Journal of Statistical Research *https://doi.org/10.3329/jsr.v59i2.88063*
2025, Vol. 59, No. 2, pp. 167-181 *ISSN 0256 - 422 X*

PREDICTING MONKEYPOX INFECTION RATIOS USING BAYESIAN AND MAXIMUM LIKELIHOOD METHODS OF PANEL DATA: EMPLOYING THE WEIGHTED EXPONENTIAL REGRESSION MODEL

SAIFALDIN HASHIM KAMAR*

Department of Accounting, College of Administration and Economics
Al-Iraqia University, Baghdad, Iraq

Email: dr.saifhkamar@gmail.com

HALA KADHUM OBEAD

Department of Banking and Finance, College of Administration and Economics
Al-Iraqia University, Baghdad, Iraq

Email: hala.obead@aliraqia.edu.iq

ASEEL NASSER SABTI

Department of Business Administration, College of Administration and Economics
Al-Iraqia University, Baghdad, Iraq

Email: aseel.sabti@aliraqia.edu.iq

SUMMARY

This study employs advanced statistical techniques, namely Maximum Likelihood Estimation (MLE) and Bayesian inference, to estimate the parameters of a weighted exponential regression model for panel data, accounting for both fixed and random effects. The empirical analysis utilizes monkeypox incidence data from the Americas, Africa, and Europe over the period 2022–2023, complemented by simulated datasets of varying sample sizes (15, 30, 45, and 60) to thoroughly evaluate the model's performance. A comparative analysis using the Mean Absolute Percentage Error (MAPE) criterion reveals that the Bayesian estimation method for random effects outperforms both the fixed effects model and the MLE approach for both fixed and random effects. Additionally, the model is used to forecast monthly infection rates over the next six months, with the Bayesian random effects model indicating a significant decline in infection rates, approaching near-zero values. This highlights the Bayesian framework's ability to accurately capture and predict the dynamics of the weighted exponential regression model in panel data contexts involving random effects.

Keywords and phrases: Maximum likelihood, Panel data, Weighted exponential regression model, Monkeypox infection.

* Corresponding author

1 Introduction

The emergence of infectious diseases, such as monkeypox, which have raised significant global health concerns since 2022, has spurred increased scholarly attention on the application of advanced statistical models in epidemiological studies. Research has largely focused on understanding the transmission dynamics of these diseases and identifying their underlying determinants, with particular emphasis on utilizing panel data. Models that incorporate temporal effects are especially valuable in this context, as they facilitate the examination of disease transmission across both time and geographical space. This growing interest spans multiple disciplines, notably economics and public health, alongside a transition from cross-sectional to panel data analysis. One key advantage of panel data is its ability to link cross-sectional observations with multiple time periods, thereby improving the precision of predictive models. The Hausman test is commonly employed for estimating fixed or random effects in panel data models, and many scholars have explored its application. Moreover, some researchers have expanded on this by using advanced estimation methods, such as Bayesian analysis, which treats parameter values as random variables influenced by prior distributions, as opposed to fixed values. In this framework, prior knowledge about the parameters can be expressed through a prior distribution, and a posterior probability function is derived by integrating available information to refine the estimation process. The weighted exponential model has emerged as a powerful tool for modeling the transmission of infectious diseases, effectively predicting nonlinear increases in infection rates across time and regions.

Morawetz (2006) emphasized the importance of Bayesian analysis for panel data, especially in scenarios with limited prior knowledge, using a simulation-based approach with a Gaussian prior distribution. Similarly, Cheruiyot et al. (2016) applied the Laplace method to estimate a two-parameter Weibull distribution using Bayesian methods, finding that the variances of the measurement scale parameter α were smaller compared to those from the Lindley approximation, and similar to those obtained via maximum likelihood estimation (MLE). To address the limitations of traditional methods like Generalized Method of Moments (GMM) and MLE, Leorato and Mezzetti (2016) introduced the use of a separable variance matrix with Bayesian methods for spatially supported panel data. Al-Noor and Hussein (2018) employed Monte Carlo simulations with symmetric and asymmetric loss functions to explore Bayesian estimates of the weighted exponential distribution parameters, using fuzzy data with reliability functions. In a similar, Zhang (2020) applied a nonparametric Bayesian approach to random effects panel data, achieving highly accurate estimates for investment rates of various enterprises in Monte Carlo simulations. Furthermore, Kasilingam et al. (2021) utilized the exponential model to forecast COVID-19 transmission rates, integrating machine learning algorithms to predict early containment indicators, considering factors such as infrastructure, regulations, and the disease itself. Recently, the Bayesian approach has gained traction in epidemiological modeling due to its flexibility in managing uncertainty and its capacity to update prior knowledge with new data. Gu and Yin (2022) demonstrated the superior stability of Bayesian estimates over conventional methods in estimating transmission coefficients for the SIR model, particularly in cases of regional heterogeneity or data scarcity. Riad et al. (2022) applied both Bayesian and classical estimation methods to estimate the parameters of a two-parameter asymmetric weighted exponential distribution. Similarly, Liao et al. (2023) used Bayesian estimates with the SIR model, applying directed acyclic graphs to assess monkeypox's fundamental reproduction number and epidemiological spread. Das (2024) and Ngungu et al. (2023) further expanded this work, creating a dimensionally consistent model to analyze monkeypox's spread in relation to non-pharmaceutical interventions and quarantine measures. Heuts et al. (2025) explored Bayesian methods in cardiovascular disease research, outlining both advantages and limitations in the context of randomized controlled trials. Abiola et al. (2025) also highlighted the robustness of integrating Bayesian hierarchical models with panel data for investigating regional variations in disease transmission. This study adopts such a framework, applying a weighted exponential regression model with both fixed and random effects to analyze monkeypox transmission. However, challenges arise from the model's sensitivity to abrupt shifts in health policies or individual behaviors, complicating its application in infectious disease modeling. To mitigate these challenges, scholars like Qian et al. (2025) have combined Bayesian methods with exponential models to improve predictive accuracy. Qian's study, using nonlinear least squares for sensitivity analysis, developed a novel model to investigate MPOX, forecasting a persistent outbreak of chickenpox in the U.S. in the near future. The existing literature underscores the potential of integrating Bayesian analysis with the weighted exponential model for panel data to provide a promising framework for understanding monkeypox transmission dynamics, considering temporal and spatial variations and the impact of various preventive measures.

This study utilizes a weighted exponential regression model, incorporating both fixed and random effects, to predict monthly monkeypox infection rates across the Americas, Africa, and Europe. The first section of the study reviews relevant prior research. The second section introduces panel data, the weighted exponential regression model, and the estimation of fixed and random effects using both Bayesian and maximum likelihood methodologies. The third section presents a comparative analysis of estimation methods through simulation. The fourth section applies the model to real-world data, presenting observed monthly infection rates alongside their corresponding forecasts. The fifth and final section discusses key findings, offering insights into the model's effectiveness and its implications for future research.

2 Materials and Methods

2.1 Panel data

Over the past decade, panel data has garnered significant attention, particularly in the fields of economics and health, due to its ability to simultaneously incorporate both time-series and cross-sectional data, thereby accounting for the effects of changes across these dimensions (Baltagi, 2015). Panel data models generally fall into four categories: pooled regression, fixed effects, random effects, and mixed effects models. In this study, the fixed or random effects of the weighted exponential regression model were identified using the Hausman test (Hausman, 1978). Since the fixed term is excluded from the model, pooled effects are not considered. The hypothesis for this analysis is as follows:

H_0 : The random effects model, rather than the fixed effects model, is appropriate

H_1 : The random effects model is not appropriate.

If the p-value is below 0.05, we reject the null hypothesis and conclude that the model with constant effects is more suitable. The parameters of the panel data model were estimated using two distinct approaches: the Bayesian estimation method, which accounts for variable parameter values, and the Maximum Likelihood Estimation (MLE) method, which assumes fixed parameter values.

2.2 Weighted exponential regression model

Consider that the probability density function for the random variable x, representing time, in the weighted exponential regression model is as follows (Nasiri et al., 2011)

$$Y_i = \frac{\alpha+1}{\alpha}\lambda e^{-\lambda x_i}(1-e^{-\alpha\lambda x_i}) + \epsilon_i,$$

where α>0 is the shape parameter, the scale parameter λ>0, and ϵ is the random error. The weighted exponential model was recast by Widiharih et al. (2013) using the following formula, which was applied in this study:

$$Y_i = e^{-\alpha x_i}(1-e^{-\lambda x_i}) + \epsilon_i. \tag{2.1}$$

Equation (2.1) can be rewritten as:

$$Y(X) = \phi(X,\theta) + \epsilon, \tag{2.2}$$

where $\theta = (\alpha, \lambda)$.

This can be further expressed using Taylor's series expansion of $\phi(X,\theta)$ at the first derivative as (Kamar et al., 2021; Obead et al., 2024):

$$E(Y_i) = \phi(x_i,\theta) = \phi(x_i,\theta_0) + \sum_{j=1}^{2} \frac{\partial\phi(x_i,\theta_0)}{\partial\theta_j}\Big|_{\theta_j=\theta_{j0}}(\theta_j = \theta_{j0}),$$

where θ_{j0} is the parameters' initial values.

Moving $\phi(x_i,\theta_0)$ to the left side of the equation, we get (Obead et al., 2024)

$$Y_i - \phi(X_i,\theta_0) = \sum_{j=1}^{2} X^*_{ij}\beta_{j0} + \epsilon_i, i = 1, 2, \ldots, n, \tag{2.3}$$

where

$$X^*_{ij} = \frac{\partial\phi(x_i,\theta)}{\partial\theta_j}\Big|_{\theta_j=\theta_{j0}} \; and \; \beta_{j0} = \theta_j - \theta_{0j}.$$

As a result, a panel model weighted exponential regression model written in matrices undergoes the following linear transformation.

$$Y_{it}^{*} = X_{it}'\beta + \epsilon_{it}, \quad t = 1, \ldots, T; i = 1, \ldots, n, \tag{2.4}$$

where Y_{it}^{*} is the vector of the response variable with rank $(n \times 1)$, X_{it} is the matrix of explanatory variables with rank $(2 \times n)$, β is the vector of parameters with rank (2×1), ϵ_{it} is the error term of rank $(n \times 1)$, and T is the number of time periods.

Model (2.4) lacks an intercept term. Consequently, instead of using pooled effects to obtain the optimal estimate for the weighted exponential regression model (Hausman, 1978), the Hausman test will be applied. While the regression models are expressed in linear form, obtaining the explanatory variable matrix requires the use of iterative methods. The non-linear Maximum Likelihood Estimation (MLE) approach can be utilized to estimate the model parameters based on this framework.

2.3 Fixed effect model with panel data using Bayesian and maximum likelihood methods

According to model (2.4) the probability function for the fixed effect model is derived from the joint probability function for random observations $(X_1, X_2, \ldots, X_n)$, as shown by Kamar and Msallam (2020):

$$L(Y^{*} \mid \beta, \sigma) \propto \sigma^{-n} \exp\left[\frac{-1}{2\sigma^2}(Y^{*} - X^{*}\beta)'(Y^{*} - X^{*}\beta)\right], \tag{2.5}$$

where L is the likelihood function, and σ is the standard deviation.

By taking the natural logarithm of (2.5), we obtain

$$\ln L(Y^{*} \mid \beta, \sigma) \propto -n\,(\ln\sigma) - \frac{1}{2\sigma^2}(Y^{*} - X^{*}\beta)'(Y^{*} - X^{*}\beta). \tag{2.6}$$

By differentiating (2.6) with respect to the vector of parameters β and setting them equal to zero, we get the following:

$$\beta_{MFE} = (\tilde{X}'\tilde{X})^{-1}\tilde{X}'\tilde{Y},$$

which represent the fixed effects model's estimates of parameters β (Obead et al., 2024), where

$$\tilde{X} = X - \bar{X}, \tilde{Y} = Y - \bar{Y}, Var(\epsilon_{it}) = \sigma_{\epsilon}^2, E(\epsilon_{it}) = 0,$$

$$Var(\beta_{MFE}) = S^2(\tilde{X}'\tilde{X})^{-1}, S^2 = \frac{\epsilon'\epsilon}{nT - k}.$$

Bayesian analysis assumes that the parameter value is not fixed but rather a random variable described by a prior distribution. It is assumed that prior knowledge about the parameter to be estimated is available. The posterior probability function is then obtained by integrating this prior knowledge with the likelihood function, resulting in a joint posterior probability function as follows:

$$P(\underline{\theta} \mid Y) \propto P(\underline{\theta})\, P(Y \mid \theta),$$

where $P(\underline{\theta})$ and $P(Y \mid \theta)$ are the prior and joint distribution.

Bayesian estimators minimize the expected loss using the following function:

$$\min_{\hat{\theta}} E(L(\hat{\theta},\theta)) = \min_{\hat{\theta}} \int L(\hat{\theta},\theta)P(\theta\,|\,Y)d\theta.$$

To estimate the fixed effect model using the Bayesian approach as described in Model (2.4), we assume a prior conjugate normal distribution for the parameters with a known σ:

$$P(\beta\,|\,\sigma) \propto \sigma^{-n} \exp\Big[-\frac{1}{2\sigma^2}(\beta-\widetilde{\beta})'\Sigma^{-1}(\beta-\widetilde{\beta})\Big], \tag{2.7}$$

where Σ^{-1} denotes the positive definite matrix that represents the variance-covariance of parameters (β), and $(\widetilde{\beta})$ signifies the arithmetic mean of the distribution.

This prior distribution is selected due to its superior prior information compared to other alternatives, whether non-informative or informative distributions, as referenced in (Murphy, 2007). The distribution (2.7) is multiplied by the likelihood function in (2.5) to obtain the posterior probability function for the parameter (β):

$$f(\beta\,|\,Y^*,\sigma) \propto \sigma^{-n} \exp\Big[-\frac{1}{2\sigma^2}(VS^2+(\beta-\widetilde{\beta})'\Sigma^{-1}(\beta-\widetilde{\beta}))\Big], \tag{2.8}$$

where $VS^2 = (Y^*-X^*\beta_{MFE})'(Y^*-X^*\beta_{MFE})$ and $V = nT-k$. The function (2.8) represents the multivariate normal distribution with the arithmetic mean $(\bar{\beta}_{BFE})$ serving as the Bayesian estimator of the parameters (β) under a weighted squared error-loss function, as follows

$$\bar{\beta}_{BFE} = (X^{*\prime}X^*+\Sigma^{-1})^{-1}(\Sigma^{-1}\widetilde{\beta}+X^{*\prime}Y^*).$$

2.4 Random effect model with panel data using Bayesian and Maximum likelihood methods

Since model (2.4) does not include intercept terms, we introduce randomness into both the intercept and slope parameters. As a result, the random effects are restricted to the slopes. This leads to the estimation of the random coefficients for the regression model in the panel data framework. The estimated slopes for model (2.4) , obtained using the Maximum Likelihood method, are presented as follows (Obead et al., 2024):

$$\beta_{MRE} = (\tilde{\tilde{X}}'\tilde{\tilde{X}})^{-1}\tilde{\tilde{X}}'\tilde{\tilde{Y}},$$

where $\tilde{\tilde{X}} = X - w\bar{X}$,$\tilde{\tilde{Y}} = Y - w\bar{Y}$. Accordingly,

$$w = 1-\sqrt{\sigma^2_{FE}/(\sigma^2_{FE}+T\Omega^2_{\mu})},$$

which was suggested by Álvarez et al. (2017), and $\Omega^2_{\mu} = \sigma^2_{POOLED} - \sigma^2_{FE}$ (Greene, 2002). Also $Var(\hat{\beta}_{MRE}) = \sigma^2_{RE}(\tilde{\tilde{X}}'\tilde{\tilde{X}})^{-1}$, where $\sigma^2_{RE} = \varepsilon'\varepsilon/(nT-k)$, and σ^2_{RE} has been suggested by Swamy and Arora (1972) using the within regression residuals. The same methodology outlined in the previous section was applied to estimate the random effects model for panel data defined by model (2.4), using the Bayesian approach to derive the corresponding posterior probability function

$$f(\beta \mid Y^*, \sigma) \propto \sigma^{-n} \exp\Big[- \frac{1}{2\sigma^2}(VS^{*2} + (\beta - \bar{\beta})'\Sigma^{-1}(\beta - \bar{\beta}))\Big], \tag{2.9}$$

where $VS^{*2} = (Y^* - X^*\beta_{MRE})'(Y^* - X^*\beta_{MRE})$ and $V = nT - k$.

The function (2.9) represents the multivariate normal distribution with the arithmetic mean $(\bar{\beta}_{BRE})$ serving as the Bayesian estimator of the parameters (β) under a weighted squared error loss function, as follows:

$$\bar{\beta}_{BRE} = (X^{*\prime}X^* + \Sigma^{-1})^{-1}(\Sigma^{-1}\bar{\beta} + X^{*\prime}Y^*).$$

Ultimately, by applying the estimator, either through Maximum Likelihood or Bayesian methods, to the initial parameter values (θ_0) in the weighted exponential regression model (2.2), we obtain the parameter estimator (θ) based on the selected method. The model (2.3) is then re-estimated to generate a second estimate of (θ). This iterative process continues until optimal, convergent parameter estimates are achieved.

3 Simulation Study

This section presents data generated using the initial parameter values of the weighted exponential regression model, as outlined in Equation (2.1) and Table (1). Under this assumption, a random error term follows a normal distribution with a mean close to zero and a variance of 0.5. Furthermore, four distinct time series were constructed to represent the three sectors, collectively forming the panel data for a simulation of actual monthly monkeypox infection rates for the Americas, Africa, and Europe during the period from 2022 to 2023.

Table 1: Initial values for the weighted exponential regression model.

Parameters		
n	b_0	b_1
15	1275	7.003
30	730.3	0.5574
45	0.573	0.0796
60	1.5	-0.0787

The implementation steps are outlined in the following algorithm:

1. Determine the sample size, and n=5,10,15,or 20 for generating data of each sector, and maximum number of iterations.

2. Determine the number of sectors $d = 3$.
3. Calculating the number of observations for all sectors $N = n \times d$.
4. Assuming initial values for the two model parameters based on the total number of observations, as shown in Table (1).
5. Assuming that the time values for the weighted exponential model take values from 1 to n.
6. Generating the random error ε as a normal distribution assuming $\sigma = 0.5$ and mean for each sector (0.04, 0.3, 0.6) respectively to ensure data diversity.
7. Based on random error and the weighted exponential regression model, the generated y values were calculated, which approximate real data about monthly monkeypox infection rates for the America, Africa, and Europe regions for the period 2022–2023.
8. Estimation is performed using the maximum likelihood method based on the FIXED model.
9. Estimation is performed using the Bayesian method based on the FIXED model.
10. Estimation is performed using the maximum likelihood method based on the RANDOM model.
11. Estimation is performed using the Bayesian method based on the RANDOM model.
12. The criterion $\left|(b_{Estimate} - b_{Initial})/b_{Initial}\right| < 0.0001$ was adopted as the stopping condition for Bayesian estimation, while for the maximum likelihood method, fitnlm function was used, which includes a convergence criterion where the error is less than 0.000001.
13. Re-estimation according to steps 8–12.
14. 14. Calculating the average of the estimated parameters according to both methods in the case of the FIXED and RANDOM model.
15. The comparison of methods using the MAPE criterion.

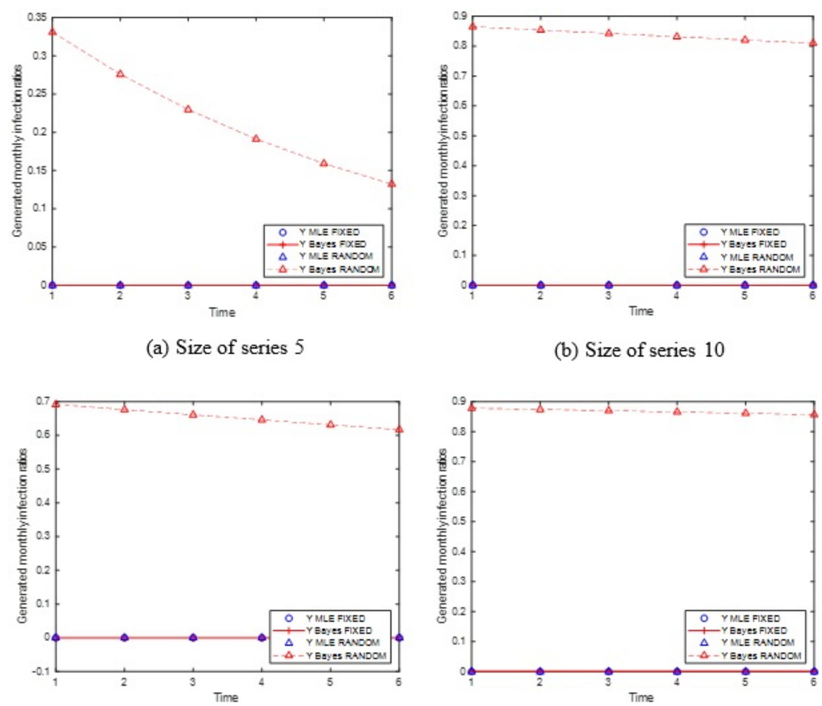
The Mean Absolute Percentage Error ($MAPE$) criterion is adopted to evaluate various strategies for estimating model parameters. Table (2) delineates the simulation outcomes of two methodologies (MLE and $Bayes$) including fixed and random factors.

Table (2) shows that the results support the use of the Bayesian method for random effects models, as it outperforms all other methods across the generated datasets, evidenced by the lowest MAPE. These findings affirm that the random effects model is optimal within the context of the weighted exponential regression model, as demonstrated by Obead et al. (2024). Moreover, the data did not exhibit a consistent trend regarding the size of the time series, whether increasing or decreasing. The model proves to be specific and robust, remaining unaffected by variations in sample size.

Figure (1) illustrates the projected infection rates for the next six months, based on the magnitude of the generated time series. Additionally, it shows a significant reduction in infection rates when

Table 2: Comparison of simulation results according to the MAPE criterion.

	MLE		*Bayes*	
n	Fixed effect	Random effect	Fixed effect	Random effect
15	93.3035	93.3330	92.9291	51.7967
30	96.4348	96.6654	86.6568	80.3608
45	98.1172	98.1172	95.2403	83.1087
60	98.3335	98.3196	98.0381	60.0471



(a) Size of series 5 (b) Size of series 10

(c) Size of series 15 (d) Size of series 20

Figure 1: Predicted monthly infection ratios based on simulated data.

applying the Bayesian method within the context of the random effects model, while the results from other approaches tend toward zero. This serves as a clear indicator of the precision and reliability of the Bayesian method.

4 Real Data and Implementation

According to the World Health Organization, monkeypox is a viral disease caused by the monkeypox virus, characterized by symptoms such as a skin rash, fever, headache, myalgia, lumbar pain, fatigue, and lymphadenopathy. Transmission can occur through human contact, contaminated objects, or infected animals. Laboratory confirmation is achieved through PCR testing. Management of monkeypox typically involves supportive care, and in some cases, vaccinations and treatments authorized for smallpox may be administered (WHO).

Over the past five decades, the monkeypox pandemic, spanning from 1970 to 2018, has been linked to zoonotic transmission. While extensive human-to-human transmission chains have not been identified, significant human-to-human spread occurs under specific conditions, leading to cases in more than 80 countries, predominantly in Europe and the Americas. The disease is primarily transmitted through direct skin contact, including respiratory secretions like mucus, fomites, or direct skin contact (Zeeshan et al., 2022).

This study utilizes data on the monkeypox virus to analyze daily infection rates across three global regions: the Americas, Africa, and Europe. Given the importance of the subject, this research applies a weighted exponential regression model to forecast monthly infection rates in these regions, with initial parameter values estimated at (0.06, 0.04). The monthly infection rates were calculated from the daily viral infection data covering the period from May 2022 to May 2023, as presented in Table (3). Source:Report of WHO.

Table 3: Confirmed monthly infection ratios of Monkeypox for the America, Africa, and Europe Regions for May 2022 to May 2023.

Month	Africa	Europe	America
05/2022	0.0199	0.0305	0.0018
06/2022	0.0997	0.1621	0.0116
07/2022	0.0592	0.3497	0.1065
08/2022	0.1630	0.3426	0.3737
09/2022	0.1249	0.0700	0.2265
10/2022	0.1208	0.0237	0.1492
11/2022	0.0733	0.0078	0.0599
12/2022	0.0733	0.0059	0.0270
01/2023	0.0886	0.0046	0.0212
02/2023	0.0446	0.0008	0.0105
03/2023	0.0223	0.0003	0.0058
04/2023	0.0645	0.0011	0.0029
05/2023	0.0821	0.0008	0.0035

The parameters of the weighted exponential regression model, both in the fixed and random effects frameworks, are estimated using Maximum Likelihood Estimation (MLE) and Bayesian methods. The estimated results are presented in Table (4). To assess the outcomes of these methods, the

Table 4: Estimation of the parameters of weighted exponential model depend on the monthly infection ratios of Monkeypox.

Method	Model	b_1	b_2
MLE	Fixed effect	-0.0335	-0.0110
	Random effect	-0.0333	-0.0110
$Bayes$	Fixed effect	0.3351	0.1273
	Random effect	0.3685	0.1382

Mean Absolute Percentage Error ($MAPE$) was used, as shown in Table (5). The Bayesian method exhibits superior performance in the random effects model, producing the lowest $MAPE$, followed by the Bayesian method in the fixed effects model

Table 5: MAPE of Fixed and Random effective models for the monthly infection ratios of Monkeypox.

Method	Model	$MAPE$
MLE	Fixed effect	4253.7175
	Random effect	4245.5539
$Bayes$	Fixed effect	549.7172
	Random effect	446.5858

The monkeypox virus infection rates were forecasted for the following six months, as shown in Table (6). The data reveals a significant decline in infection rates, dropping to 0.0026. This suggests that the virus has nearly disappeared, with infection rates approaching zero in the targeted regions. These results highlight the effectiveness of the treatments that have been implemented. From Table (6), the infection rates of monkeypox can be seen to have decreased in the following six months, and they have continued to fall further as time approached zero.

Figure (2) shows the decline in monthly virus infection rates based on the Bayesian random-effects model.

Table 6: Prediction the monthly infection ratios of Monkeypox by using Bayesian method.

Month	Fixed effect	Random effect
1	0.0076	0.0049
2	0.0056	0.0035
3	0.0041	0.0025
4	0.0030	0.0017
5	0.0022	0.0012
6	0.0016	0.0009
Mean	0.0040	0.0026

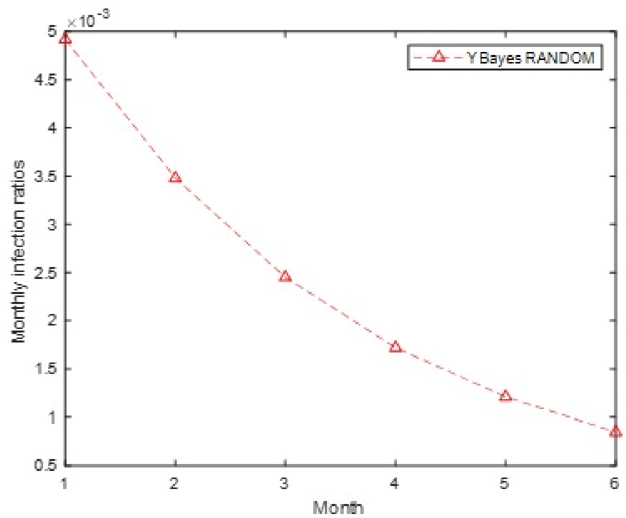


Figure 2: Predicted monthly infection ratios of Monkeypox.

5 Conclusion

The simulation results of the study confirm that the Bayesian method, applied within the random effects model, provided the most accurate parameter estimates for the weighted exponential regression model across all generated datasets. Additionally, the findings showed no consistent trend in the size of the time series, whether increasing or decreasing, demonstrating the model's stability. This stability remains unaffected by sample size, although it is more efficient with smaller samples.

An analysis of actual monthly monkeypox infection rates across the Americas, Africa, and Europe, using a random effects model within a weighted exponential regression framework and estimated via Bayesian methods, reveals a significant decline in infection levels, approaching nearly zero (0.0026). This trend reflects a substantial reduction in the disease's prevalence, largely attributed to effective treatment measures. However, continued vigilance is necessary due to the severity of the disease and its potential for resurgence. This underscores the importance of prioritizing public health policies and allocating resources to address any emergencies that could trigger a resurgence in infections.

Acknowledgements

The authors gratefully acknowledge everyone who helped them to complete this research.

References

Abiola, O. L., Garba, M. K., Adeshola, A. D., and Abayomi, O. S. (2025), "Modelling Dynamic Panel Data Using Hierarchical Bayesian Approach," *International Journal of Science for Global Sustainability (IJSGS)*, 11.

Al-Noor, N. H. and Hussein, L. K. (2018), "Weighted exponential distribution: approximate Bayes estimations with fuzzy data," *Al-Nahrain Journal of Science*, 174–185.

Baltagi, B. H. (2015), *The Oxford handbook of panel data*, Oxford University Press.

Cheruiyot, K. W., Ouko, A., and Kirimi, E. (2016), "Bayesian Inferences for Two Parameter Weibull Distribution," *International Journal of Mathematics and Statistics Studies*, 4, 24–33.

Das, H. K. (2024), "Exploring the Dynamics of Monkeypox Transmission with Data-Driven Methods and a Deterministic Model," *Frontiers in Epidemiology*, 4, 1334964.

Greene, W. H. (2002), "Greene econometric analysis," .

Gu, J. and Yin, G. (2022), "Bayesian SIR Model with Change Points with Application to the Omicron Wave in Singapore," *Scientific Reports*, 12, 20864.

Hausman, J. A. (1978), "Specification tests in econometrics," *Econometrica: Journal of the econometric society*, 1251–1271.

Heuts, S., Kawczynski, M. J., Sayed, A., Urbut, S. M., Albuquerque, A. M., Mandrola, J. M., Kaul, S., Harrell, F. E. J., Gabrio, A., and Brophy, J. M. (2025), "Bayesian Analytical Methods in Cardiovascular Clinical Trials: Why, When, and How," *Canadian Journal of Cardiology*, 41, 30–44.

Kamar, S. H. and Msallam, B. S. (2020), "Comparative study between generalized maximum entropy and Bayes methods to estimate the four parameter weibull growth model," *Journal of Probability and Statistics*, 2020, 7967345.

Kamar, S. H., Obead, H. K., and Shakir, A. M. (2021), "Combine maximum entropy with Fourier series residual to estimate the modified exponential growth model," *Mathematics in Engineering, Science and Aerospace*, 12, 19–28.

Kasilingam, D., Sathiya Prabhakaran, S. P., Rajendran, D. K., Rajagopal, V., Santhosh Kumar, T., and Soundararaj, A. (2021), "Exploring the Growth of COVID-19 Cases Using Exponential Modelling across 42 Countries and Predicting Signs of Early Containment Using Machine Learning," *Transboundary and Emerging Diseases*, 68, 1001–1018.

Leorato, S. and Mezzetti, M. (2016), "Spatial panel data model with error dependence: A Bayesian separable covariance approach," *Bayesian Analysis*, 11, 1035–1069.

Liao, L.-C., Hsu, C.-Y., Chen, H.-H., and Lai, C.-C. (2023), "Estimating the Global Spread of Epidemic Human Monkeypox with Bayesian Directed Acyclic Graphic Model," *Vaccines*, 11, 468.

Morawetz, U. (2006), "Bayesian modelling of panel data with individual effects applied to simulated data," Tech. rep., Diskussionspapier, [Online]. Available: `https://hdl.handle.net/10419/236556`.

Murphy, K. P. (2007), "Conjugate Bayesian Analysis of the Gaussian Distribution," Technical Report, University of British Columbia, `https://www.cse.iitk.ac.in/users/piyush/courses/tpmi_winter19/readings/bayesGauss.pdf`.

Nasiri, P., Makhdoom, I., and Yaghoubian, B. (2011), "Estimation parameters of the weighted exponential distribution," *Australian Journal of Basic and Applied Sciences*, 5, 2007–2014.

Ngungu, M., Addai, E., Adeniji, A., Adam, U. M., and Oshinubi, K. (2023), "Mathematical Epidemiological Modeling and Analysis of Monkeypox Dynamism with Non-Pharmaceutical Intervention Using Real Data from United Kingdom," *Frontiers in Public Health*, 11, 1101436.

Obead, H. K., Kamar, S. H., and Msallam, B. S. (2024), "Estimate the Parameters of the Weighted Exponential Regression Model for Panel Data," *Iraqi Journal of Science*, 65, 2703–2711.

Qian, M., Li, D., Hao, Z., Hu, S., and Li, W. (2025), "An Epidemiological Model of Monkeypox: Model Prediction and Control Application," *BMC Infectious Diseases*, 25, 485.

Riad, F. H., Hussam, E., Gemeay, A. M., Aldallal, R. A., and Afify, A. Z. (2022), "Classical and Bayesian Inference of the Weighted-Exponential Distribution with an Application to Insurance Data," *Mathematical Biosciences and Engineering*, 19, 6551–6581.

Swamy, P. A. V. B. and Arora, S. S. (1972), "The exact finite sample properties of the estimators of coefficients in the error components regression models," *Econometrica: journal of the Econometric Society*, 40, 261–275.

Widiharih, T., Haryatmi, S., et al. (2013), "D-optimal designs for weighted exponential and generalized exponential models," *Applied Mathematical Sciences*, 7, 1067–1079.

Zeeshan, H. M., Rubab, A., Dhlakama, H., Ogunsakin, R. E., and Okpeku, M. (2022), "Global research trends on monkeypox virus: a bibliometric and visualized study," *Tropical Medicine and Infectious Disease*, 7, 402.

Zhang, B. (2020), "Forecasting with Bayesian Grouped random effects in panel data," *SSRN Electronic Journal*.

Álvarez, I. C., Barbero, J., and Zofío, J. L. (2017), "A panel data toolbox for MATLAB," *Journal of Statistical Software*, 76, 1–27.

Received: March 30, 2025

Accepted: November 16, 2025