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The Impact of Couple Stress on Thermosolutal Instability of Visco-Elastic Nanofluids in Porous Medium

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Abstract

This work aims to investigate the impact of couple stress on the thermosolutal convection of viscoelastic nanofluid saturated in the porous medium. The rheological behavior of a viscoelastic nanofluid is characterized by the Rivlin-Erickson model. We study the linear stability analysis using the normal mode analysis method and examined analytically and graphically using MATLAB the impact of non-dimensional factors such as the couple stress, the concentration Rayleigh number, the solutal Rayleigh number, the thermos-nanofluid Lewis number, the thermosolutal Lewis number, Dufour and the Soret parameters and found that the couple stress, nanofluid Lewis number and modified diffusivity ratio enhance the instability of thermosolutal convection.

Keywords: Thermosolutal instability; Nanofluid; Rivlin-Erikson model; Porous medium; Couple stress.

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1. Introduction

Nanofluids are colloidal suspensions of nanoparticles in a base fluid, characterized by particles with dimensions ranging from 1 to 100 nanometers. These fluids are composed of a base fluid, such as oil, glycol, water, or ethylene, with nanoparticles dispersed within. The term nanoparticle was initially proposed by Choi [1] and has since been widely used in the development of heat transfer fluids. Buongiorno [2] developed a model for nanofluids and studied the effects of Brownian diffusion and thermophoresis. Eastman *et al.* [3] observed thermal conductivity increases by 40 % when copper nanoparticles were added to ethylene glycol. Chandrashekhar examined the thermal instability of a newtonian fluid under a range of hydrodynamic and hydromagnetic assumptions [4].

Due to its diverse range of applications, the study of thermosolutal convection in porous media has attracted significant attention from academics in recent decades. These applications include chemical research, oceanography, cancer therapy, bioengineering, engineering, food processing, and nuclear industries. Shivakumara *et al.* [5] investigated the impact of rotating couple stress fluid on the electrohydrodynamic convection and found

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that the presence of couple stress causes the fluid layer to become unstable. Kuznetsov and Nield [6] conducted an analytical and numerical study of the thermosolutal instability in a horizontal layer of porous media saturated with nanofluid. Pundir *et al.* [7] examined the impact of rotation on thermosolutal convection in a visco-elastic nanofluid and observed that an increment in Taylor number enhances the stability of stationary convection.

Umawati and Beg [8] used a non-Newtonian nanofluid, explored the onset of thermosolutal convection with a porous medium, and observed that the couple stress parameter was stabilizing in both oscillatory and stationary convections. The study of thermosolutal convection of a couple stress rotatory fluid in porous media by Kumar and Mohan [9] was relevant to astrophysics and geophysics and posed intricate challenges as a double-diffusion phenomenon. Singh and Nisar [10] examined the thermal instability of magnetohydrodynamic couple stress nanofluid in a rotating porous medium. Kumar *et al.* [11] explored the effect of magnetic field, rotation, and suspended dust particles on the onset of double diffusive convection in a couple-stress fluid saturated with a porous medium. They established the conditions that determine whether overstability exists or not. Chand *et al.* [12] focused on the thermal instability of a couple stress nanofluid saturated in a horizontal layer of porous medium. Malashetty *et al.* [13] used linear and weak nonlinear stability analyses. They studied the onset of double-diffusive convection with a couple-stress fluid in a porous medium. They discovered that the solute Rayleigh number and the couple stress parameter were critical in stabilizing stationary and oscillatory convection.

Devi *et al.* [14] examined the impact of variable gravity fields on the convective stability of a coupled stress fluid for three different combinations of bounding surfaces. They observed that variable gravity can either enhance or reduce the stability of the system, depending on the direction of the gravity variation. Choudhary *et al.* [15] investigated the effect of variable viscosity on the stability of couple stress fluid layers for different conducting boundaries and found that the couple stress has a stabilizing effect, while the viscosity variation has a destabilizing effect on the system. Bishnoi and Kumar [16] studied the combined impact of Hall currents and salt gradients on an elastic-viscous nanofluid and discovered that Hall currents possess a dual influence on the system in the presence of salt. Kumar *et al.* [17] studied the effect of a magnetic field on the onset of thermal convection in a porous layer saturated with Jeffrey nanofluid and found that the Chandrasekhar number delayed the onset of convection, while the Jeffrey parameter accelerated it. Sharma *et al.* [18] explored how variable gravity affects the thermal instability of rotating Jeffrey nanofluids in porous media and discovered the necessary condition for overstability of oscillatory convection. Sharma and Kumar [19] explored the combined effect of rotation and magnetic field on the onset of convection of Jeffery nanofluid saturated in a porous medium. Arora *et al.* [20] examined the combined impact of magnetic field, viscosity, and rotation on the onset of convection in magnetic nanofluids and found that increasing the Taylor number stabilizes the system. Yadav *et al.* [21] conducted a study using two types of boundaries: in the first, the top plane is isothermal and the bottom boundary plane is insulated, and in second, lower and upper boundary planes are isothermal, and rotation on the Casson fluid generated by purely internal heating in a porous layer. They noticed that the Casson parameter exhibits a dual impact on the system, and the Taylor Darcy number has a stabilizing effect; they also observed that if both boundary planes are isothermal, then the structure is more stable.

Kuiry and Vishwakarma [22] investigated the effect of MHD flow of viscous, incompressible fluid with small electrical conductivity in porous medium and found that the fluid velocity decreases due to the increasing value of kinematic viscosity and constant suction velocity. Munshi *et al.* [23] studied the physical properties of water as the base fluid and copper as the nanoparticles. It has been found that the Darcy number is a good control parameter for heat transfer in fluid flow through a porous medium. Gani *et al.* [24] studied the effects on unsteady MHD flow of a nanofluid for free convection past an Inclined plate.

As per the above literature survey, no assessment has yet been conducted to assess the impact of couple stress on visco-elastic nanofluid. The present study focuses on exploring the impact of couple stress on thermosolutal convection of visco-elastic nanofluid in the presence of a porous medium and the Soret factor on Rivlin-Erickson nanofluid. Using MATLAB software, the stationary convection is discussed analytically and graphically.

2. Methodology and Mathematical Model

Rivlin and Erickson [25] proposed a constitutive equation given as:
$\tau_{ij} = 2\left(\mu + \mu' \frac{\partial}{\partial t}\right) e_{ij}$; $e_{ij} = \frac{1}{2}\left(\frac{\partial q_i}{\partial x_j} + \frac{\partial q_j}{\partial x_i}\right)$ where τ_{ij} is a shearing stress, e_{ij} is the rate of strain tensor, μ is viscosity, μ' is viscoelasticity, q_i is a velocity vector and x_i is a position vector.

Here, an infinite horizontal layer of a Rivlin-Erikson nanofluid of width d considered and bounded by the planes $z = 0$ and $z = d$. The layer is solved and heated from below and acted upon by a gravitational force $\boldsymbol{g} = (0, 0, -g)$ in z direction, C is concentration, T is temperature, and φ is volumetric fraction of nanoparticles. We assumed that the concentration, temperature, and volumetric fraction of nanoparticles at lower and upper boundaries are C_0, T_0, φ_0 and C_1, T_1 φ_1 respectively. Following Nield and Kuznetsov [26], Chand [27], and Bishnoi *et al.* [28], the equations of conservation of mass, momentum, thermal energy, and nanoparticles for Rivlin-Ericksen fluid using the Boussinesq approximation are taken as

$$\nabla \boldsymbol{q} = 0 \tag{1}$$

$$\frac{\rho}{\varepsilon}\left(\frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\boldsymbol{q}.\nabla)\right)\boldsymbol{q} = -\nabla p - \frac{1}{k_1}\left(\mu + \mu' \frac{\partial}{\partial t}\right)\boldsymbol{q} + \frac{1}{k_1}\left(\mu + \mu_c \frac{\partial}{\partial t}\right)\boldsymbol{q}$$
$$+\boldsymbol{g}\big(\varphi\rho_p + \rho_f(1-\varphi)\{\,(1 - \alpha_T(T - T_0) - \alpha_C(C - C_0))\}\big) \tag{2}$$

$$\left(\frac{\partial}{\partial t} + \frac{1}{\varepsilon}\boldsymbol{q}.\nabla\right)\varphi = D_B\nabla^2\varphi + \frac{D_T}{T_1}\nabla^2 T \tag{3}$$

$$\left((\rho_c)_m \frac{\partial}{\partial t} + (\rho_c)_f \boldsymbol{q}.\nabla\right)T = k\nabla^2 T + \varepsilon(\rho_c)_p\left[D_B\nabla\varphi.\nabla T + \frac{D_T}{T_1}\nabla T.\nabla T\right]$$
$$+ (\rho_c)_f D_{TC}\nabla^2 C \tag{4}$$

$$\left(\frac{\partial}{\partial t}+\frac{1}{\varepsilon}\boldsymbol{q}.\nabla\right)C = D_S\nabla^2 C + D_{CT}\nabla^2 T \tag{5}$$

where $\boldsymbol{q}, \varepsilon, \rho, p, k_1, \mu, \mu', \mu_c, \boldsymbol{g}, \rho_p, \rho_f, \alpha_T, \alpha_c$ denote Darcy velocity vector, porosity of porous medium, the density, pressure, medium permeability of fluid, viscosity, viscoelasticity, couple stress viscosity, gravitational acceleration, density of nanoparticles, reference density of nanofluid, coefficient of thermal expansion, coefficient of solute concentration, respectively. k is the thermal conductivity of the fluid, D_T is the thermoporetic diffusion coefficient, D_B is the Brownian diffusion coefficient, D_S is the solutal diffusivity, $(\rho_c)_p$ is the heat capacity of nanoparticles, $(\rho_c)_f$ is the heat capacity of the fluid, $(\rho_c)_m$ is the heat capacity of the fluid in a porous medium, D_{TC} is the Dufour diffusivity of the porous medium and D_{CT} is the Soret diffusivity of the porous medium.

We assume that the volumetric fraction and temperature of the nanoparticles are constant on the boundaries, given by Nield and Kuznetsov [29].

$$w = 0, \quad T = T_0, \quad C = C_0, \quad D_B\frac{\partial\varphi}{\partial z}+\frac{D_T}{T_1}\frac{\partial T}{\partial z} = 0 \quad \text{at } z = 0 \tag{6}$$

$$w = 0, \quad T = T_1, \quad C = C_1, \quad D_B\frac{\partial\varphi}{\partial z}+\frac{D_T}{T_1}\frac{\partial T}{\partial z} = 0 \quad \text{at } z = d \tag{7}$$

Introducing non-dimensional parameters as follows:

$$(x', y', z') = \frac{(x,y,z)}{d}, \quad (u', v', w') = \frac{(u,v,w)}{k_f}d, \quad t' = \frac{k_f}{\sigma d^2}t, \quad p' = \frac{k_1}{\mu k_f}p,$$

$$C' = \frac{C-C_1}{C_0-C_1}, \quad \varphi' = \frac{\varphi-\varphi_0}{\varphi_1-\varphi_0}, \quad T' = \frac{T-T_1}{T_0-T_1}, \tag{8}$$

where $\sigma = \frac{(\rho_c)_m}{(\rho_c)_f}$ is the thermal capacity ratio and $k_m = \frac{k}{(\rho_c)_f}$ is the thermal diffusivity of the fluid.

Equations (1) to (5) are obtained in a non-dimensional form (dash has been dropped for simplicity), as follows:

$$\nabla\boldsymbol{q} = 0 \tag{9}$$

$$\frac{1}{\sigma Va}\left(\frac{\partial \boldsymbol{q}}{\partial t}\right) = -\nabla p - \left(1 + F\frac{\partial}{\partial t}\right)\boldsymbol{q} - (1+\eta\nabla^2)\boldsymbol{q} - R_m\hat{k} - R_n\varphi\hat{k} + R_aT\hat{k} + \frac{R_s}{Ln}C\hat{k} \tag{10}$$

$$\frac{1}{\sigma}\frac{\partial\varphi}{\partial t}+\frac{1}{\varepsilon}\boldsymbol{q}.\nabla\varphi = \frac{1}{Ln}\nabla^2\varphi + \frac{N_A}{Ln}\nabla^2 T \tag{11}$$

$$\frac{\partial T}{\partial t} + \boldsymbol{q}.\nabla T = \nabla^2 T + \frac{N_B}{Ln}\nabla\varphi\nabla T + \frac{N_A N_B}{Ln}\nabla T\nabla T + N_{TC}\nabla^2 C \tag{12}$$

$$\frac{1}{\sigma}\frac{\partial C}{\partial t}+\frac{1}{\varepsilon}\boldsymbol{q}.\nabla C = \frac{1}{Le}\nabla^2 C + N_{CT}\nabla^2 T \tag{13}$$

The following boundary conditions are in their non-dimensional form:

$$w = 0, \quad \frac{\partial w}{\partial z} = 0, \quad T = 1, \quad C = 1, \quad D_B\frac{\partial\varphi}{\partial z} + N_A\frac{\partial T}{\partial z} = 0 \quad \text{at } z = 0 \tag{14}$$

$$w = 0, \quad \frac{\partial w}{\partial z} = 0, \quad T = 0, \quad C = 0, \quad D_B\frac{\partial\varphi}{\partial z} + N_A\frac{\partial T}{\partial z} = 0 \quad \text{at } z = 1 \tag{15}$$

The non-dimensional numbers are used in the equations. (9) to (13) are given as:

$Ln = \frac{k_f}{D_B}$ is the nanofluid Lewis number, $F = \frac{\mu' k_f}{\mu \sigma d^2}$ is a kinematic viscoelastic parameter, $Le = \frac{k_f}{D_s}$ is the thermosolutal Lewis number, $N_A = \frac{D_T(T_0 - T_1)}{D_B T_1(\varphi_1 - \varphi_0)}$ is the modified diffusivity ratio number, $N_B = \frac{\varepsilon(\rho_c)_p}{(\rho_c)_f}(\varphi_1 - \varphi_0)$ is the modified particle density increment number, $R_m = \left(\rho_p \varphi_0 + (1 - \varphi_0)\rho_f\right)\frac{g k_1 d}{\mu k_f}$ is the basic density Rayleigh number, $R_n = \left(\rho_p - \rho_f\right)(\varphi_1 - \varphi_0)\frac{g k_1 d}{\mu k_f}$ is the concentration Rayleigh number, $R_a = \rho_f (T_0 - T_1)\alpha_T \frac{g k_1 d}{\mu k_f}$ is the Rayleigh number, $R_s = \rho_f (C_0 - C_1)\alpha_c \frac{g k_1 d}{\mu D_s}$ is the solutal Rayleigh number, $N_{TC} = \frac{D_{CT}(C_0 - C_1)}{k_f(T_0 - T_1)}$ is the Soret parameter, $N_{CT} = \frac{D_{CT}(T_0 - T_1)}{k_f(C_0 - C_1)}$ is Dufour parameter, $Da = \frac{k_1}{d^2}$ is the Darcy number and $Va = \frac{\varepsilon Pr}{Da}$ is Vadasz's number.

2.1. *Basic solution*

Suppose that the basic state of C, T, and φ are not dependent on time, given as follows:

$$\boldsymbol{q} = 0, \quad p = p_b(z), \quad T = T_b(z), \quad \varphi = \varphi_b(z), \quad C = C_b(z)$$

By using boundary conditions, and get the approximation solution is given as follows:

$$T_b(z) = (1 - z), \quad \varphi_b(z) = \varphi_0 + N_A z \text{ and } C_b(z) = (1 - z) \tag{16}$$

2.2. *Perturbation solution*

Introducing perturbations which dependent on time onto the basic state and given as:

$$\boldsymbol{q} = \boldsymbol{q}^*, \; T = T_b + T^*, \; \varphi = \varphi_b + \varphi^*, \; C = C_b + C^*, \; p = p_b + p^* \tag{17}$$

Using equation (17) in eqs. (9) to (13) under linear stability theory, neglecting the product of prime quantities. After dropping asterisks, we get the subsequent equations:

$$\nabla \boldsymbol{q} = 0 \tag{18}$$

$$\frac{1}{\sigma Va}\left(\frac{\partial \boldsymbol{q}}{\partial t}\right) = -\nabla p - \left(1 + F\frac{\partial}{\partial t}\right)\boldsymbol{q} - (1 + \eta\nabla^2)\boldsymbol{q} - R_n \varphi \hat{k} + R_a T \hat{k} + \frac{R_s}{Le} C \hat{k} \tag{19}$$

$$\frac{1}{\sigma}\frac{\partial \varphi}{\partial t} + \frac{N_A}{\varepsilon} w = \frac{1}{Ln}\nabla^2 \varphi + \frac{N_A}{Ln}\nabla^2 T \tag{20}$$

$$\frac{\partial T}{\partial t} + w = \nabla^2 T - \frac{\varepsilon N_B}{Ln}\left[N_A \frac{\partial T}{\partial z} + \frac{\partial \varphi}{\partial z}\right] + N_{TC}\nabla^2 C \tag{21}$$

$$\frac{1}{\sigma}\frac{\partial C}{\partial t} + \frac{1}{\varepsilon} w = \frac{1}{Le}\nabla^2 C + N_{CT}\nabla^2 T \tag{22}$$

The boundary conditions are:

$$w = 0, \quad T = 0, \quad C = 0, \quad \frac{\partial \varphi}{\partial z} + N_A \frac{\partial T}{\partial z} = 0 \quad \text{at } z = 0 \text{ and } z = 1 \tag{23}$$

Using the identity $curl\; curl = grad\;(div) - \nabla^2$ for operate $\hat{k}.curl\; curl$ on eq. (19), we get

$$\left[\frac{1}{\sigma Va}\frac{\partial}{\partial t} + \left(1 + F\frac{\partial}{\partial t}\right) + (1 + \eta\nabla^2)\right]\nabla^2 w = R_a \nabla_H^2 T + \frac{R_s}{Ln}\nabla_H^2 C - R_n \nabla_H^2 \varphi \tag{24}$$

Where $\nabla_H^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

3. Normal Mode Analysis

The normal mode approach is applied by assuming that minor perturbations are represented by wave-like components with amplitudes in the z-direction, as follows:

$$(w, T, C, \varphi) = [W(z), \Theta(z), \Gamma(z), \Phi(z)]exp(ilx + imy + nt) \tag{25}$$

where n is the disturbance growth rate, $a^2 = l^2 + m^2$ is the resulting wave number, l and m are the wave numbers along the x and y directions, respectively.
Using equation (25) in equations (24), (20), (21), and (22) becomes

$$\left[\frac{n}{\sigma Va} + (1 + Fn) + \left(1 + \eta(\mathrm{D}^2 - \mathrm{a}^2)\right)\right](\mathrm{D}^2 - \mathrm{a}^2)W + \mathrm{a}^2 R_a\Theta + \mathrm{a}^2\frac{R_s}{Le}\Gamma - \mathrm{a}^2 R_n\Phi = 0 \tag{26}$$

$$\frac{N_A}{\varepsilon}W - \frac{N_A}{Ln}(\mathrm{D}^2 - \mathrm{a}^2)\Theta - \left(\frac{(\mathrm{D}^2-\mathrm{a}^2)}{Ln} - \frac{n}{\sigma}\right)\Phi = 0 \tag{27}$$

$$W + \left[(\mathrm{D}^2 - \mathrm{a}^2) - n - \frac{\varepsilon N_A N_B}{Ln}D\right]\Theta + N_{TC}(\mathrm{D}^2 - \mathrm{a}^2)\,\Gamma - \frac{\varepsilon N_B}{Le}D\,\Phi = 0 \tag{28}$$

$$\frac{1}{\varepsilon}W + N_{CT}(\mathrm{D}^2 - \mathrm{a}^2)\,\Theta + \left[\frac{1}{Le}(\mathrm{D}^2 - \mathrm{a}^2) - \frac{n}{\sigma}\right]\Gamma = 0 \tag{29}$$

$$W = 0, \quad \Theta = 0, \quad \Gamma = 0, \quad D\Phi + N_A \mathrm{D}\Theta = 0 \text{ at } z = 0 \text{ and } z = 1. \tag{30}$$

Where $D = \frac{d}{dz}$ and $a^2 = l^2 + m^2$.

After normal mode analysis, the solutions W, Θ, Φ, and Γ consider in the form as follows:

$$W = W_0 sin\pi z,\ \Theta = \Theta_0 sin\pi z, \Phi = \Phi_0 sin\pi z, \Gamma = \Gamma_0 sin\pi z \tag{31}$$

Substituting equation (31) into the equations (26) – (29) and integrating with respect to z from $z = 0$ to $z = 1$. We obtain the following matrix:

$$\begin{bmatrix} \left(1 + n\left(F + \frac{1}{\sigma Va}\right) - \eta J^2\right)J^2 & -\mathrm{a}^2 R_a & -\mathrm{a}^2\frac{R_s}{Le} & \mathrm{a}^2 R_n \\ \frac{N_A}{\varepsilon} & \frac{N_A}{Ln}J^2 & 0 & \frac{J^2}{Ln} + \frac{n}{\sigma} \\ -1 & J^2 + n & J^2 N_{TC} & 0 \\ -\frac{1}{\varepsilon} & N_{CT}J^2 & \frac{J^2}{Le} + \frac{n}{\sigma} & 0 \end{bmatrix}\begin{bmatrix} W_0 \\ \Theta_0 \\ \Gamma_0 \\ \Phi_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{32}$$

Where $J^2 = \pi^2 + a^2$ is the wave number.

After putting the above matrix equal to zero, get a non-trivial solution, so obtained the eigenvalue equation as follows:

$$R_a = \frac{1}{(J^2\sigma\varepsilon + n\varepsilon Le - N_{TC}J^2\sigma Le)}\left[\frac{\varepsilon}{\mathrm{a}^2}\left(1 + n\left(F + \frac{1}{\sigma Va}\right) - \eta J^2\right)\left((J^2 + n)(J^2\sigma + nLe) - J^4\sigma Le N_{TC}N_{CT}\right) + \sigma R_s(J^2\varepsilon N_{CT} - (J^2 + n)) - \frac{\sigma R_n}{(J^2\sigma + nLn)}\{(J^2\sigma + nLe)(N_A Ln(J^2 + n) + \varepsilon N_A J^2) - N_A J^4\sigma Le N_{TC}(Ln N_{CT} + \varepsilon)\}\right] \tag{33}$$

4. Stationary Convection

Putting $n = 0$ in equation (33), we get

$$R_a = \frac{1}{(\varepsilon - N_{TC}Le)}\left[\frac{\varepsilon(\pi^2+a^2)}{a^2}\left(1-\eta(\pi^2+a^2)\right)(1-LeN_{TC}N_{CT}) + R_s(\varepsilon N_{CT}-1) - R_n(N_A Ln + \varepsilon N_A - N_A LeN_{TC}(LnN_{CT}+\varepsilon))\right] \quad (34)$$

Equation (34) contains the kinematic viscoelastic parameter F. This equation is expressed in terms of the thermal Rayleigh number, which is an expression of $a, N_{CT}, N_{TC}, Le, N_A, R_s, R_n, Ln,$ and ε.

Substitute $a^2 = x\pi^2$ in equation (33), in the absence of Soret and Dufour parameters, we get:

$$R_a = \frac{(1+x)}{x}(1-\eta(1+x)\pi^2) - \frac{R_s}{\varepsilon} - \frac{R_n}{\varepsilon}(N_A Ln + \varepsilon N_A) \quad (35)$$

We examine the derivative of R_a with respect to η

$$\frac{\partial R_a}{\partial \eta} = \frac{-\varepsilon(1-LeN_{TC}N_{CT})}{(\varepsilon - N_{TC}Le)}\left(\frac{(\pi^2+a^2)^2}{a^2}\right)$$

is positive if $1 - LeN_{TC}N_{CT}$ and $\varepsilon - N_{TC}Le$ both have opposite signs, then couple stress increases the stability.

The derivative R_a with respect to R_s examine

$$\frac{\partial R_a}{\partial R_s} = \frac{(\varepsilon N_{CT} - 1)}{(\varepsilon - N_{TC}Le)}$$

is positive if $(\varepsilon N_{CT} - 1)$ and$(\varepsilon - N_{TC}Le)$ both have the same sign then R_s increase the stability.

The derivative R_a with respect to R_n examine

$$\frac{\partial R_a}{\partial R_n} = \frac{-(N_A Ln + \varepsilon N_A - N_A LeN_{TC}(LnN_{CT}+\varepsilon))}{(\varepsilon - N_{TC}Le)}$$

is negative if $N_A Ln + \varepsilon N_A - N_A LeN_{TC}(LnN_{CT}+\varepsilon)$ and $\varepsilon - N_{TC}Le$ both have the same sign then R_n decrease the instability.

The derivative R_a with respect to Ln examine

$$\frac{\partial R_a}{\partial Ln} = \frac{R_n N_A(LeN_{TC}N_{CT} - 1)}{(\varepsilon - N_{TC}Le)}$$

is positive if $LeN_{TC}N_{CT} - 1$ and $\varepsilon - N_{TC}Le$ both have the same sign, then Ln increases the instability.

We examine the derivative R_a with respect to N_A

$$\frac{\partial R_a}{\partial N_A} = \frac{R_n(LeN_{TC}(LnN_{CT}+\varepsilon) - Ln - \varepsilon)}{(\varepsilon - N_{TC}Le)}$$

is positive if $LeN_{TC}(LnN_{CT}+\varepsilon) - Ln - \varepsilon$ and $\varepsilon - N_{TC}Le$ both have the same sign then N_A increase the instability.

5. Oscillatory Convection

In equation (33) put $n = i\omega$ and get

$$R_a = \frac{1}{(J^2\sigma\varepsilon - N_{TC}J^2\sigma Le + i\omega\varepsilon Le)}\left[\frac{\varepsilon}{\mathrm{a}^2}\left(1 + i\omega\left(F + \frac{1}{\sigma Va}\right) - \eta J^2\right)\left((J^2 + i\omega)(J^2\sigma + i\omega Le) - J^4\sigma Le N_{TC}N_{CT}\right) + \sigma R_s(J^2\varepsilon N_{CT} - (J^2 + i\omega)) - \frac{\sigma R_n}{(J^2\sigma + i\omega Ln)}\{(J^2\sigma + i\omega Le)(N_A Ln(J^2 + i\omega) + \varepsilon N_A J^2) - N_A J^4\sigma Le N_{TC}(Ln N_{CT} + \varepsilon)\}\right] \quad (36)$$

In the absence of Soret and Dufour parameters, taking heat capacity ratio σ as unity, we get

$$R_a = \frac{1}{(J^2\varepsilon + i\omega\varepsilon Le)}\left[\frac{\varepsilon}{\mathrm{a}^2}\left(1 + i\omega\left(F + \frac{1}{Va}\right) - \eta J^2\right)(J^2 + i\omega)(J^2\sigma + i\omega Le) - R_s(J^2 + i\omega) - \frac{R_n}{(J^2 + i\omega Ln)}\{(J^2 + i\omega Le)(N_A Ln(J^2 + i\omega) + \varepsilon N_A J^2)\}\right] \quad (37)$$

After separating real and imaginary parts of equation (36), we get in the form

$$R_a = \Delta_1 + i\omega\Delta_2$$

where $\Delta_1 = J^2\left[\frac{\varepsilon}{\mathrm{a}^2}\left\{(1 - \eta J^2)(J^4 - \omega^2 Le) - \omega^2\left(F + \frac{1}{Va}\right)(1 + Le)J^2\right\} - R_s J^2 - \frac{R_n N_A J^2}{(J^4 + \omega^2 Ln^2)}\{J^4(Ln + \varepsilon) - \omega^2 Le Ln\} - \frac{R_n N_A J^2 Ln\omega^2}{(J^4 + \omega^2 Ln^2)}\{Le(Ln + \varepsilon) + Ln\}\right] + \omega^2 Le\left[\frac{\varepsilon}{\mathrm{a}^2}\left\{\left(F + \frac{1}{Va}\right)(J^4 - \omega^2 Le) + (1 - \eta J^2)(1 + Le)J^2\right\} - R_s + \frac{R_n N_A Ln}{(J^4 + \omega^2 Ln^2)}\{J^4(Ln + \varepsilon) - \omega^2 Le Ln\} - \frac{R_n N_A J^4}{(J^4 + \omega^2 Ln^2)}\{Le(Ln + \varepsilon) + Ln\}\right]$ and

$$\Delta_2 = -Le\left[\frac{\varepsilon}{\mathrm{a}^2}\left\{(1 - \eta J^2)(J^4 - \omega^2 Le) - \omega^2\left(F + \frac{1}{Va}\right)(1 + Le)J^2\right\} - R_s J^2 - \frac{R_n N_A J^2}{(J^4 + \omega^2 Ln^2)}\{J^4(Ln + \varepsilon) - \omega^2 Le Ln\} - \frac{R_n N_A J^2 Ln\omega^2}{(J^4 + \omega^2 Ln^2)}\{Le(Ln + \varepsilon) - Ln\}\right] + J^2\left[\frac{\varepsilon}{\mathrm{a}^2}\left\{\left(F + \frac{1}{Va}\right)(J^4 - \omega^2 Le) + (1 - \eta J^2)(1 + Le)J^2\right\} - R_s + \frac{R_n N_A Ln}{(J^4 + \omega^2 Ln^2)}\{J^4(Ln + \varepsilon) - \omega^2 Le Ln\} - \frac{R_n N_A J^4}{(J^4 + \omega^2 Ln^2)}\{Le(Ln + \varepsilon) - Ln\}\right]$$

With oscillatory onset $\Delta_2 = 0$ and $\omega \neq 0$, this gives the relation

$$a_1(\omega^2)^2 + a_2\omega^2 + a_3 = 0 \quad (38)$$

where $a_1 = \frac{\varepsilon Le Ln^2}{\mathrm{a}^2}\{Le(1 - \eta J^2) + (1 + Le)J^2\}$,

$$a_2 = \frac{\varepsilon J^2}{\mathrm{a}^2}\left[J^2\left\{(1 - \eta J^2) + \left(F + \frac{1}{Va}\right)(1 + Le)J^2\right\} + Ln^2\left\{\left(F + \frac{1}{Va}\right)J^4 + (1 - \eta J^2)(1 + Le)J^2\right\} - Le Ln^2(1 - \eta J^2)J^2 - Le\left(F + \frac{1}{Va}\right)J^4\right] + R_s Ln^2 J^2(Le - 1 + R_n N_A J^2 Le^2 Ln(-Le + (Le + \varepsilon) - Ln),$$

$$a_3 = -\frac{\varepsilon Le J^8}{a^2}(1-\eta J^2) + R_s J^2(Le-1) + R_n J^6 N_A\{(Ln+\varepsilon)(1+Ln) - Le(Ln+\varepsilon) + Ln\} + \frac{\varepsilon J^6}{a^2}\left\{\left(F+\frac{1}{Va}\right)J^4 + (1-\eta J^2)(1+Le)J^2\right\}.$$

Then

$$R_{osc} = J^2\left[\frac{\varepsilon}{a^2}\left\{(1-\eta J^2)(J^4-\omega^2 Le) - \omega^2\left(F+\frac{1}{Va}\right)(1+Le)J^2\right\} - R_s J^2 - \frac{R_n N_A J^2}{(J^4+\omega^2 Ln^2)}\{J^4(Ln+\varepsilon) - \omega^2 Le Ln\} - \frac{R_n N_A J^2 Ln\omega^2}{(J^4+\omega^2 Ln^2)}\{Le(Ln+\varepsilon)+Ln\}\right] + \omega^2 Le\left[\frac{\varepsilon}{a^2}\left\{\left(F+\frac{1}{Va}\right)(J^4-\omega^2 Le) + (1-\eta J^2)(1+Le)J^2\right\} - R_s + \frac{R_n N_A Ln}{(J^4+\omega^2 Ln^2)}\{J^4(Ln+\varepsilon) - \omega^2 Le Ln\} - \frac{R_n N_A J^4}{(J^4+\omega^2 Ln^2)}\{Le(Ln+\varepsilon)+Ln\}\right] \quad (39)$$

For an oscillatory neutral solution, the positive root of ω^2 exits. If positive roots exist, then the critical thermal Rayleigh number for oscillatory convection can be derived by numerically minimizing (38) with respect to wave number, and if positive roots do not exist, then oscillatory convection is not possible.

6. Results and Discussion

Here, the stationary convection and the impact of various parameters in the nanofluid discussed as follows:

Fig. 1 for the critical Rayleigh number vs wave number a for $\varepsilon = 0.5, Le = 500, R_s = 500, N_{TC} = 5, N_{CT} = 10, R_n = -1, N_A = 5, Ln = 100$ and varying $\eta = 0.2, \eta = 0.4, \eta = 0.6$, and observed that the curve of the Rayleigh number R_a is increasing by η is increasing. Thus, couple stress has a stabilizing effect.

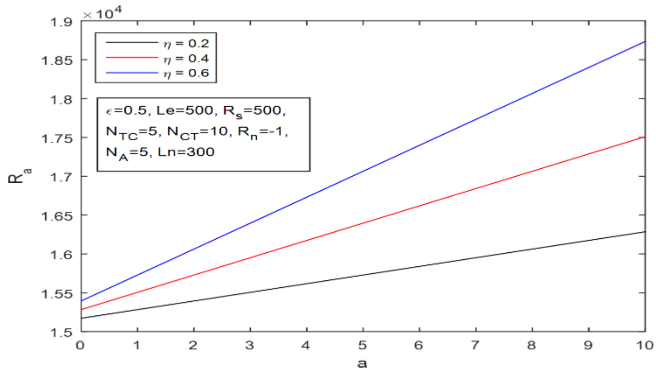

Fig. 1. Rayleigh number variation with wave number for various pair stress η values.

Fig. 2 for the critical Rayleigh number vs wave number a for $\varepsilon = 0.5, \eta = 0.2, Le = 500, N_{TC} = 5, N_{CT} = 10, R_n = -1, N_A = 5, Ln = 100$ and varying $R_s = 1000, R_s = 1500, R_s = 2000$, and observed that the curve of the Rayleigh number R_a is decreasing by R_s is increasing. So, the solutal Rayleigh number has a destabilizing effect.

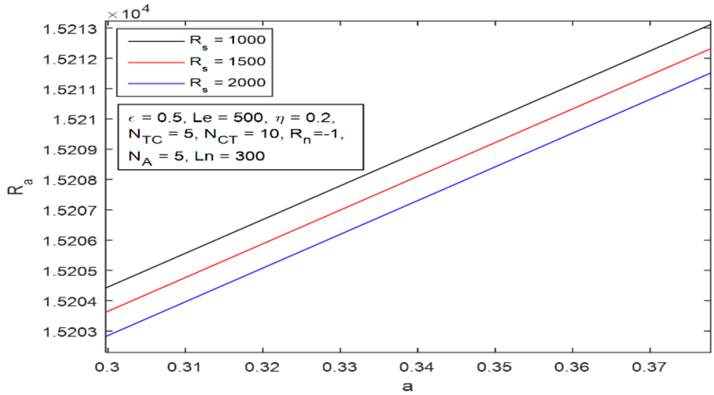

Fig. 2. Rayleigh number variation with wave number for various pairs of solutal Rayleigh numbers R_s values.

Fig. 3 for the critical Rayleigh number vs wave number a for $\varepsilon = 0.5, \eta = 0.2$ $Le = 500, N_{TC} = 5, N_{CT} = 10, R_n = -1, N_A = 5, R_s = 500$ and varying $Ln = 100, Ln = 200, Ln = 300$, and observed that the curve of the Rayleigh number R_a is increasing by Ln is increasing. So, the nanofluid Lewis number has a stabilizing effect.

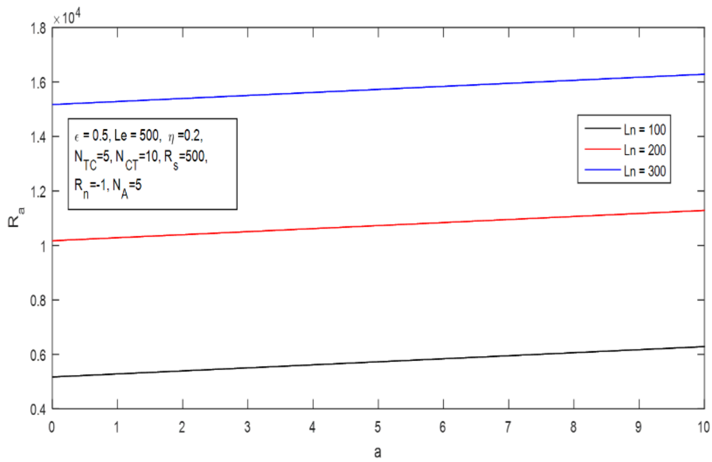

Fig. 3. Rayleigh number variation with wave number for various pair nanofluid Lewis number Ln values.

Fig. 4 for the critical Rayleigh number vs wave number a for $\varepsilon = 0.5, \eta = 0.2$ $Le = 500, N_{TC} = 5, N_{CT} = 10, N_A = 5, R_s = 500, Ln = 100$ and varying $R_n = -1, R_n = 0, R_n = 1$, and observed that the curve of the Rayleigh number R_a is decreasing by R_n is increasing. So, the concentration Rayleigh number has a destabilizing effect.

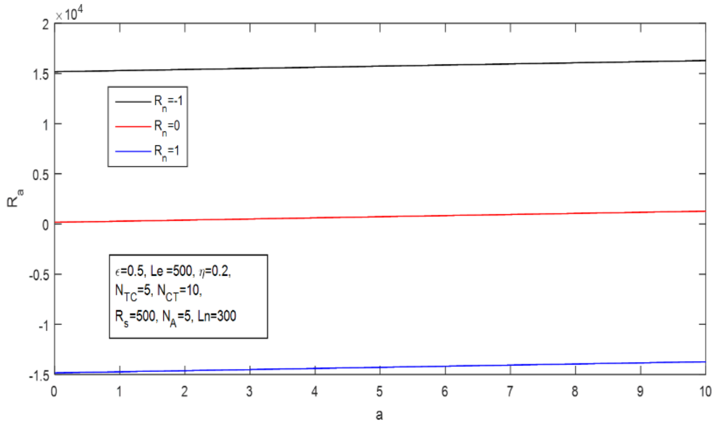

Fig. 4. Rayleigh number variation with wave number for various pair concentrations. Rayleigh number R_n values.

Fig. 5 for the critical Rayleigh number vs wave number a for $\varepsilon = 0.5, \eta = 0.2, N_{TC} = 5, N_{CT} = 10, N_A = 5, R_n = -1, R_s = 500, Ln = 100$ and varying $Le = 10, Le = 50, Le = 100$, and observed that the curve of the Rayleigh number R_a is decreasing by R_n is increasing. So thermosolutal Lewis number has a destabilizing effect.

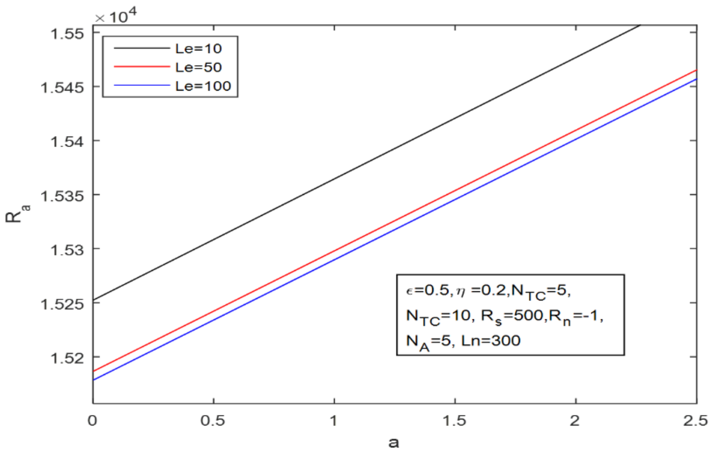

Fig. 5. Rayleigh number variation with wave number for various pair thermosolutal Lewis number Le values.

Fig. 6 for the critical Rayleigh number vs wave number a for $\varepsilon = 0.5, \eta = 0.2, N_{TC} = 5, N_{CT} = 10, Le = 500, R_n = -1, R_s = 500, Ln = 100$ and varying $N_A = 5, N_A = 10, N_A = 15$, and observed that the curve of the Rayleigh number R_a is increasing by N_A is increasing. So modified diffusivity ratio number N_A has a stabilizing effect.

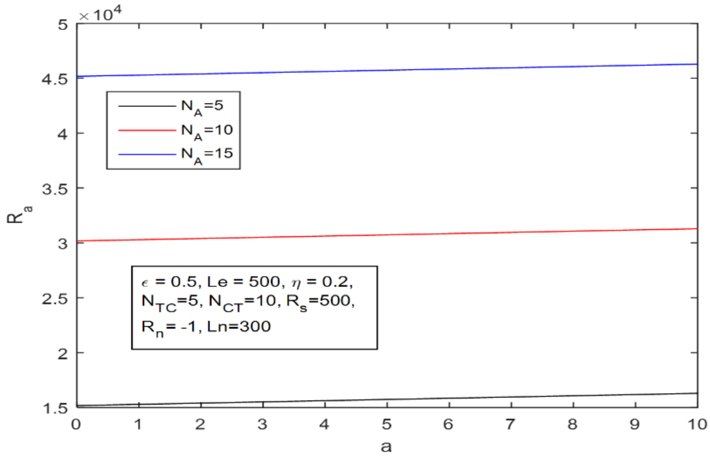

Fig. 6. Rayleigh number variation with wave number for various pairs of modified diffusivity ratio numbers N_A values.

Fig. 7 for the critical Rayleigh number vs wave number a for $\varepsilon = 0.5, \eta = 0.2, N_A = 5, N_{CT} = 10, Le = 500, R_n = -1, R_s = 500, Ln = 100$ and varying $N_{TC} = 1, N_{TC} = 5, N_{TC} = 10$, and observed that the curve of the Rayleigh number R_a is decreasing by N_{TC} is increasing. So, the Soret parameter number N_{TC} has a destabilizing effect.

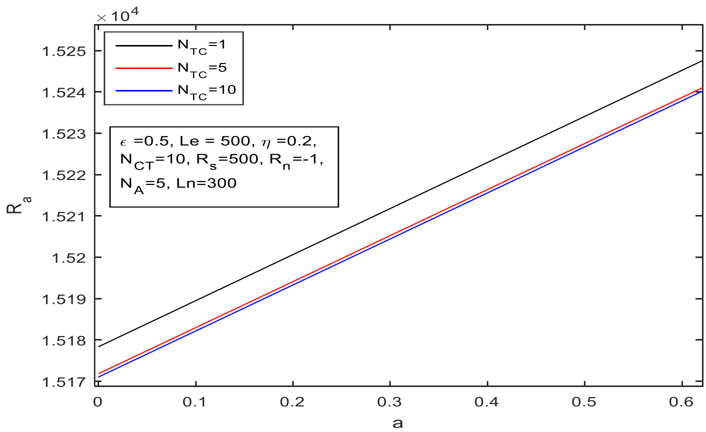

Fig. 7. Rayleigh number variation with wave number for various pair Soret parameter number N_{TC} values.

7. Conclusion

The effect of couple stress, Thermosolutal instability of Rivlin-Erikson nanofluids saturated in a horizontal layer with a porous medium heated from below, was studied under the boundary conditions. In linear stability analysis, and examined the effect of various parameters.

(i) The Rayleigh number depends on concentration, temperature, couple stress, and nanoparticle parameters but does not depend on the kinematic viscoelasticity parameter.
(ii) The Rayleigh number does not depend on the modified particle density number.
(iii) The couple stress η, nanofluid Lewis number Ln, and modified diffusivity ratio N_A enhance the instability of thermosolutal convection.
(iv) Solutal Rayleigh number R_s, concentration Rayleigh number R_n, thermosolutal Lewis number Le has a destabilizing effect on the system.
(v) Soret parameter N_{TC} has destabilized the system.

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