Publications

Available Online

J. Sci. Res. **17** (3), 727-736 (2025)

JOURNAL OF SCIENTIFIC RESEARCH

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Heat and Mass Transfer Effects on Flow Past Parabolic Started Vertical Plate with Chemical Reaction and Thermal Radiation

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Received 20 September 2024, accepted in final revised form 12 March 2025

Abstract

In this paper an attempt has been made to study the unsteady flow past a parabolic starting motion of the infinite vertical plate, in the presence of heat source parameter, thermal radiation and chemical reaction. The plate temperature and concentration level near the plate raised uniformly. The non-dimensional governing equations are solved with the help of Finite Difference method. The effects of radiation, heat source parameter, chemical reaction, Schmidt number, Prandtl number on the characteristics of the flow are displayed graphically and the variation in skin friction, Nusselt number and Sherwood number are shown in table and the results are discussed. The result indicates that the rising values of thermal radiation and chemical reaction parameter decrease the fluid velocity. It is also found that temperature decreased under the effect of the radiation parameter whereas chemical reaction contributes to decrease the concentration profile.

Keywords: Heat and mass transfer; Parabolic starting motion; Heat flux, Thermal radiation.

doi: https://dx.doi.org/10.3329/jsr.v17i3.76213 J. Sci. Res. **17** (3), 727-736 (2025)

1. Introduction

The study on heat and mass transfer effects on flow past parabolic motion has received a considerable amount of attention in recent years. It is important to study parabolic flow because it helps to reduce energy losses in fluid flow by reducing viscous interactions between near layers of fluid and the pipe wall. Compared to unstable flow, parabolic flow is easier to model and compute which includes the fluid's random and unpredictable motion. Theoretical study of heat transfer effects on flow past a parabolic started vertical plate in the presence of chemical reaction of first order investigated by Muthucumaraswamy and Velmurugan [1]. Muralidharan and Rajamanickam discussed parabolic started flow past an infinite vertical plate with uniform heat flux and variable mass diffusion [2]. Das *et al.* [3] studied unsteady free convection flow past a vertical plate with heat and mass fluxes in presence of thermal radiation. Selvaraj *et al.* [4] discussed on Mhd-parabolic flow past an accelerated isothermal vertical plate with heat and mass diffusion in the presence of rotation.

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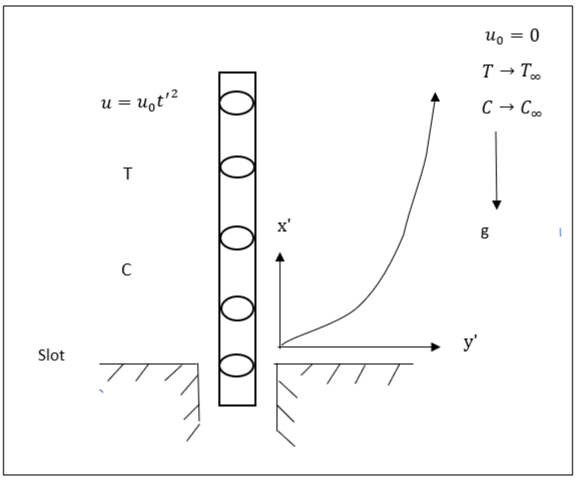
Armstrong *et al.* [5] studied on MHD flow past a parabolic started vertical plate with variable temperature and mass diffusion. Jose *et al.* [6] discussed heat and mass transfer effects on parabolic flow past an accelerated isothermal vertical plate in the presence of chemical reaction and Hall current. Visalakshi and Vasanthabhavam [7] studied heat and mass transfer effects on flow parabolic started vertical plate with constant heat flux. Agrawal *et al.* [8] discussed the effects of heat and mass transfer past a parabolic started infinite vertical plate. Muthucumaraswamy and Geeta [9] studied the effects of parabolic motion on an isothermal vertical plate with constant mass flux. Muthucumaraswamy and Velmurugan [10] discussed heat and mass transfer effects on flow parabolic starting motion of the infinite isothermal vertical plate with uniform mass diffusion, in presence of a homogeneous chemical reaction of the first order. Recently Nath *et al*. [11] discussed on the effect of thermal stratification on Unsteady Parabolic Flow past an infinite vertical plate with chemical reaction. But they neglected thermal radiation and heat source parameter in their discussions.

As far our knowledge, recently enough research has not been made on the effects of heat and mass transfer on unsteady flow past a parabolic starting motion of an infinite vertical plate There are many applications for parabolic motion such as solar cooker, solar concentrators, and parabolic trough solar collector. Motivated by the numerous applications, this work investigates the effects of thermal radiation, heat source parameter, chemical reaction on the unsteady flow past a parabolic starting motion of an infinite vertical plate.

2. Mathematical Formulation

The unsteady flow of a viscous incompressible fluid past an infinite vertical plate with uniform diffusion, in the presence of heat source and chemical reaction have been considered. The x' -axis is taken along the plate in the vertically upward direction and the y'-axis is taken normal to the plate.



Physical model of the problem

The plate and fluid are at the same temperature T_∞ and concentration C_∞, at time $t' \leq 0$. At time $t' > 0$, the plate is started with a velocity $u = u_0 t'^2$ in its own plane against gravitational field. The temperature from the plate is raised to T_w and the concentration level near the plate also raised to C_w.
The equations governing the fluid flow are as follows [10]:

$$\frac{\partial u}{\partial t'} = \vartheta \frac{\partial^2 u}{\partial y'^2} + g\beta(T - T_\infty) + g\beta_c(C - C_\infty) \tag{2.1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = \kappa \frac{\partial^2 T}{\partial y'^2} - \frac{\partial q'}{\partial y} - Q_0(T - T_\infty) \tag{2.2}$$

$$\frac{\partial C}{\partial t'} = D_M \frac{\partial^2 C}{\partial y'^2} - K_1(C - C_\infty) \tag{2.3}$$

The boundary conditions are.

$$\begin{aligned} &u = 0:\ T = T_\infty ,C = C_\infty\ for\ all\ y', t' \leq 0 \\ &t' > 0: u = u_0 t'^2, \mathrm{T} = T_w,\ ,C = C_w \quad at\ y' = 0 \\ &u \to 0:\ T \to T_\infty ,C \to C_\infty\ as\ y' \to \infty \end{aligned} \tag{2.4}$$

The radiative heat flux q′ is given by

$$\mathrm{q}' = -\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y'},$$

where σ is the Stefan-Boltzmann constant and k^* is the Rosseland mean absorption coefficient.

As it is considered that the temperature difference within the flow is sufficiently small, so that T^4 can be expressed in Taylor series about T_∞ and neglecting higher order terms,

$$T^4 = 4T_\infty{}^3 T - 3T_\infty{}^4$$

In order to make the mathematical model normalize, we introduce the following non-dimensional quantities [10].

$$U = u(\frac{u_0}{\vartheta^2})^{\frac{1}{3}}, t = (\frac{u_0{}^2}{\vartheta})^{\frac{1}{3}} t', Y = y'(\frac{u_0}{\vartheta^2})^{\frac{1}{3}}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \emptyset = \frac{C - C_\infty}{C_w - C_\infty}$$

$$Gr = \frac{g\beta_c(T - T_\infty)}{(\vartheta u_0)^{\frac{1}{3}}},\ Gc = \frac{g\beta_c(C_w - C_\infty)}{(\vartheta u_0)^{\frac{1}{3}}}\ \mathrm{Sc} = \frac{\vartheta}{D_M}, K = K_1\left(\frac{\vartheta}{u_0{}^2}\right)^{\frac{1}{3}}, Pr = \frac{1}{\kappa}\vartheta\rho C_p,$$

$$N = \frac{16\sigma T_\infty{}^3}{3k^*\kappa}, Q = Q_0\frac{1}{\rho C_p}(\frac{\vartheta^2}{u_0{}^2})^{\frac{1}{3}} \tag{2.5}$$

Using the above non-dimensional quantities, the equations (2.1)-(2.3) in non-dimensional form are:

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial Y^2} + Gr\theta + Gc\varphi \tag{2.6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{N\theta}{Pr} - Q\theta \tag{2.7}$$

$$\frac{\partial \emptyset}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \emptyset}{\partial Y^2} - K\emptyset \tag{2.8}$$

The corresponding initial and boundary conditions in dimensionless form are as follows:

$$\begin{aligned} &U = 0:\ \theta = 0, \emptyset = 0 for\ all\ Y, t \leq 0 \\ &t > 0: U = t^2, \theta = 1, \emptyset = 1\ at\ Y = 0 \\ &U \to 0:\ \theta \to 0, \emptyset \to 0\ as\ Y \to \infty \end{aligned} \tag{2.9}$$

3. Method of Solution

The finite difference scheme is used to solve the above (2.6)-(2.8) partial differential equations along with the initial and boundary conditions (2.9). The technique given below is followed to convert the set of PDE to set of algebraic equations,

$$\frac{\partial z}{\partial t} = \frac{z(i+1,j) - z(i,j)}{\Delta t}$$

$$\frac{\partial z}{\partial Y} = \frac{z(i,j+1) - z(i,j)}{\Delta Y}$$

$$\frac{\partial^2 z}{\partial Y^2} = \frac{z(i,j+1) - 2z(i,j) + z(i,i-1)}{(\Delta Y)^2}$$

Where z stands for U,θ *and* $\emptyset$; i and j are node locations for t and y directions respectively.

The obtained algebraic equations are,

$$U(i,j+1) = \frac{\Delta t U(i+1,j) + ((\Delta Y)^2 - 2\Delta t)U(i,j) + \Delta t U(i-1,j) + Gr\Delta t(\Delta Y)^2\theta(i,j) + Gc\Delta t(\Delta Y)^2\varphi(i,j)}{(\Delta Y)^2}$$

$$\theta(i,j+1) = \frac{\Delta t\theta(i+1,j) + (Pr(\Delta Y)^2 - 2\Delta t)\theta(i,j) + \Delta t\theta(i-1,j) - N\Delta t(\Delta Y)^2\theta(i,j) - PrQ\Delta t(\Delta Y)^2\theta(i,j)}{Pr(\Delta Y)^2}$$

$$\emptyset(i,j+1) = \frac{\Delta t\emptyset(i+1,j) + (Sc\Delta Y^2 - 2\Delta t)\emptyset(i,j) + \Delta t\emptyset(i-1,j) - ScK\Delta t\Delta Y^2\theta(i,j)}{Sc(\Delta Y)^2}$$

The above algebraic equations from (2.6) - (2.8) with the boundary condition (2.9) were solved in MATLAB.

To observe the physical significance of this study, the changes of the velocity profile, temperature profile and concentration profile with respect to the changes in the parameters, Heat source Parameter (Q), Chemical reaction (K), Radiation Parameter (N), Prandtl number (Pr), Schmidt number (Sc), Grashof number (Gr) and mass Grashof number (Gm) are depicted in graphs in Figs. 1-12. The variation in skin friction, Nusselt number and Sherwood number are shown in Table 1. The values of Sc are chosen water vapor (0.62), ammonia (0.78) and Propyl Benzene (2.62) the values of Pr are chosen as 0.71 and 7.0 which represent air and water. The values of other parameters are chosen arbitrarily.

Figs. 1 and 2 explore the effect of radiation parameter (N) on the velocity and temperature profiles respectively. As shown in Fig. 1, the velocity decreased under the effect of the radiation parameter. The radiation parameter decelerates the fluid velocity. Physically it is true because increasing radiation parameter makes the fluid thick and causes the velocity and the momentum boundary layer thickness to decrease

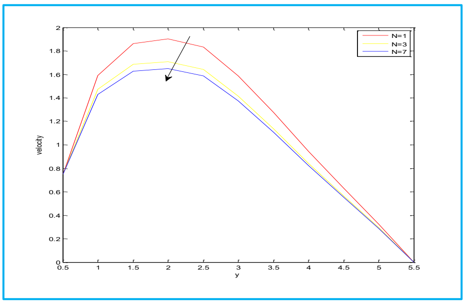


Fig. 1. Velocity profile for different values of N, Gr=5, Gm=6, Pr=0.71, Q=1, Sc=0.62.

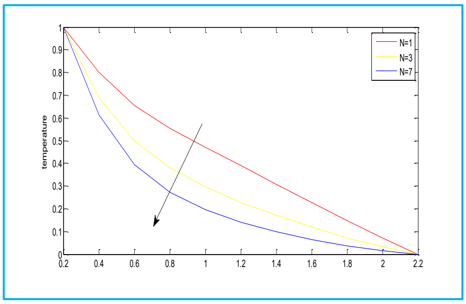


Fig. 2. Temperature profile for different values of N when Gr=5, Gm=6, Pr=0.71, Q=1, Sc=0.62.

In the presence of radiation, thermal boundary layer always found to thicken which shows that radiation provides an additional means to diffuse energy. That means that thermal boundary layer decreases and more uniform temperature distribution across the boundary layer.

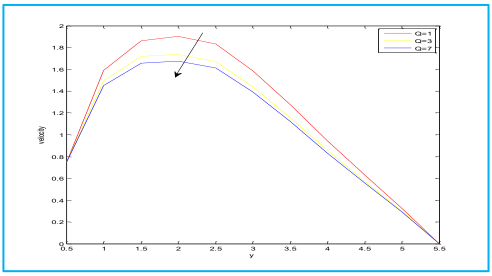


Fig. 3. Velocity profile for different values of Q when Gr=5, Gm=6, Pr=0.71, Sc=0.62, N=1.

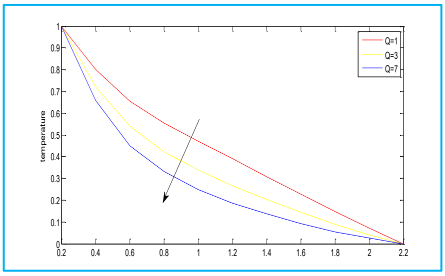


Fig. 4. Temperature profile for different values of Q when Gr=5, Gm=6, Pr=0.71, Sc=0.62, N=1.

The effect of heat generation parameters on the velocity and temperature field are shown in Figs. 3 and 4. It is noticed that the velocity and temperature decrease with the increase of Heat source parameter Q.

For different values of Pr, the velocity and temperature profiles are shown in Figs. 5 and 6. An increase in Prandtl number is led to a decrease in the velocity and temperature profile. It is physically true. As Prandtl number describes the ratio between momentum diffusivity and thermal diffusivity, Pr controls the relative thickening of the momentum and thermal boundary layers.

Again, as Pr increases, the viscous forces dominate the thermal diffusivity and hence decreases the velocity. In Fig. 6, the temperature decreased under the effect of the Pr. Because smaller values of Prandtl number equate to an increase in the thermal conductivity of the fluid, therefore, heat is able to diffuse away from the heated surface more rapidly for higher values of Prandtl number. Therefore, in case of smaller Prandtl numbers, the thermal boundary layer is thicker, and the heat transfer is reduced. So, the fluid temperature decreases with an increase in Pr.

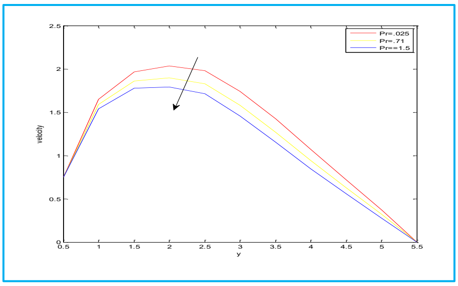


Fig. 5. Velocity profile for different values of Pr when Gr=5, Gm=6, Q=1, Sc=0.62, N=1.

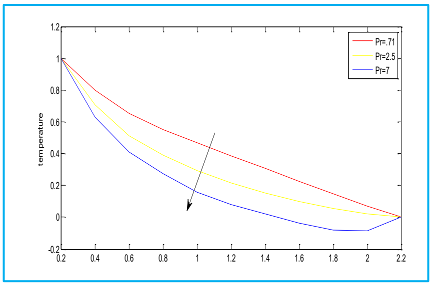


Fig. 6. Temperature profile for different values of Pr when Gr=5, Gm=6, Q=1, Sc=0.62, N=1.

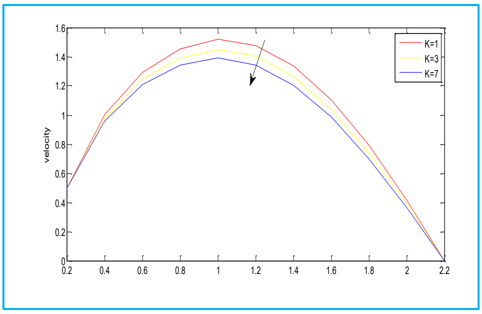


Fig. 7. Velocity profile for different values of K when Gr=5, Gm=6, Pr=0.71, Q=1, Sc=0.62, N=1.

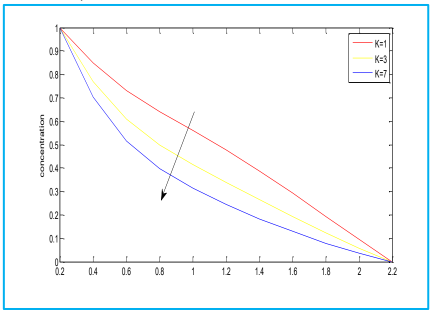


Fig. 8. concentration profile for different values of K when Gr=5, Gm=6, Pr=0.71, Q=1, Sc=0.62, N=1.

The influence of Chemical reaction parameters on velocity and concentration profiles are shown in Figs. 7 and 8. It is noticed that the velocity and concentration decrease with the increased value of chemical reaction parameter K.

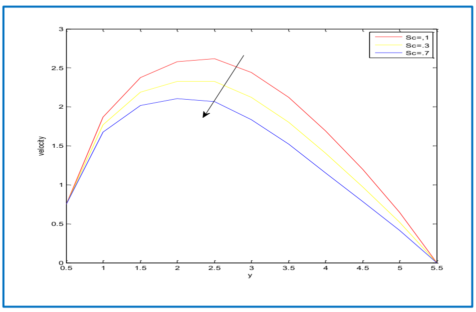


Fig. 9. Velocity profile for different values of Sc when Gr=5, Gm=6, Pr=0.71, Q=1, N=1, K=0.5.

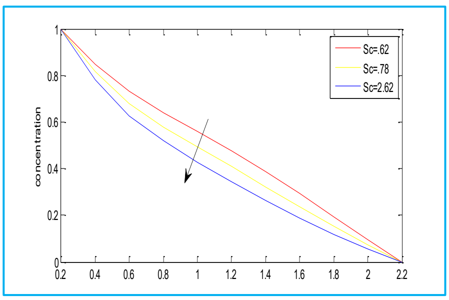


Fig. 10. Concentration profile for different values of Sc when Gr=5, Gm=6, Pr=0.71, Q=1, N=1.

It is observed from Fig. 9 that the velocity decreased with the increase of Schmidt Number. Physically, it is true as because the increased value of Sc decreases chemical species molecular diffusivity, which reduces the velocity of the fluid. Fig 10 illustrates the effect of Schmidt number on concentration. It is noticed that there is a decreasing trend in the concentration with an increase in the Schmidt number.

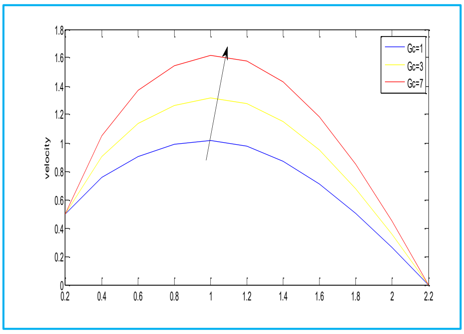


Fig. 11. Velocity profile for different values of Gr when Gm=6, Pr=0.71, Q=1, Sc=0.62, N=1.

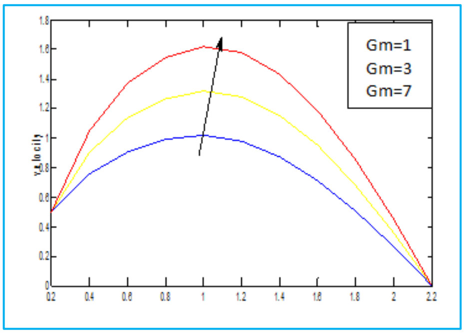


Fig. 12. Velocity profile for different values of Gm when Gr=5,Pr=.71,Q=1, Sc=.62, N=1.

From Figs. 11 and 12, it is observed that the velocity increases with the increase of thermal and mass Grashof numbers Gr and Gm. Grashof number describes the buoyancy forces to viscous forces. Increasing this parameter leads to an increase in the thermal and mass buoyancy forces. Hence the velocity increases with the increase of this parameter. Physically, Gr>0 means heating of the fluid of cooling boundary surface and Gm>0 means the chemical species concentration in the free stream region is less the concentration at the boundary surface.

The changes of skin friction, Nusselt number and Sherwood number along with the increasing values of N, Q, Pr, K, Sc, Gr and Gm are shown in Table1.

Table 1. Numerical values of skin friction, Nusselt number and Sherwood number for different values of N, Q, Pr, K, Sc, Gm and Gr.

N	Q	Pr	K	Sc	Gr	Gm	c_f	Nu	Sh
1	1	0.71	1	1	1	1	2.096067	- 0.924891	-0.670930
3	1	0.71	1	1	1	1	1.943897	-1.508682	-0.670930
7	1	0.71	1	1	1	1	1.825684	-1.930130	-0.670930
1	1	0.71	1	1	1	1	2.096067	-0.924891	-0.670930
1	3	0.71	1	1	1	1	1.985428	-1.354115	-0.670930
1	7	0.71	1	1	1	1	1.893170	-1.692805	-0.670930
1	1	0.71	1	1	1	1	2.095768	-0.925926	-0.670930
1	1	2.5	1	1	1	1	2.010864	-1.242444	-0.670930
1	1	7	1	1	1	1	1.900221	-1.718533	-0.670930
1	1	0.71	1	1	1	1	2.096067	-0.924891 -	-0.670930
1	1	0.71	3	1	1	1	1.966110	0.924891	-1.105042
1	1	0.71	7	1	1	1	1.856143	-0.924891	-1.453132
1	1	0.71	1	0.62	1	1	2.096067	-0.924891	-0.670930

1	1	0.71	1	0.78	1	1	2.068555	-0.924891	-0.749624
1	1	0.71	1	2.62	1	1	2.038691	-0.924891	-0.839363
1	1	0.71	1	1	1	1	1.052653	-0.924891	-0.670930
1	1	0.71	1	1	2	1	1.835214	-0.924891	-0.670930
1	1	0.71	1	1	3	1	2.617774	-0.924891	-0.670930
1	1	0.71	1	1	1	1	0.728888	-0.924891	-0.670930
1	1	0.71	1	1	1	2	1.549196	-0.924891 -	-0.924891 -
1	1	0.71	1	1	1	3	2.369503	0.670930	0.670930

It is noticed that the skin-friction coefficient and the local heat transfer rates at the plate decrease with an increase in N, Q and Pr. It is also found that the local mass transfer rate and Skin-friction coefficient decreases with an increase in K or Sc. Also, it was noticed that the local skin-friction coefficient increases with an increase in the thermal Grashof number and mass Grashof number

2. Conclusion

In this paper an unsteady flow past a parabolic starting motion of the infinite vertical plate with constant heat flux, in presence heat source parameter, thermal radiation and chemical reaction has been studied numerically. The non-dimensional governing equations are solved with the help of Finite Difference method. The results are presented graphically and analyzed. The present investigation leads to the conclusion as follows:

The fluid velocity increases with the increasing values of the thermal and mass Grashof number but decreases with an increase of thermal radiation, heat source parameter, Prandtl number and Schmidt number. The temperature decreases with raising values of heat source parameter and Prandtl number. Chemical reaction Schmidt number contributes to decrease the concentration profile.

This work can further be extended by considering some more relevant fluid parameters like Soret number, DuFour number etc. The geometry of the model can also be changed to accommodate some flows with important applications

Code Validation

For the accuracy of the results, the present study is compared with the previous study of Muthucumaraswamy and Velmurugan [10] as shown in Fig. 13. It is observed that, in the absence of thermal radiation and heat source parameter, the result obtained in our present work are in good agreement with that of Muthucumaraswamy and Velmurugan [10]. It is observed that both the figures exactly coincide, depicting similar result that the velocity increases with the increasing values of the thermal Grashof number or mass Grashof number.

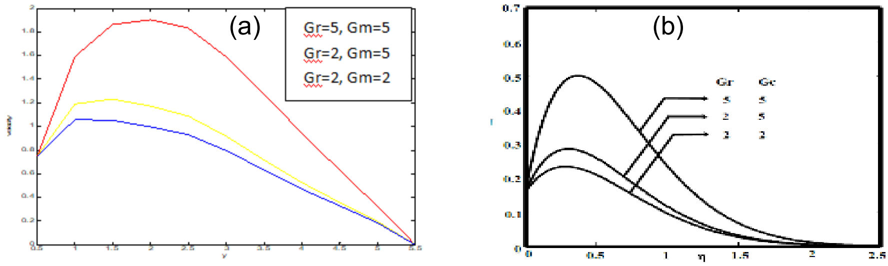


Fig. 13. Effects of thermal Gc, Gm on Velocity Profile of (a) present study and (b) previous study by Muthucumaraswamy and Velmurugan, when Sc= 0.6, K= 0.2, Gm=5, Gr=5, N=0, Q=0.

Nomenclature

g	Acceleration due to gravity
T and C	Dimensional fluid temperature and concentration
T_∞ and C_∞	Dimensional fluid temperature and concentration at the free stream
q'	Radiative heat flux,
u	Velocity of the fluid in the x - direction
u_0	Velocity of the plate
U	Dimensionless velocity
Q_0	Heat absorption,
D_M	Mass diffusion coefficient
K_1	Chemical reaction parameter
Pr	Prandtl number
Sc	Schmidt number
M	Magnetic parameter
K	Chemical reaction parameter
Gr	Thermal Grashof number
N	Radiation parameter
Q	Heat source parameter

Greek symbols

ϑ	Kinematic viscosity,
β	Volumetric coefficient of thermal expansion
β_c	Volumetric coefficient of expansion with species concentration
ρ	Fluid density
κ	Thermal conductivity

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