, M., and Az-Zo’bi, E. (2025d), “New Software Reliability Growth Model: Piratical Swarm Optimization -Base Parameter Estimation in Environments with Uncertainty and Dependent Failures,” *Statistics, Optimization & Information Computing*, 13(1), 209–221. <https://doi.org/10.19139/soic-2310-5070-2109>.

- Hussain, A., Pati, K., Atiyah, A., and Tashtoush, M. (2025e), "Rate of occurrence estimation in geometric processes with Maxwell distribution: A comparative study between artificial intelligence and classical methods," *International Journal of Advances in Soft Computing and Its Applications*, 17(1), 1–15. <https://doi.org/10.15849/IJASCA.250330.01>.
- Hussain, A. S., Saadoon, N. Q., Abdulghafour, A. S., and Abduirazzaq, N. T. (2025f), "Parameters estimation of a suggested model non-homogeneous Poisson process: A comparison of conventional and artificially intelligent approaches," *AIP Conference Proceedings*, 3282(1), p. 020023. <https://doi.org/10.1063/12.XXXXXXX>.
- Hussein, L., Alqarni, K., Hilmi, M., Agina, M., Shirawia, N., Abdelreheem, K., Hassan, T., and Tashtoush, M. (2024), "The mediating role of learning management system use in enhancing system effectiveness," *WSEAS Transactions on Business and Economics*, 21, 2067–2078. <https://doi.org/10.37394/23207.2024.21.169>.
- Ibrahim, I., Taha, W., Dawi, M., Jameel, A., Tashtoush, M., and Az-Zo'bi, E. (2024), "Various closed-form solitonic wave solutions of conformable higher-dimensional Fokas model in fluids and plasma physics," *Iraqi Journal For Computer Science and Mathematics*, 5(3), 401–417.
- Khair, U., Fahmi, H., al Hakim, S., and Rahim, R. (2017), "Forecasting error calculation with mean absolute deviation and mean absolute percentage error," *Journal of Physics: Conference Series*, 930(1), Article 012002. <https://doi.org/10.1088/1742-6596/930/1/012002>.
- Lawson, C. (2001), "Statistical methods for the reliability of repairable systems," *Journal of Quality Technology*, 33(4), 527–528. <https://doi.org/10.1080/00224065.2001.11980114>.
- Li, Q., Liu, L., Li, T., and Yao, K. (2023), "Bayesian change-points detection assuming a power law process in the recurrent-event context," *2025 Communications in Statistics-Simulation and Computation*, 52(12), 6011–6033. <https://doi.org/10.1080/03610918.2021.2002350>.
- Mohsin, S., Hameed, K., Az-Zo'bi, E., and Tashtoush, M. (2025), "Discretization of the Inverse Rayleigh-G Family: Theoretical Properties, Machine Learning-Based Parameter Estimation, and Practical Applications," *Statistics, Optimization & Information Computing*, 14(3), 1174–1197. <https://doi.org/10.19139/soic-2310-5070-2618>.
- Muralidharan, K. (2018), "Repairable system modeling using power law process," *Journal of Quality Technology System reliability management*, (pp. 79–101). CRC Press. <https://doi.org/10.1080/00224065.2001.11980114>.
- Pozza, F., Durante, D., and Szabo, B. (2024), "Skew-symmetric approximations of posterior distributions," *Water Resources Management*, arXiv preprint. <https://arxiv.org/abs/2409.14167>.
- Prabhas, K., Basem, A., Lakshmi, L., Talha, A., Mohammed, S., Khan, M., and Khedher, N. (2025), "A Deep Learning Framework for Brain Tumor Detection Using CNNs and Transfer Learning on MRI Scans," *Systems and Soft Computing*, , p.200389.

- Ramesh, N., Rode, G., and Onof, C. (2022), "A Cox process with State-Dependent exponential pulses to model rainfall," *Water Resources Management*, 36(1), 297–313. <https://doi.org/10.1007/s11269-021-03028-6>.
- Schwarz, G. (1978), "Estimating the dimension of a model," *The annals of statistics*, 461–464.
- Sharma, A., Aravindan, N., Ignatia, K., Areche, F., Loganathan, G., Taha, A., Dwivedi, P., and Yellapragada, R. (2025), "Hybrid multi criteria decision making methodology for improving sustainability of the manufacturing sector," *International Journal on Interactive Design and Manufacturing (IJIDeM)*, 19(2), pp.695–704.
- Shilpa, B. and Chohan, J., Beemkumar, N., Kulshreshta, A., Ghodhbani, R., Othman, N., Abdullaeva, B., and Khan, M. (2025), "A novel machine learning approach for numerical simulation on the hybrid nanofluid flow past a converging/diverging channel: Properties of tantalum and alumina nanoparticles," *Partial Differential Equations in Applied Mathematics*, 13, 101063.
- Shinohara, S., Manome, N., Nakajima, Y., Gunji, Y. P., Moriyama, T., Okamoto, H., Mitsuyoshi, S., and Chung, U. I. (2021), "Power laws derived from a Bayesian decision-making model in non-stationary environments," *Symmetry*, 13(4), 718. <https://doi.org/10.3390/sym13040718>.
- Shirawia, N., Kherd, A., Bamsaoud, S., Tashtoush, M., Jassar, A., and Az-Zo'bi, E. (2024), "Dejdmrong collocation approach and operational matrix for a class of second-order delay IVPs: Error analysis and applications," *WSEAS Transactions on Mathematics*, 23, 467–479.
- Sulaiman, M., Hussain, A., Abu Hammad, M., Tashtoush, M., Az-Zo'bi, E., and Abbas, R. (2025), "Parameters Estimation in a Non-Homogeneous Poisson Process with Fuzzy Data through a Modified Fuzzy Maximum Likelihood Approach Using Artificial Intelligence," *2025 12th International Conference on Information Technology (ICIT 2025): Innovation Technologies, Amman, Jordan, May 27–30, 2025*, pp. 188–195, DOI: 10.1109/ICIT64950.2025.11049166.
- Van-Dyck, J. and Verdonck, T. (2014), "The Precision of power-law NHPP estimates for multiple systems with known failure rate scaling," *Reliability Engineering & System Safety*, 126, 143–152. <https://doi.org/10.1016/j.ress.2014.01.019>.
- Veza, I., Afzal, A., Mujtaba, M. A., Hoang, A. T., Balasubramanian, D., Sekar, M., Fattah, I., Soudagar, M. E., EL-Seesy, A. I., Djamari, D. W., and Hananto, A. L. (2022), "Review of artificial neural networks for gasoline, diesel and homogeneous charge compression ignition engine," *Alexandria Engineering Journal*, 61(11), 8363–8391. <https://doi.org/10.1016/j.aej.2022.01.072>.

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