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Cosmic Acceleration in an Anisotropic Universe with Bulk Viscosity in $f(Q,T)$ Gravity Theory

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Abstract

The $f(Q,T)$ gravity theory has been studied in the context of a spatially homogeneous and anisotropic Bianchi type-VI space-time in the presence of bulk viscous fluid. The field equations are solved explicitly with the help of hyperbolic hybrid scale factor $\mathcal{R} = e^{at}[\tanh(t)]^b$. The non-linear functional forms of $f(Q,T)$ gravity: $f(Q,T) = Q + \alpha Q^2 + \beta T$ where Q and T are non-metricity scalar and trace of energy momentum tensor respectively is considered. Some physical and geometrical properties are calculated and plotted their graphs in terms of time. For the considered model it is found that the coefficient of bulk viscosity appears to be positive and decreases over time. The cosmological behaviour of energy density, effective pressure, Equation of state parameter, and deceleration parameter are quite in good agreement with recent findings of cosmology. The energy conditions of the model are also studied.

Keywords: $f(Q,T)$ gravity; Bulk viscous; Bianchi type-VI; Equation of state (EoS) parameter.

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1. Introduction

According to the Big Bang cosmology, our universe emerged from a singularity and is around 13.8 billion years old. Cosmologists have been drawn to modified gravity theories to comprehend dark energy's role better. Numerous studies show that modified theories of gravity may explain the acceleration of the cosmos in both early and late times. In modified theories of gravity, which are geometrical generalizations of Einstein's general theory of relativity, cosmic acceleration can be achieved by rearranging Einstein-Hilbert action by substituting the curvature scalar R for a more generalized function. This may correspond to a curvature scalar or a different function with matter-geometry coupling. Some extensively used modified gravity theories are $f(R)$ gravity [1-4], $f(R,T)$ gravity [5-8], $f(G)$ gravity [9-11], $f(T)$ gravity [12-15], $f(Q)$ gravity [16-18], $f(Q,T)$ gravity [19].

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Among this geometrically modified theory of gravity, the $f(Q,T)$ gravity proposed by Xu *et al*. [19] has drawn a lot of interest in recent years. Here, Q and T represent the non-metricity and trace of the energy-momentum tensor, respectively. The $f(Q,T)$ gravity is an extension of $f(Q)$ gravity, which is based on the non-minimal coupling between the non-metricity Q and the trace T of the matter-energy momentum tensor. This non-minimal coupling results in the non-conservation of the energy-momentum tensor potentially providing mechanisms for Matter creation or decay in the early or late universe and explaining cosmic acceleration without dark energy. Motivated by this, we utilize the $f(Q,T)$ gravity theory to investigate possible explanations for various cosmological phenomena occurring in the universe. As an extension of symmetric teleparallel gravity theory, the $f(Q,T)$ gravity theory is also constrained by the curvature free and torsion free conditions, i.e., $R^{\rho}{}_{\sigma\mu\nu} = 0$ and $T^{\rho}{}_{\mu\nu} = 0$. However, in a torsionless space, gravity is driven by the non-metricity, which is defined as $Q_{\alpha\mu\nu} = \nabla_{\alpha} g_{\mu\nu}$. Several authors have explored the applications of this theory in many contexts such as Arora *et al*. [20] explored the bulk viscosity in modified $f(Q,T)$ gravity theory within Friedmann-Lemaitre-Robertson-Walker metric (FLRW). Pati *et al*. [21] investigated some rip cosmological models, Shiravand *et al*. [22] studied cosmological inflation by choosing a linear combination of Q and T. Loo *et al*. [23] construct the correct energy-balanced equation with the covariant formulation in the $f(Q,T)$ theory. Gadbail *et al*. [24], Pradhan *et al*. [25], Tayde *et al*. [26] and Narzary and Dewri [27] studied some cosmological models in $f(Q,T)$ gravity in different context and discussed the dynamical aspects of the model.

Cosmological models with bulk viscosity have gained importance in recent years. In this paper, we consider the universe is filled with bulk viscous fluid. Since the Dissipative forces, including Bulk viscosity, play an important role during the early stages of cosmic evolution. Matter behaved like a viscous fluid during the neutrino decoupling in the early phase of Universe [28,29]. Bulk Viscosity develops whenever a fluid expands too quickly and loses thermodynamic equilibrium. It is a measure of the pressure required to restore equilibrium to a compressed or expanding system [30-32]. The total effective negative pressure, resulting in a repulsive gravitational force due to bulk viscosity, counteracts the attractive gravitational pull of matter and provides a driving force for the universe's rapid expansion. Within the field of cosmology, the expansion scalar θ measures how quickly the volume of the fluid is expanding. The pressure resulting from bulk viscosity is directly related to the expansion scalar. Therefore, the universe's expansion itself plays a role in generating effective pressure through viscosity. The coefficient of viscosity is known to decrease as the universe expands [33,34]. Numerous researchers have examined the impact of bulk viscosity on cosmological evolution in different space-time: Tiwari *et al*. [35] investigated Bianchi Type-V cosmological models with time-varying; Mishra *et al*. [36] studied the dynamical behaviour of Bianchi type VI_h universe in $f(R,T)$ gravity. Arora *et al*. [37] investigated late time acceleration with viscosity $\xi = \xi_0 + \xi_1 H + \xi_2 H^2$ and Koussour and Bennai [38] presented cosmological models on a Bianchi type-I space-time in the framework of $f(R,T)$ modified theory. Mete and Dudhe [39] studied FRW cosmological model with bulk viscosity in the context of $f(R)$ gravity. Also, Dewri [40] and Brahma and Dewri [41] studied dark energy with electromagnetic field in different context of modified gravity in different space-time. Basumatary and Dewri [42] explored Bianchi type VI_0 cosmological model in the framework of Sen-Dunn theory of gravitation. Kumawat *et al*. [43] discussed the exact solution of Einstein's field equations for anisotropic Bianchi type VI_0 cosmological model in the framework of the Sáez-Ballester theory of gravitation for barotropic fluid distribution.

Motivated by the above works, this article investigates a cosmological model within a spatially homogeneous and anisotropic Bianchi type VI framework, incorporating the presence of bulk viscosity. This presents a new cosmological model with bulk viscosity in the Bianchi type VI framework, addressing a significant gap in the literature and enhancing our understanding of cosmological dynamics in $f(Q,T)$ gravity.

2. Basic Formalism in $f(\mathbf{Q},\mathbf{T})$ Gravity

The $f(Q,T)$ gravity is constrained with the curvature and torsion-free assumptions, i.e., $R^{\rho}{}_{\sigma\mu\nu} = 0$ and $T^{\rho}{}_{\mu\nu} = 0$. The general action for $f(Q,T)$ gravity [19] is given as,

$$S = \int \sqrt{(-g)}\left(\frac{1}{16\pi} f(Q,T) + \mathcal{L}_m\right) d^4 x \tag{1}$$

where Q stands for the non-metricity scalar, T for the trace of the stress-energy momentum tensor, $\mathcal{L}_m$ for the matter lagrangian and $g \equiv det(g_{\mu\nu})$. Here, the energy-momentum tensor can be defined as, $T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\,\mathcal{L}_m)}{\delta g^{\mu\nu}}$

Further, the non-metricity scalar is defined as

$$Q \equiv -g^{\mu\nu}\left(L^{\alpha}{}_{\beta\mu}L^{\beta}{}_{\nu\alpha} - L^{\alpha}{}_{\beta\alpha}L^{\beta}{}_{\mu\nu}\right) \tag{2}$$

where $L^{\alpha}{}_{\beta\mu}$ is the deformation tensor written as

$$L^{\alpha}{}_{\beta\mu} = -\frac{1}{2}g^{\alpha\lambda}\left(\nabla_{\mu}g_{\beta\lambda} + \nabla_{\beta}g_{\lambda\mu} - \nabla_{\lambda}g_{\mu\beta}\right) = \frac{1}{2}g^{\alpha\lambda}\left(Q_{\mu\beta\lambda} + Q_{\beta\mu\lambda} - Q_{\alpha\beta\mu}\right) \tag{3}$$

As for the non-metricity tensor $Q_{\gamma\mu\nu}$ is expressed as $Q_{\gamma\mu\nu} \equiv \nabla_{\gamma}g_{\mu\nu}$

The superpotential tensor, known as non-metricity conjugate, can be expressed by

$$P^{\alpha}{}_{\mu\nu} = -\frac{1}{2}L^{\alpha}_{\mu\nu} + \frac{1}{4}\left(Q^{\alpha} - \tilde{Q}^{\alpha}\right)g_{\mu\nu} - \frac{1}{4}\delta^{\alpha}{}_{(\mu}Q_{\nu)} \tag{4}$$

Now, varying the gravitational action (1) w.r.t the metric tensor $g_{\mu\nu}$ the corresponding field equations of $f(Q,T)$ gravity is obtained as,

$$-\frac{2}{\sqrt{-g}}\nabla_{\alpha}\left(f_Q\sqrt{-g}\,P^{\alpha}_{\mu\nu}\right) - \frac{1}{2}f g_{\mu\nu} + f_T\left(T_{\mu\nu} + \Theta_{\mu\nu}\right) - f_Q\left(P_{\mu\alpha\beta}Q_{\nu}^{\alpha\beta} - 2Q^{\alpha\beta}{}_{\mu}P_{\alpha\beta\nu}\right) = 8\pi T_{\mu\nu} \tag{5}$$

where $f_Q \equiv \frac{\partial f}{\partial Q}$, $f_T \equiv \frac{\partial f}{\partial T}$.

3. Bianchi Type VI Universe in $f(\mathbf{Q},\mathbf{T})$ Gravity

The anisotropic nature of Bianchi Type spacetime implies expansion rates are direction-dependent, spatial geometry is homogeneous but not isotropic. To explore the limits of isotropy in cosmology and understand how anisotropies might decay or persist in the universe's evolution, we consider the universe is described by Bianchi type VI space-time which is spatially homogeneous and anisotropic, and is given by,

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2mx}dy^2 + C^2e^{2mx}dz^2 \tag{6}$$

where the scale factors A, B and C are the functions of cosmic time t and m is a non-zero constant.

The non-metricity scalar for Bianchi type-VI space-time becomes,

$$Q = 2\left[\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{m^2}{A^2}\right] \tag{7}$$

Here, we consider the universe is filled with bulk viscous fluid. Therefore, the energy-momentum tensor $T_{\mu\nu}$ can be parametrized as,

$$T_{\mu\nu} = (\rho + \bar{p})u_\mu u_\nu + pg_{\mu\nu} \tag{8}$$

where $\bar{p} = p - \xi\theta.$ (9)

Here $\rho, \mathrm{p}, \bar{p}, \xi$ and θ are the energy density, isotropic pressure, bulk viscous pressure, bulk viscosity, and expansion scalar, respectively. The four-velocity vector u^μ is presumed to satisfy $u^\mu u_\mu = -1$. By the definition of $T_{\mu\nu}$, the $\Theta_{\mu\nu}$ can be expressed $\Theta_{\mu\nu} = \bar{p}g_{\mu\nu} - 2T_{\mu\nu}$. For the specification of ξ, we assume that the fluid obeys a linear equation of state $p = \gamma\rho, 0 \le \gamma \le 1$.

The field equations of $f(Q,T)$ gravity (5) for the Bianchi type-VI space-time can be obtained with the help of Eqs. (6) and (8) in Eq. (5) as, [44]

$$\frac{f(Q,T)}{2} - f_Q\left[\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{2\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{2m^2}{A^2}\right] - \dot{f_Q}\left[\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right] = -8\pi\bar{p} \tag{10}$$

$$\frac{f(Q,T)}{2} - f_Q\left[\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{2\dot{A}\dot{C}}{AC}\right] - \dot{f_Q}\left[\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right] = -8\pi\bar{p} \tag{11}$$

$$\frac{f(Q,T)}{2} - f_Q\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC}\right] - \dot{f_Q}\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right] = -8\pi\bar{p} \tag{12}$$

$$\frac{f(Q,T)}{2} - 2f_Q\left[\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC}\right] = 8\pi\rho + 8\pi G(\rho + \bar{p}) \tag{13}$$

$$mf_Q\left[\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right] = 0 \tag{14}$$

the overhead dot represents the derivative with respect to cosmic t. In this case, f_Q and $8\pi G \equiv f_T$ represent differentiation with respect to Q and T respectively.

The following physical parameters were defined as they play crucial role in solving field equations and in cosmological analysis. The spatial volume V and average scale factor $'\mathcal{R}'$ are defined as

$$V = \mathcal{R}^3 = ABC \tag{15}$$

The Hubble parameter is defined as

$$H = \frac{\dot{\mathcal{R}}}{\mathcal{R}} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{16}$$

The deceleration parameter, expansion scalar θ and shear scalar σ^2 are defined as

$$q = -\frac{\mathcal{R}\ddot{\mathcal{R}}}{\dot{\mathcal{R}}^2} = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) \tag{17}$$

$$\theta = 3H \tag{18}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right) \tag{19}$$

The anisotropy parameter A_m of the expansion is characterized by the directional Hubble parameters, and the mean Hubble parameter is given as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{2\sigma^2}{3H^2} \tag{20}$$

where H_1, H_2 and H_3 are directional Hubble parameters in the direction of x, y and $z-axis$, respectively, and $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$.

4. Solutions of Bianchi Type VI Model

The solution of Eq. (14) yields

$$C = kB \tag{21}$$

where $k > 0$ is the constant of integration. Without loss of generality, we take $k = 1$ for the sake of simplicity. Using the value of C in the above Eqs. (10)-(13), we obtain

$$\frac{f(Q,T)}{2} - f_Q\left[\frac{2\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{2\dot{B}^2}{B^2} + \frac{2m^2}{A^2}\right] - 2\dot{f}_Q\frac{\dot{B}}{B} = -8\pi\bar{p} \tag{22}$$

$$\frac{f(Q,T)}{2} - f_Q\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{3\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2}\right] - \dot{f}_Q\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right] = -8\pi\bar{p} \tag{23}$$

$$\frac{f(Q,T)}{2} - 2f_Q\left[\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2}\right] = 8\pi\rho + 8\pi G(\rho + \bar{p}) \tag{24}$$

From Eq. (22) and (23) we get

$$\frac{f(Q,T)}{2} - \frac{f_Q}{2}\left[\frac{\ddot{A}}{A} + \frac{3\ddot{B}}{B} + \frac{5\dot{A}\dot{B}}{AB} + \frac{3\dot{B}^2}{B^2} + \frac{2m^2}{A^2}\right] - \frac{\dot{f}_Q}{2}\left[\frac{3\dot{B}}{B} + \frac{\dot{A}}{A}\right] = -8\pi\bar{p} \tag{25}$$

Therefore, Eq. (24) becomes

$$\frac{f(Q,T)}{2} - \frac{2f_Q}{1+G}\left[\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2}\right] - \frac{f_Q G}{2(1+G)}\left[\frac{\ddot{A}}{A} + \frac{3\ddot{B}}{B} + \frac{5\dot{A}\dot{B}}{AB} + \frac{3\dot{B}^2}{B^2} + \frac{2m^2}{A^2}\right] - \frac{\dot{f}_Q G}{2(1+G)}\left[\frac{3\dot{B}}{B} + \frac{\dot{A}}{A}\right] = 8\pi\rho \tag{26}$$

We only get two independent field equations with five unknown parameters A, B, ρ, p and f. Therefore, the system of Eqs. (25)-(26) is undetermined and supplementary equations relating these parameters are needed to obtain explicit solutions of this system. We assumed that scalar expansion is proportional to the shear scalar. i.e., $\theta \propto \sigma$, which leads to a relation between the metric functions as follows

$$A = B^n \tag{27}$$

where $n \neq 1$ is a positive constant. Using this relation in Eqs. (25)-(26), follows that

$$f(Q,T) - f_Q\left[\frac{\dot{B}^2}{B^2}(n^2 + 4n + 3) + \frac{\ddot{B}}{B}(n+3) + \frac{2m^2}{B^{2n}}\right] - \frac{\dot{f}_Q\dot{B}}{B}(n+3) = -16\pi\bar{p} \tag{28}$$

$$f(Q,T)-\frac{2f_Q\dot{B}^2}{(1+G)B^2}(4n+2)-\frac{f_Q G}{(1+G)}\left[\frac{\dot{B}^2}{B^2}(n^2+4n+3)+\frac{\ddot{B}}{B}(n+3)+\frac{2m^2}{B^{2n}}\right]-\frac{\dot{f_Q}\dot{B}G}{(1+G)B}(n+3)=16\pi\rho. \tag{29}$$

In order to attain exact solutions for the energy density and pressure and to investigate the characteristics of a dark energy (DE) model, we consider an assumed dynamics to obtain the dynamically changing equation of state parameter. Here, the physical variation of the scale factor is considered as

$$\mathcal{R}(t)=e^{at}[\tanh(t)]^b \tag{30}$$

Where a and b are positive constants. Various scale factors play distinct roles in the analysis of cosmic dynamics; for instance, the exponential scale factor predominates during the late phase of the universe, while the hybrid scale factor facilitates a transition from early deceleration to late-time cosmic acceleration [45]. In this work, we have utilized a combination of exponential and hyperbolic functions, referred to as the hyperbolic hybrid scale factor. However, limited research has been conducted on this hyperbolic hybrid scale factor in relation to the cosmic dynamics of the universe. Jokweni *et al.* [46] explored locally rotationally symmetric (LRS) Bianchi type-I in general relativity and in $f(R,T)$ gravity and solutions have been found by means of a special Hubble parameter, yielding a hyperbolic hybrid scale factor. Basumatary and Dewri [47] studied a cosmological model in $f(G)$ gravity within a Bianchi type-III space time by considering same hyperbolic hybrid scale factor. Consequently, motivated by the above discussions the scale factor Eq. (30) has been chosen to investigate the cosmic dynamics further. The scale factor is zero at $t=0$ and the model becomes singular at $t=0$, featuring a point-type singularity where the model starts to expand from the Big Bang at $t=0$, as illustrated in Fig. 1.

Using Eqs. (27) and (30) in Eq. (15) we obtained the expressions

$$A(t)=[e^{at}\{\tanh(t)\}^b]^{\frac{3n}{n+2}} \tag{31}$$

$$B(t)=[e^{at}\{\tanh(t)\}^b]^{\frac{3}{n+2}} \tag{32}$$

$$C(t)=[e^{at}\{\tanh(t)\}^b]^{\frac{3}{n+2}}. \tag{33}$$

Now, using Eqs. (31)- (33) in equation (6), we can write the Bianchi type-VI model in the present case as

$$ds^2=-dt^2+[e^{at}\{\tanh(t)\}^b]^{\frac{6n}{n+2}}dx^2+[e^{at}\{\tanh(t)\}^b]^{\frac{6}{n+2}}(e^{-2mx}dy^2+e^{2mx}dz^2) \tag{34}$$

5. Cosmological Models

To investigate viable cosmological scenarios in the framework of $f(Q,T)$ gravity theory, certain assumed forms of the functional $f(Q,T)$ must be considered. In this paper, to get the viable cosmological model, we consider the functional form of $f(Q,T)$ gravity as $f(Q,T)=Q+\alpha Q^2+\beta T$ where $\alpha\neq 0$ *and* β are the free parameters [22].

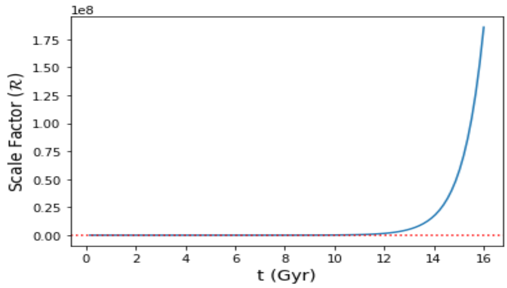

Fig. 1. Scale factor ($\mathcal{R}$) vs. time (Gyr).

5.1. *Model:* $f(Q,T) = Q + \alpha Q^2 + \beta T$

For the non-linear form of the functional in the form $f(Q,T) = f(Q,T) = Q + \alpha Q^2 + \beta T$, we have

$F = \frac{\partial f}{\partial Q} = 1 + 2\alpha Q, \dot{F} = 2\alpha\dot{Q},\ \ Q = \frac{6(n+10)}{n+2}\left(a + \frac{2b}{\sinh(2t)}\right)^2 + 2m^2\{e^{at}(\tanh t)^b\}^{\frac{-6n}{n+2}}$ and $G = \frac{\beta}{8\pi}$ and $G_1 = \frac{G}{1+G}$.

So, the bulk viscous pressure and energy density can be obtained as,

$$\bar{p} = \frac{-1}{32\pi(1+2G)}\left[\begin{array}{c} 2(1+\alpha)Q + (GG_1 - 2 - G)\left\{(n+3\}\left(\frac{F\ddot{B}}{B} + \frac{\dot{F}\dot{B}}{B}\right) + (n^2+4n+3)F\left(\frac{\dot{B}}{B}\right)^2 + \frac{2Fm^2}{B^{2n}}\right\} \\ +2F(4n+2)G_1\left(\frac{\dot{B}}{B}\right)^2 \end{array}\right] \quad (35)$$

$$\rho = \frac{1}{32\pi(1+2G)}\left[\begin{array}{c} 2(1+\alpha)Q + (3G - (2+3G)G_1)\left\{(n+3\}\left(\frac{F\ddot{B}}{B} + \frac{\dot{F}\dot{B}}{B}\right) + (n^2+4n+3)F\left(\frac{\dot{B}}{B}\right)^2 + \frac{2Fm^2}{B^{2n}}\right\} \\ -2F(4n+2)\left(\frac{2}{1+G} + 3G_1\right)\left(\frac{\dot{B}}{B}\right)^2 \end{array}\right] \quad (36)$$

For this choice of model, the bulk viscosity coefficient ξ and the isotropic pressure p can be obtained as

$$\xi = \frac{1}{96\pi(1+2G)H}[2(1+\alpha)\,Q(\gamma-1) + \{(3\gamma-1)G - (2\gamma+3\gamma G - G)G_1 - 2\}\left\{(n+3\}\left(\frac{F\ddot{B}}{B} + \frac{\dot{F}\dot{B}}{B}\right) + (n^2+4n+3)F\left(\frac{\dot{B}}{B}\right)^2 + \frac{2Fm^2}{B^{2n}}\right\} - 2F(4n+2)\left(\frac{2\gamma}{1+G} + (1+3\gamma)G_1\right)\left(\frac{\dot{B}}{B}\right)^2, \quad (37)$$

$$p = \gamma\rho = \frac{\gamma}{32\pi(1+2G)}[2(1+\alpha)\,Q + (3G - (2+3G)G_1)\left\{(n+3\}\left(\frac{F\ddot{B}}{B} + \frac{\dot{F}\dot{B}}{B}\right) + (n^2+4n+3)F\left(\frac{\dot{B}}{B}\right)^2 + \frac{2Fm^2}{B^{2n}}\right\} - 2F(4n+2)\left(\frac{2}{1+G} + 3G_1\right)\left(\frac{\dot{B}}{B}\right)^2. \quad (38)$$

Consequently, the equation of state parameter can be obtained from the above expressions of pressure and density as,

$$\omega = -\frac{\left[2(1+\alpha)Q + (GG_1-2-G)\left\{(n+3\}\left(\frac{F\ddot{B}}{B}+\frac{\dot{F}\dot{B}}{B}\right) + (n^2+4n+3)F\left(\frac{\dot{B}}{B}\right)^2 + \frac{2Fm^2}{B^{2n}}\right\} + 2F(4n+2)G_1\left(\frac{\dot{B}}{B}\right)^2\right]}{\left[2(1+\alpha)Q + (3G-(2+3G)G_1)\left\{(n+3\}\left(\frac{F\ddot{B}}{B}+\frac{\dot{F}\dot{B}}{B}\right) + (n^2+4n+3)F\left(\frac{\dot{B}}{B}\right)^2 + \frac{2Fm^2}{B^{2n}}\right\} - 2F(4n+2)\left(\frac{2}{1+G}+3G_1\right)\left(\frac{\dot{B}}{B}\right)^2\right]}. \quad (39)$$

The values of $B, \dot{B}$ and $\ddot{B}$ are $B = [e^{at}\{\tanh(t)\}^b]^{\frac{3}{n+2}}$, $\dot{B} = \frac{3\,B}{n+2}\left(a + \frac{2b}{\sinh 2t}\right)$ and $\ddot{B} = \frac{9B}{(n+2)^2}\left(a + \frac{2b}{\sinh 2t}\right)^2 - \frac{3B}{n+2}\left(\frac{4b\cosh 2t}{(\sinh 2t)^2}\right)$

which can be substituted in the Eqs. (35)- (39) to get the respective expressions for effective pressure, energy density, coefficient of bulk viscosity, isotropic pressure and equation of state (EoS) parameter. The graphical behaviour of the effective pressure for the model is shown in Fig. 2. It can be seen that the effective pressure $(\overline{p})$ starts from the high negative value and has settled down to near zero. At the same time, the energy density ρ evolves from a high positive value to a small positive value in the present and late times. It remains in the positive domain during the whole cosmic evolutionary process, which can be seen from Fig. 2. Fig. 3 shows the positively decreasing behaviour of p for the considered model. Therefore, the presented model describes the evolution of the universe in a way that is consistent with present-day accelerating universe. From the Fig. 4, it can be observed that the coefficient of bulk viscous (ξ) is a decreasing function of time, indicating viscous effects were present near Big Bang singularity and remained in the positive domain throughout the cosmic evolution. The evolution trajectory of the EoS parameter (ω) is shown in the Fig. 5. From the graphical representation, it can be seen that the EoS parameter is in the negative domain, i.e., in the region $-1 \leq \omega < 0$. The numerical value of EoS parameter ω is constrained by several cosmological observations such as Supernovae Cosmology Project [48], $\omega = -1.035^{+0.055}_{-0.079}$; observations of the Cosmic Microwave Background radiation obtained by the Wilkinson Microwave Anisotropy Probe satellite (WMAP+CMB) [49], $\omega = -1.073^{\{+0.090\}}_{\{-0.089\}}$; Planck 2018 [50], $\omega = -1.03 \pm 0.03$. The present value of EoS parameter ω for this model corresponds to the parameters of the model, $\omega_0 = -0.9205$, which is in agreement with cosmological observations. Thus, we can conclude that the behaviour of EoS parameter favours a quintessence evolutionary phase.

Fig. 6 illustrates the squared sound stability of the model with respect to time. In the universe, there are three different kinds of particles, i.e., sub-luminal, luminal, and super-luminal. The sub-luminal particles move relatively slowly in comparison to the speed of light, while the luminal particles travel at the same speed as the speed of light. In contrast, the super-luminal particles move faster than the speed of light. Super-luminal particles could either not exist at all, or if they do, they do not interact with ordinary matter. When the speed of sound is less than the local light speed, $C^2_{s(t)} \leq 1$, we can conclude about the non-violation of causality. The positive sound speed $\left(C^2_{s(t)} > 1\right)$ is necessary for the classical stability of the universe [51,52]. From Fig. 6, it can be observed that the squared sound speed is less than -1 in the present and late time cosmic evolution, the model remains unstable with the expansion of the Universe.

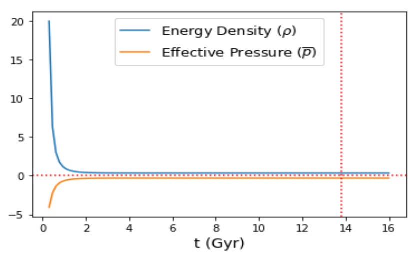

Fig. 2. Energy Density (ρ) and Effective Pressure $(\bar{p})$ vs. time (Gyr).

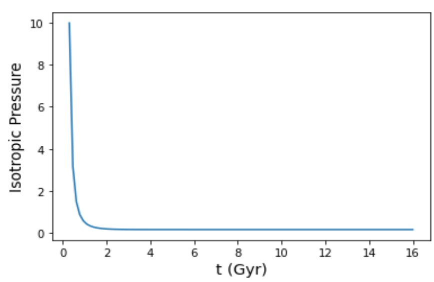

Fig. 3. Isotropic Pressure (p) vs. time (Gyr).

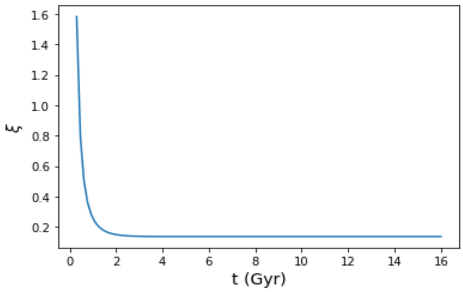

Fig. 4. Coefficient of Bulk Viscous (ξ) vs. time (Gyr).

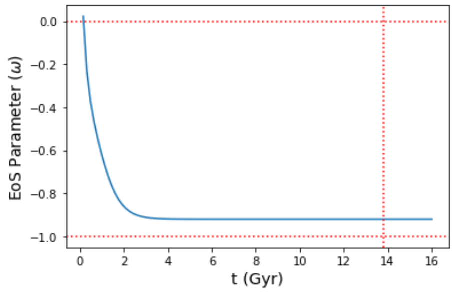

Fig. 5. Equation of state parameter (ω) vs. time (Gyr).

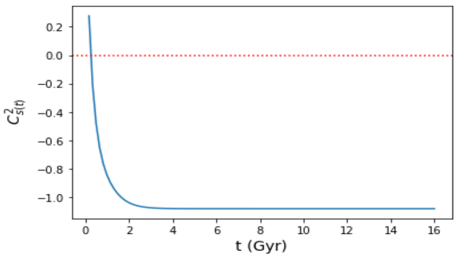

Fig. 6. $C^2_{s(t)}$ vs. time (Gyr).

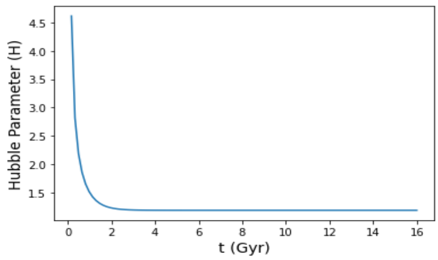

Fig. 7. Hubble Parameter (H) vs. time (Gyr).

6. Physical Behavior of the Model

The physical quantities of observational interests in cosmology such as Hubble parameter (H), Deceleration parameter (q), Spatial volume of scale factor (V), scalar of expansion (θ), Shear scalar $(\sigma)$and mean anisotropy parameter (A_m) are given in the following:
The Hubble parameter can be written as

$$\mathrm{H} = \mathrm{a} + \frac{2\mathrm{b}}{\sinh(2\mathrm{t})}. \tag{40}$$

The deceleration parameter can be obtained as

$$q = -1 + \frac{4b\cosh(2t)}{(a\sinh(2t) + 2b)^2}. \tag{41}$$

The spatial volume, expansion scalar, shear scalar and anisotropy parameter of the model are given respectively,

$$V = [e^{at}\{\tanh(t)\}^b]^3\,, \theta = 3\left(a + \frac{2b}{\sinh(2t)}\right),\ \sigma^2 = 3\left[a + \frac{2b}{\sinh(2t)}\right]^2\left(\frac{n-1}{n+2}\right)^2$$

and $\quad A_m = 2\left(\frac{n-1}{n+2}\right)^2.$ (42)

The graphical representation of the Hubble parameter is shown in Fig. 7, and the Volume, Shear scalar, and Expansion scalar of the cosmological model is shown in Fig. 8. It can be observed that the Hubble parameter (H),and σ parameters take large positive values at $t = 0$ and take the small positive values as $t \to \infty$. The volume is directly proportional to time, which can be seen from Fig. 8. Also, it can be observed from Eq. (42)

that spatial volume (V) is zero at $t = 0$ while the expansion scalar θ is infinite. This suggests that the universe starts evolving with zero volume at $t = 0$, which is a Big Bang scenario. The mean anisotropy $A_m \neq 0$, the model does not approach isotropy for $n \neq 1$. In other words, as long as $n \neq 1$, our model is direction-dependent. However, for $n = 1$, there is no shear indicating that the model is a isotropy for all t. The signature of the deceleration parameter describes the acceleration or deceleration Universe, i.e., when $q > 0$, then the expansion phase of the universe is decelerating; when $-1 \leq q < 0$, then the expansion phase of the universe is exponential expansion (for $q = -1$ is known as de-sitter expansion) and when $q < -1$ expansion phase of the universe is super exponential. For the model, the behaviour of the deceleration parameter from Fig. 9 depicted that the universe is transitioning from the deceleration phase to the acceleration phase and the present value of q, $q_0 = -1$. Therefore, the expansion phase of our model Universe is de-sitter expansion.

7. Energy Conditions and Statefinder Parameter

In this work, we consider energy conditions to test the validity of the models in the context of cosmic acceleration. There are several forms of energy conditions, such as null energy conditions (NEC), weak energy conditions (WEC), strong energy conditions (SEC), and dominant energy conditions (DEC) are given for the content of the universe in the form of a viscous fluid in $f(Q,T)$ gravity as follows [53]:

i. Null energy conditions (NEC) $\Leftrightarrow \rho + \bar{p} \geq 0$,
ii. eak energy conditions (WEC) $\Leftrightarrow \rho + \bar{p} \geq 0$ and $\rho \geq 0$,
iii. Strong energy conditions (SEC) $\Leftrightarrow \rho + 3\bar{p} \geq 0$, and
iv. Dominant energy conditions (DEC) $\Leftrightarrow \rho - |\bar{p}| \geq 0$ and $\rho \geq 0$,

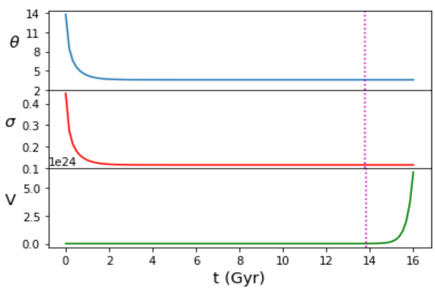

Fig. 8. Volume, Shear scalar, and Expansion scalar vs. time.

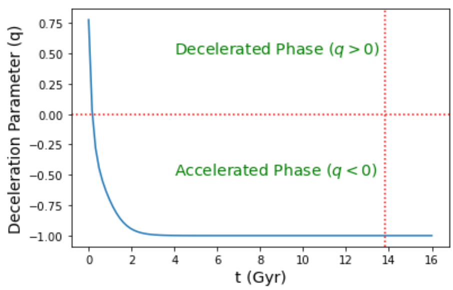

Fig. 9. Deceleration Parameter (q) vs. time.

The graphical representation of energy conditions is shown in Fig. 10. For the considered model, it can be seen that WEC, NEC, and DEC are obeying as they remain in the positive domain while SEC remains in the positive domain in early cosmic evolution. However, as the universe evolves, the SEC evolves into a negative domain. The violation of SEC illustrates the accelerating behaviour of the model.

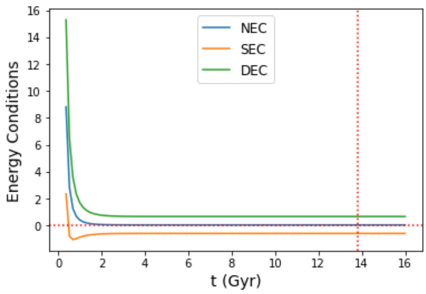

Fig. 10. Energy Conditions vs. time.

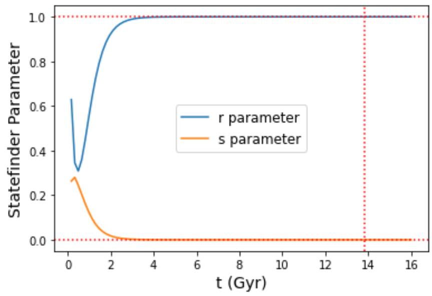

Fig. 11. Statefinder Parameter vs. time.

The Statefinder is a geometrical diagnostic and allows us to characterize the properties of dark energy in an independent manner. Sahini *et al*. [54] introduced the diagnostic parameter (r, s) is called statefinder parameter which is defined as

$$r = \frac{\dddot{R}}{RH^3} = 1 + 3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3}, s = \frac{r-1}{3\left(q - \frac{1}{2}\right)} \tag{43}$$

The statefinder parameter pair (r, s) of the concerned model is calculated to demonstrate the behaviour of the model with bulk viscosity, which is obtained as

$$r = 1 - \frac{12b \cosh 2t}{(a \sinh 2t + 2b)^2} + \frac{16b(\cosh 2t)^2 - 8b(\sinh 2t)^2}{(a \sinh 2t + 2b)^3} \tag{44}$$

$$s = \frac{-12b \cosh 2t + (a \sinh 2t + 2b)^2 + \frac{(16b \cosh 2t)^2 - 8b(\sinh 2t)^2}{(a \sinh 2t + 2b)^3}}{12b \cosh 2t - \frac{9}{2}(a \sinh 2t + 2b)} \tag{45}$$

By using Eqs. (44) and (45), the graph of statefinder parameter r, s is shown in the Fig. 11. It can be observed that the trajectory starts evolving from the region $r < 1, s > 0$, which represents the quintessence model of dark energy. Eventually, it approaches $(r, s) \rightarrow (1,0)$ w.r.t. time which represents the ΛCDM model [55]. However, the model behaves like ΛCDM model at present and in the late time cosmic evolution.

8. Conclusion

In the present work, the Bianchi type-VI cosmological model was investigated in the presence of bulk viscosity within the framework of $f(Q, T)$ gravity. We considered the non-linear $f(Q, T)$ function as $f(Q, T) = Q + \alpha Q^2 + \beta T$ where $\alpha \neq 0$ and β are the free parameters. With the help of hyperbolic hybrid scale factor as $\mathcal{R}(t) = e^{at}[\tanh(t)]^b$ the field equations have been precisely solved. The parameters of the model are chosen as $\alpha = -0.102, \beta = 2, \gamma = 0.5, a = 1.19, b = 0.563, m = 1$ and $n = 1.658$ to provide a physically acceptable energy density. Based on these data, it has been found that the Hubble parameter, expansion scalar, and shear scalar decrease positively as time tends to ∞, and the volume is found to be directly proportional to time. It has been observed that the deceleration parameter of our model Universe shows the signature flipping behaviour, i.e., it shows positive in the early phase and negative for the present and late time universe. From the graph the energy density is observed to be a decreasing function of time and remains in the positive domain. For the bulk viscous pressure, it is negative throughout the cosmic evolution. The negativity of $\bar{p}$ ensures an accelerating universe at the present epoch. In the absence of bulk viscosity, the pressure remains as $\bar{p} = p$ since $\xi = 0$, lacking any extra damping or acceleration terms associated with viscosity. This simplifies the system and reduces its dissipative characteristics. The behaviour of the bulk viscosity coefficient (ξ) is a decreasing function of time, and it is consistent with thermodynamics. Further, the behaviour of the EoS parameter for the considered model depicts the quintessence model in present and in late time cosmic evolution as $\omega \equiv -1$. From the energy conditions, it can be concluded that the violation of SEC depicts the accelerating behaviour of the model. The physical and geometrical behaviour discussed in this paper with hyperbolic hybrid scale factor in the aid of bulk viscous unveiled intriguing dynamics and the solutions given in this work may be useful for a better understanding of the characteristics of the Bianchi type-VI model in $f(Q,T)$ gravity.

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