

## **Gravitoelectromagnetism and Gravitomagnetic Clock Effect**

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### **Abstract**

We present a new and elegant method of derivation of the gravitomagnetic clock effect using gravitational Lorentz force, which appears in gravitoelectromagnetism. The result is precisely the same as that of general relativity in the Kerr field.

*Keywords:* Gravitomagnetic clock effect; Gravitoelectromagnetism; Lorentz force.

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### **1. Introduction**

The most successful theory of gravity is the general relativity of Einstein. It is a theory based on differential geometry. According to the theory, a massive body influences the space-time around it, making it curved. Other bodies move in that curved space-time following the curvature. For recent critical overviews of general relativity, see, for example, [1-3] and the references therein.

A theory of gravity intermediate between general relativity and Newtonian gravity is gravitoelectromagnetism. Gravitoelectromagnetism [4-8] is a theory of gravity where a set of Maxwellian equations prevail. Several efforts have been made to measure different aspects of gravitomagnetism [9-16]. Gravitomagnetism is parallel to electromagnetism, where gravitoelectric and gravitomagnetic fields provide the full gravity field. It is established that a gravitomagnetic field exists around a rotating astrophysical mass. The Gravity Probe B [17,18] mission has conclusively detected the gravitomagnetic field of Earth.

In gravitoelectromagnetism, a source body of inertial mass  $M$  is assumed to have a gravitoelectric charge of  $M$  and a gravitomagnetic charge of  $2M$ . The test body possesses here similar charges but with opposite signs. This is done to make gravity always attractive, and the field has spin 2. In our previous papers [19,20], we have put arguments that the assignment of oppositely signed charges to the source and test bodies is seemingly justified but conceptually unacceptable. This is because there is no experimental difference between the source and test bodies except for the large and smallness of the masses. To save a theory like gravitoelectromagnetism, we assume the source and test the

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body's symmetrical gravitoelectric and gravitomagnetic charges. Here, both source and test bodies have positive gravitoelectric charges and negative gravitomagnetic charges. So, a test body of inertial mass  $m$  possesses gravitoelectric charge  $(+m)$  and gravitomagnetic charge  $(-2m)$ . In this short article, we shall derive the expression for the gravitomagnetic clock effect, which is by now a well-studied phenomenon that appears in the orbit of a test body around a Kerr black hole. The general relativistic gravitomagnetic clock effect indicates the shift between proper periods of two counter-revolving satellites. This time shift may be used to verify the presence of the terrestrial gravitomagnetic field by means of orbiting gravitomagnetic clocks. These clocks are used to calculate the effect of a global navigation satellite system compared to a geostationary satellite. One can explore the possibility of using space-borne standard clocks for detecting the gravitational field on the Earth with the gravitomagnetic clock effect [21-28]. It is shown in Refs. [29-33] that two freely counter revolving test particles in the field of a rotating mass take different periods to complete the same full orbit. Let  $t_+$  ( $t_-$ ) denote the period of prograde (retrograde) motion along an equatorial orbit according to asymptotically static inertial observers; then  $t_+ - t_- = \frac{4\pi a}{c}$ , where  $a = \frac{J}{Mc}$ , the specific spin of the central rotating mass  $M$ . The path to this result is solely general relativistic. Here, in this article, we shall employ the Lorentz force law of gravitoelectromagnetism which is

$$\vec{F} = m\vec{E}_N - 2m\frac{\vec{v}}{c} \times \vec{B}_g \quad (1)$$

where,  $\vec{F} = -m\omega^2 r \hat{r}$ ,  $\vec{E}_N = -\frac{GM}{r^2} \hat{r}$  and  $r = (x^2 + y^2)^{1/2}$ , i.e., the motion of the test particle is on the equatorial plane, and  $\omega$  is the angular velocity of the test particle. Note that the mass  $m$  in the first term on the right-hand side of Eq. (1) is usually written as  $(-m)$  in gravitoelectromagnetism. But in our case here, this is different due to the assignment of positive gravitoelectric charge to the test body. It is to be noted that there are other alternative derivations of the gravitomagnetic clock effect [24,27]. However, for the simplest case of the circular orbit of a test body around a spinning central body, the expression for the gravitomagnetic clock effect is the same as the one mentioned. In the next section, we will present the derivation of the gravitomagnetic clock effect using Eq. (1).

## 2. Gravitomagnetic Clock Effect according to Gravitational Lorentz Force

According to gravitoelectromagnetic physics, the vector potential  $\vec{A}$  at positions far away from the central body is given by

$$\vec{A} = \frac{G}{c} \frac{\vec{J} \times \vec{r}}{r^3} \quad (2)$$

where,  $\vec{J} = \frac{2}{5}MR^2\Omega\hat{z}$  is the angular momentum of the central body of mass  $M$ , radius  $R$ , and angular velocity  $\Omega$ . Here, we have assumed  $\vec{J}$  to point along the  $z$ -axis. Moreover,  $\vec{r} = (\hat{x}x + \hat{y}y)$ , the angular velocity of the test particle in prograde orbit is  $\vec{\omega} = \omega\hat{z}$  and in a retrograde orbit, it is  $\vec{\omega} = -\omega\hat{z}$ . Now, the gravitomagnetic field  $\vec{B}_g$  is given by

$$\vec{B}_g = \vec{\nabla} \times \vec{A} \tag{3}$$

After some algebra, we find that

$$\vec{B}_g = -\frac{G}{c} \frac{\vec{J}}{r^3} \tag{4}$$

So, we can write

$$-m\omega^2 r \hat{r} = -\frac{GMm}{r^2} \hat{r} - \frac{2m}{c} (\vec{\omega} \times \vec{r}) \times \left(-\frac{G}{c} \frac{\vec{J}}{r^3}\right) \tag{5}$$

In prograde orbit

$$(\vec{\omega} \times \vec{r}) \times (\Omega \hat{z}) = \Omega \omega \vec{r} \tag{6}$$

and in a retrograde orbit

$$(\vec{\omega} \times \vec{r}) \times (\Omega \hat{z}) = -\Omega \omega \vec{r} \tag{7}$$

Using all the facts in Eq. (5), we can write for the prograde orbit

$$m\omega^2 r = \frac{GMm}{r^2} - \frac{4GMm R^2}{5c^2 r^2} \Omega \omega \tag{8}$$

and for retrograde orbit

$$m\omega^2 r = \frac{GMm}{r^2} + \frac{4GMm R^2}{5c^2 r^2} \Omega \omega \tag{9}$$

Therefore, we obtain for prograde and retrograde orbits

$$\omega^2 = \frac{GM}{r^3} \left[ 1 \mp \frac{4R^2 \Omega \omega}{5c^2} \right] \tag{10}$$

where, inside the bracket (-) sign corresponds to prograde and (+) sign corresponds to retrograde orbits.

Now,  $\omega = \frac{d\phi}{dt}$ , where  $\phi$  is the azimuthal angle in the equatorial plane and for a prograde orbit

$$\left(\frac{d\phi}{dt}\right)_+ = +\omega_k \left[ 1 - \frac{4R^2 \Omega \omega}{5c^2} \right]^{1/2} \tag{11}$$

and for retrograde orbit

$$\left(\frac{d\phi}{dt}\right)_- = -\omega_k \left[ 1 + \frac{4R^2 \Omega \omega}{5c^2} \right]^{1/2} \tag{12}$$

where,  $\omega_k = \sqrt{\frac{GM}{r^3}}$  is the Keplerian angular velocity. Taking the inverse of Eq. (11) and Eq. (12) and using the binomial theorem, we get approximately

$$\left(\frac{dt}{d\phi}\right)_+ = \frac{1}{\omega_k} \left[ 1 + \frac{2R^2 \Omega \omega}{5c^2} \right] \tag{13}$$

$$\text{and } \left(\frac{dt}{d\phi}\right)_- = -\frac{1}{\omega_k} \left[ 1 - \frac{2R^2 \Omega \omega}{5c^2} \right] \tag{14}$$

Integrating Eqs. (13) and (14) from 0 to  $2\pi$  for prograde orbit and from 0 to  $(-2\pi)$  for retrograde orbit, we get the periods as

$$t_+ = \frac{2\pi}{\omega_k} \left[ 1 + \frac{2R^2\Omega\omega}{5c^2} \right] \quad (15)$$

$$\text{and } t_- = \frac{2\pi}{\omega_k} \left[ 1 - \frac{2R^2\Omega\omega}{5c^2} \right] \quad (16)$$

where, we have assumed  $\omega$  on the right-hand side to be constant. Subtracting Eq. (16) from Eq. (15), we obtain

$$t_+ - t_- = \frac{2\pi}{\omega_k} \left( \frac{2R^2\Omega\omega}{5c^2} \right) + \frac{2\pi}{\omega_k} \left( \frac{2R^2\Omega\omega}{5c^2} \right) \quad (17)$$

Now,  $\omega_k \approx \omega$ . Therefore, we obtain

$$t_+ - t_- = \frac{8\pi R^2\Omega}{5c^2} \quad (18)$$

Using  $R^2\Omega = \frac{5}{2} \frac{J}{M}$ , the relation (18) reduces to

$$t_+ - t_- = \frac{4\pi J}{Mc^2} \quad (19)$$

which is the standard expression for the gravitomagnetic clock effect [34]. Thus, we have found a new path to the gravitomagnetic clock effect using gravitoelectromagnetism, especially the Lorentz force. The result here is an approximation, but in general, relativity the Eq. (19) is exact. This is important because the gravitomagnetic clock effect has already crossed the boundary of classical mechanics and entered the quantum regime. (see, for example, references [35,36].

### 3. Conclusion

The expression for the gravitomagnetic clock effect is derived by employing the Lorentz force law of gravitoelectromagnetism. Usually, the expression for the gravitomagnetic clock effect is found using general relativity. Here, we employ a different route to calculate the gravitomagnetic clock effect: the Lorentz force law of gravitoelectromagnetism. In gravitoelectromagnetism, the test and source bodies are assumed to have the opposite gravitoelectric and gravitomagnetic charges. But, here, we have employed a new approach: the source body and test body should have symmetrical gravitoelectric and gravitomagnetic charges. Thereby, the source body possesses gravitoelectric charge  $M$  and gravitomagnetic charge  $(-2M)$ . The test mass has a gravitoelectric charge  $(m)$  and gravitomagnetic charge  $(-2m)$ . With this provision, we have a Lorentz force law which is a little bit different from that of gravitoelectromagnetism described by the Lorentz force law, where the first term contains a positive mass  $(+m)$ , whereas, in gravitoelectromagnetism, it is  $(-m)$ . The final result, i.e., the gravitomagnetic clock effect is similar to general relativity. But our result is

approximate. We have used an approximation to general relativity, i.e., gravitoelectromagnetism. Thus, our work sheds new light on the theory of gravity.

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