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Limits on Effective Masses of Light and Heavy Majorana Neutrinos for Positron Emitting Modes of Double Beta Decay

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Abstract

Double beta decay is a rare weak interaction process in which two identical nucleons inside the nucleus decay with or without the emission of neutrinos. If the neutrinoless double beta decay is observed, the (e^+DBD) processes will play a crucial role in discriminating the finer issues like the dominance of Majorana neutrino mass or the right-handed current. In the present work, we have obtained the limits on the effective mass of light and heavy Majorana neutrinos for the electron-positron conversion and double positron-emitting modes of ^{96}Ru, ^{106}Cd, ^{124}Xe, and ^{130}Ba isotopes, using the nuclear transition matrix elements NTMEs $M^{(0\nu)}$ and $M^{(0N)}$ for light and heavy Majorana neutrinos obtained in projected Hartree-Fock Bogoliubov (PHFB) model. The predicted half-lives and corresponding extracted limits on heavy neutrino mass $\langle M_N \rangle$ is discussed. We have also calculated nuclear sensitivities $\xi^{(0\nu)}$ and $\xi^{(0N)}$ due to the light and heavy neutrino exchange, respectively. Finally, the mass limits are obtained using various phase space factors (PSF), and the effect of this PSF on mass limits is discussed.

Keywords: Double beta decay; Nuclear transition matrix elements; Majorana neutrino mass; Positron emitting modes; Nuclear sensitivities.

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1. Introduction

The nuclear double beta $(\beta\beta)$ decay is characterized by two modes. They are the two-neutrino double-beta $(2\nu\beta\beta)$ decay and the neutrinoless double beta $(0\nu\beta\beta)$ decay. These modes can further be classified into double electron $(\beta^-\beta^-)$ emission, double positron $(\beta^+\beta^+)$ emission, electron-positron conversion $(\varepsilon\beta^+)$ and double electron capture (ECEC). The latter three processes are energetically competing, and we refer to them as positron double beta decay (e^+DBD) modes as given in Eqs. (1) and (2).

$$\left.\begin{array}{r} {}^{A}_{Z}X \rightarrow {}_{Z+2}{}^{A}Y + 2e^- + 2\nu_e \quad (\beta^+\beta^+)_{2\nu} mod\,e \\ e^- + {}^{A}_{Z}X \rightarrow {}_{Z-2}{}^{A}Y + e^+ + 2\nu_e \quad (\varepsilon\beta^+)_{2\nu}\ mod\,e \\ 2e^- + {}^{A}_{Z}X \rightarrow {}_{Z-2}{}^{A}Y + 2\nu_e \quad (\mathrm{ECEC})_{2\nu} mod\,e \end{array}\right] \tag{1}$$

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$$\left.\begin{array}{r} {}^{A}_{Z}X \rightarrow {}_{Z+2}{}^{A}Y + 2e^{-} \quad (\beta^{+}\beta^{+})_{0\nu} mod\, e \\ e^{-} + {}^{A}_{Z}X \rightarrow {}_{Z-2}{}^{A}Y + e^{+} \quad (\varepsilon\beta^{+})_{0\nu}\ mod\, e \\ 2e^{-} + {}^{A}_{Z}X \rightarrow {}_{Z-2}{}^{A}Y \quad (\mathrm{ECEC})_{0\nu} mod\, e \end{array}\right] \tag{2}$$

The experimental and theoretical study of the (e+DBD) decay has been excellently reviewed [1-13]. Also, the observation of flavor oscillation of neutrinos at solar neutrino [14,15], atmospheric neutrino [16-18], and reactor neutrino experiments [19,20] confirmed the conclusion that neutrinos are massive particles. However, it is generally agreed that the observation of $0\nu\beta\beta$ decay can clarify a number of issues regarding the nature of neutrinos, namely the origin of neutrino mass (Dirac vs. Majorana), the absolute scale on neutrino mass, the type of a hierarchy, and CP violation in the leptonic sector, etc. [21]. On the other hand, observing $(e^{+}\mathrm{DBD})_{0\nu}$ decay modes will help decide issues such as the dominance of the mass mechanism or right-handed currents [22]. The experimental and theoretical study of $(\mathrm{ECEC})_{0\nu}$ decay mode has not been attempted in a great deal so far, even though the kinetic energy released in this decay mode is the largest. The emission of one real photon is forbidden for the $0^{+} \rightarrow 0^{+}$ transition if atomic electrons are absorbed from the *K*-shell. Therefore, one must consider various processes such as internal pair production, internal conversion, etc. [4]. The decay rates of the processes mentioned above have to be calculated at least by the third-order perturbation theory. Hence, the experimental, as well as theoretical study of (e+DBD)0ν decay, has been restricted to only $(\beta^{+}\beta^{+})_{0\nu}$ and $(\varepsilon\beta^{+})_{0\nu}$ modes. The complex structure of nuclei in general, and of mass region 96 < A < 156 in particular, is due to the subtle interplay of pairing and multipolar correlations present in the effective two-body interaction. The mass region $A \approx 100$ offers a nice example of shape transitions at $N = 60$. The nuclei are soft vibrators for neutron numbers $N < 60$ quasirotors for $N > 60$. Nuclei with neutron numbers $N = 60$ are transitional nuclei. Hence, it is clear that deformation plays a crucial role in reproducing the properties of these nuclei. The effects of pairing and quadrupolar correlations on the NTMEs of the $(\beta^{-}\beta^{-})_{0\nu}$ mode have been studied in the interacting shell model (ISM) [23]. In the projected Hartree-Fock Bogoliubov (PHFB) model, the role of deformation effects due to quadrupolar [24] and multipolar correlations [25] has also been studied.

In the PHFB model, the interplay of pairing and deformation degrees of freedom is treated simultaneously and on equal footing. However, the structure of the intermediate odd *Z*-odd *N* nuclei, which provide information on the single-β decay rates and the distribution of GT strengths, cannot be studied in the present version of the PHFB model. In spite, the PHFB model along with the Pairing plus quadrupole (PQQ) interaction [26] in conjunction with the summation method has been successfully applied to study the $(e^{+}\mathrm{DBD})_{2\nu}$ decay of ^{96}Ru, ^{106}Cd, ^{124}Xe and ^{130}Ba [27] isotopes for the $0^{+} \rightarrow 0^{+}$ transition, not in isolation but together with other observed nuclear spectroscopic properties, namely yeast energy spectra, reduced $B(E2{:}0^{+} \rightarrow 2^{+})$ transition probabilities, quadrupole moments $Q(2^{+})$, and gyromagnetic factors $g(2^{+})$. This success of the PHFB model has prompted us to apply the same to study the $0^{+} \rightarrow 0^{+}$ transition of $(\varepsilon\beta^{+})_{0\nu}$and $(\beta^{+}\beta^{+})_{0\nu}$ modes for the nuclei mentioned above. The article is organized as follows. A brief description of theoretical formalism is presented in section 2. In section 3, we have calculated limits on effective masses of light and heavy Majorana neutrinos and analyzed them along with nuclear sensitivities. The different phase space factors and their effect on light and heavy Majorana neutrino mass limits are discussed. Finally, some concluding remarks are presented in section 4.

2. Theoretical Formalism

In the neutrino mass mechanism, the half-lives $T_{1/2}^{0\nu}$ for the $0^+ \rightarrow 0^+$transition of $(\varepsilon\beta^+)_{0\nu}$ and $(\beta^+\beta^+)_{0\nu}$ modes in the 2n mechanism are given by [4,28]

$$\left[T_{1/2}^{0\nu}(\beta)\right]^{-1} = G_{01}(\beta) g_A^4 \left|\frac{\langle m_\nu \rangle}{m_e} M^{(0\nu)} + \frac{m_p}{\langle M_N \rangle} M^{(0N)}\right|^2 \tag{3}$$

Here, β denotes the $(\varepsilon\beta^+)_{0\nu}$ /$(\beta^+\beta^+)_{0\nu}$ modes. G_{01} is the integrated kinematical factor and m_p (m_e) being the proton (electron) mass. The effective light and heavy neutrino masses are given by

$$\langle m_\nu \rangle = \sum{}_i^{'} U_{ei}^2 m_i \,, m_i < 10 \ eV \ \ ; \langle M_N \rangle^{-1} = \sum{}_i^{''} U_{ei}^2 m_i^{-1}, m_i > 1GeV \tag{4}$$

where U_{ei} is the unitary mixing matrix, and m_i is the Majorana neutrino mass.

Also $M^{(K)} = M_{GT}^{(K)} - M_F^{(K)}$ (5)

Here, K = 0ν, $0N$ denotes the exchange of light and heavy Majorana neutrino mechanism. In the closure approximation, Fermi and Gamow-Teller, nuclear transition matrix elements (NTMEs),$M_F^{(K)}$and $M_{GT}^{(K)}$are written as

$$M_F^{(0\nu)} = \left(\frac{g_V}{g_A}\right)^2 \sum_{n,m} \left\langle 0_F^+ \parallel H(r) \tau_n^- \tau_m^- \parallel 0_I^+ \right\rangle \tag{6}$$

$$M_{GT}^{(0\nu)} = \sum_{n,m} \left\langle 0_F^+ \parallel \sigma_n . \sigma_m H(r) \tau_n^- \tau_m^- \parallel 0_I^+ \right\rangle \tag{7}$$

$$M_F^{(0N)} = 4\pi (m_p m_e)^{-1} \left(\frac{g_V}{g_A}\right)^2 \sum_{n,m} \left\langle 0_F^+ \parallel \delta(r) \tau_n^- \tau_m^- \parallel 0_I^+ \right\rangle \tag{8}$$

$$M_{GT}^{(0N)} = 4\pi (m_p m_e)^{-1} \sum_{n,m} \left\langle 0_F^+ \parallel \sigma_n . \sigma_m \delta(r) \tau_n^- \tau_m^- \parallel 0_I^+ \right\rangle \tag{9}$$

Here I,F denote the initial and final states.

The neutrino potential H(r) arising due to the exchange of light neutrino [29] is defined as

$$H(r) = \frac{2\pi R}{(2\pi)^3} \int d^3 q \, \frac{\exp(i\mathrm{q.r})}{\omega(\omega + \overline{A})} \tag{10}$$

With $\overline{A} = \langle E_N \rangle - \frac{1}{2} \langle E_I + E_F \rangle$ (11)

E_N, E_I, E_F being the energies of intermediate, initial, and final states. The effect due to the finite size of nucleons (FNS) is taken into account by a dipole type of form factor, making the replacement

$$g_v(q^2) = g_v\left(\frac{\Lambda^2}{\Lambda^2+q^2}\right)^2 ; g_A(q^2) = g_A\left(\frac{\Lambda^2}{\Lambda^2+q^2}\right)^2 \tag{12}$$

with Λ = 850 MeV and $g_V(q^2)$, $g_A(q^2)$ are real functions of a Lorenz scalar q^2. The values of these form factors in the zero momentum transfer limit are the vector and axial-vector coupling constants, respectively. The values taken for these coupling constants are g_v =1.0 and g_A =1.254. In the PHFB model, configuration mixing takes care of long-range correlations. The short-range correlation (SRC) effect, arising mainly from the repulsive nucleon-nucleon potential due to exchange of ρ and ω mesons, is incorporated through phenomenological Jastrow type of correlation using the Miller and Spencer parameterization by the prescription [30]

$$\left\langle j_1^\pi j_2^\pi J \left| O \right| j_1^\nu j_2^\nu J' \right\rangle \rightarrow \left\langle j_1^\pi j_2^\pi J \left| fOf \right| j_1^\nu j_2^\nu J' \right\rangle \tag{13}$$

where $f(r) = 1 - ce^{-ar^2}(1 - br^2)$ (14)

with a = 1.1 fm^{-2}, b = 0.68 fm^{-2} and c = 1.0

In the PHFB model, the calculation of the NTMEs $M^{(K)}$ of the $(\varepsilon\beta^+)_{0\nu}$and $(\beta^+\beta^+)_{0\nu}$ modes are carried out as follows. The two basic ingredients of the PHFB model are an independent quasiparticle mean-field solution and the projection technique. To start with, amplitudes (u_{im}, v_{im}) and expansion coefficients $C_{ij,m}$ required to specify the axially symmetric HFB intrinsic state $|\Phi_0\rangle$ with K = 0, is obtained by carrying out the HFB calculation by minimizing the expected value of the effective Hamiltonian. Subsequently, states with good angular momentum **J** are obtained from $|\Phi_0\rangle$ using the standard projection technique [31] given by

$$\left|\Psi_{00}^J\right\rangle = \frac{(2J+1)}{8\pi^2}\int D_{00}^J(\Omega)R(\Omega)\left|\Phi_0\right\rangle d\Omega \tag{15}$$

where $R(\Omega)$ and $D_{00}^J(\Omega)$ are the rotation operator and the rotation matrix, respectively. Further

$$\left|\Phi_0\right\rangle = \prod_{im}(u_{im} + v_{im}b_{im}^\dagger b_{\overline{im}}^\dagger)\left|0\right\rangle \tag{16}$$

with the creation operators$b_{im}^\dagger$ and $b_{i\bar{m}}^\dagger$ defined as

$$b_{im}^\dagger = \sum_\alpha C_{i\alpha,m}a_{\alpha m}^\dagger \text{ and } b_{\overline{im}}^\dagger = \sum_\alpha (-1)^{l+j-m}C_{i\alpha,m}a_{\alpha,-m}^\dagger \tag{17}$$

Finally, the NTMEs $M^{(K)}$ for the $(\varepsilon\beta^+)_{0\nu}$ and $(\beta^+\beta^+)_{0\nu}$ modes are calculated by using the closure approximation in the PHFB model as

$$M^{(K)} = \left\langle \Psi_{00}^{J_f=0} \left\| O^{(K)} \tau^- \tau^- \right\| \Psi_{00}^{J_i=0} \right\rangle$$
$$= \left[n_{Z,N}^{J_i=0}\, n_{Z-2,N+2}^{J_f=0} \right]^{-1/2} \int_0^{\pi} n_{(Z,N),(Z-2,N+2)}(\theta) \sum_{\alpha\beta\gamma\delta} \left(\alpha\beta \left| O^{(K)} \tau^- \tau^- \right| \gamma\delta \right)$$
$$\times \sum_{\varepsilon\eta} \frac{\left(f_{Z-2,N+2}^{(\nu)*} \right)_{\varepsilon\beta}}{\left[1 + F_{Z,N}^{(\nu)}(\theta) f_{Z-2,N+2}^{(\nu)*} \right]_{\varepsilon\alpha}} \frac{\left(F_{Z,N}^{(\pi)*} \right)_{\eta\delta}}{\left[1 + F_{Z,N}^{(\pi)}(\theta) f_{Z-2,N+2}^{(\pi)*} \right]_{\gamma\eta}} \sin\theta\, d\theta \quad (18)$$

where

$$n^J = \int_0^{\pi} \{\det[1 + F^{(\pi)}(\theta) f^{(\pi)\dagger}]\}^{1/2} \times \{\det[1 + F^{(\nu)}(\theta) f^{(\nu)\dagger}]\}^{1/2} d_{00}^J(\theta) \sin(\theta) d\theta \quad (19)$$

and

$$n_{(Z,N),(Z-2,N+2)}(\theta) = \left\{ \det\left[1 + F_{Z,N}^{(\pi)}(\theta) f_{Z-2,N+2}^{(\pi)\dagger} \right] \right\}^{1/2} \times \left\{ \det\left[1 + F_{Z,N}^{(\nu)}(\theta) f_{Z-2,N+2}^{(\nu)\dagger} \right] \right\}^{1/2} \quad (20)$$

The л(ν) represents the proton (neutron) on nuclei involved in the $(\varepsilon\beta^+)_{0\nu}/(\beta^+\beta^+)_{0\nu}$ mode.
The matrices $f_{Z,N}$ and $F_{Z,N}$ (θ) are given by

$$[f_{Z,N}]_{\alpha\beta} = \sum_i C_{ij_\alpha,m_\alpha} C_{ij_\beta,m_\beta} (v_{im_\alpha}/u_{im_\alpha}) \delta_{m_\alpha,-m_\beta} \quad (21)$$

$$[F_{Z,N}(\theta)]_{\alpha\beta} = \sum_{m'_\alpha m'_\beta} d_{m_\alpha m'_\alpha}^{j_\alpha}(\theta) d_{m_\beta m'_\beta}^{j_\beta}(\theta) f_{j_\alpha m'_\alpha, j_\beta m'_\beta} \quad (22)$$

The NTMEs $M^{(K)}$ for the $(\varepsilon\beta^+)_{0\nu}$and $(\beta^+\beta^+)_{0\nu}$ modes are calculated by evaluating the matrices $[f_{Z,N}]_{\alpha\beta}$ and $[F_{Z,N}(\theta)]_{\alpha\beta}$ using expressions (21) and (22) respectively and then using these values in equation (18), the required NTMEs are obtained with 20 Gaussian quadrature points in the range (0,π).

3. Results and Discussion

In our model space, the doubly even ^{76}Sr ($N = Z = 38$) and ^{100}Sn ($N = Z = 50$) nuclei were treated as inert cores for the nuclei in the mass region A = 96–108 namely ^{96}Ru and ^{106}Cd nuclei and for ^{124}Xe and ^{130}Ba nuclei in the mass region A = 124–130, respectively. With ^{76}Sr core, the single particle orbits used are $1p_{1/2}$, $2s_{1/2}$, $1d_{3/2}$, $1d_{5/2}$, $0g_{7/2}$, $0g_{9/2}$ and $0h_{11/2}$ with SPEs as -0.8 MeV, 6.4 MeV, 7.9 MeV, 5.4 MeV, 8.4 MeV, 0.0 MeV and 8.6 MeV respectively. Similarly, with ^{100}Sn core, single particle orbits taken are $2s_{1/2}$, $1d_{3/2}$, $1d_{5/2}$, $1f_{7/2}$, $0g_{7/2}$, $0h_{9/2}$ and $0h_{11/2}$ with energies 1.4 MeV, 2.0 MeV, 0.0 MeV, 12.0 MeV, 4.0 MeV, 12.5 MeV and 6.5 MeV respectively. The HFB wave functions were generated by using an effective Hamiltonian with a PQQ type of effective two-body interaction [26] given by

$$H = Hsp + V(P) + V(QQ) \quad (23)$$

where *Hsp*, *V* (*P*), and *V* (*QQ*) represent the single-particle Hamiltonian, the pairing, and quadrupole-quadrupole part of the effective two-body interaction, respectively.

3.1. *Results of $(\varepsilon\beta^{+})_{0\nu}$ and $(\beta^{+}\beta^{+})_{0\nu}$ modes*

In Table 1, we have presented the obtained limits on effective light and heavy Majorana neutrino masses using Eq (3) through available experimental half-lives for ^{96}Ru, ^{106}Cd, ^{124}Xe and ^{130}Ba isotopes using the phase space factors G_{01} as 9.62 x10^{-18} yr^{-1} (8.45 x10^{-19} yr^{-1}), 1.30 x10^{-17} yr^{-1} (9.6 x10^{-19} yr^{-1}), 1.97 x10^{-17} yr^{-1} (1.14 x10^{-18} yr^{-1}), 1.76 x10^{-17} yr^{-1} (2.57 x10^{-19} yr^{-1}) for $(\varepsilon\beta^{+})_{0\nu}$ ($(\beta^{+}\beta^{+})_{0\nu}$) mode in case of these four isotopes respectively [32].

Table 1. Experimental half-lives $T_{1/2}^{0\nu}$ $(exp.)$ (at 90% C. L.) and limits on the effective masses $\langle m_\nu\rangle$ and $\langle M_N\rangle$ of Majorana neutrinos for the $(\varepsilon\beta^{+})_{0\nu}$and $(\beta^{+}\beta^{+})_{0\nu}$ modes of ^{96}Ru, ^{106}Cd, ^{124}Xe and ^{130}Ba isotopes.

Nuclei	$T_{1/2}^{0\nu}$ (exp) (yr)		Ref.	$M^{(0\nu)}$	$M^{(0N)}$	$\langle m_\nu\rangle$ (eV)		$\langle M_N\rangle$ (GeV)	
	$\varepsilon\beta^{+}$	$\beta^{+}\beta^{+}$				$\varepsilon\beta^{+}$	$\beta^{+}\beta^{+}$	$\varepsilon\beta^{+}$	$\beta^{+}\beta^{+}$
^{96}Ru	>7.7x10^{19}	>1.3x10^{20}	[33]	2.274	66.196	5.3x10^{3}	1.4x10^{4}	2.7x10^{3}	1.0x10^{3}
^{106}Cd	>1.4x10^{22}	>5.9x10^{21}	[34]	4.007	123.597	1.9x10^{2}	1.1x10^{3}	7.8x10^{3}	1.4x10^{4}
^{124}Xe	>1.2x10^{18}	>4.2x10^{17}	[35]	1.969	61.339	3.4x10^{4}	2.4x10^{5}	4.4x10^{2}	62.6
^{130}Ba	>4.0x10^{21}	>4.0x10^{21}	[36]	1.698	54.227	7.2x10^{2}	5.9x10^{3}	2.1x10^{4}	2.6x10^{3}

The matrix elements $M^{(0\nu)}$and $M^{(0N)}$for light and heavy Majorana neutrinos are calculated in the PHFB model using short-range correlation and finite-size effect and are given in columns 5 and 6, respectively. From the Table, it is seen that the extracted limits on m_ν and M_N are not so much stringent as in the case of $(\beta^{-}\beta^{-})_{0\nu}$ decay. It is observed that better limits are obtained for $(\varepsilon\beta^{+})_{0\nu}$ mode even if the limits on half-lives of $(\varepsilon\beta^{+})_{0\nu}$ and $(\beta^{+}\beta^{+})_{0\nu}$ modes are same. The best-obtained limits are $\langle m_\nu\rangle$ < 1.9x10^{2} eV for ^{106}Cd isotope and $\langle M_N\rangle$>> 2.12x10^{4} GeV for ^{130}Ba isotope in case of $(\varepsilon\beta^{+})_{0\nu}$ mode while $\langle m_\nu\rangle$ < 1.07x10^{3} eV and $\langle M_N\rangle$>> 1.37x10^{4} GeV for ^{106}Cd isotope in case of $(\beta^{+}\beta^{+})_{0\nu}$ mode. As the extracted limits on the effective neutrino masses $\langle m_\nu\rangle$ and $\langle M_N\rangle$ are not stringent enough, it is more meaningful to calculate half-lives of $(\varepsilon\beta^{+})_{0\nu}$ and $(\beta^{+}\beta^{+})_{0\nu}$ modes, which will be helpful in the design of future experimental setups. Hence, the half-lives of $(\varepsilon\beta^{+})_{0\nu}$ and $(\beta^{+}\beta^{+})_{0\nu}$ modes for $\langle m_\nu\rangle$ = 50 meV are calculated, and we have extracted corresponding limits on heavy neutrino mass $\langle M_N\rangle$, as shown in Table 2.

Table 2. Predicted half-lives $T_{1/2}^{0\nu}$, corresponding extracted effective mass of heavy Majorana neutrino $\langle M_N\rangle$, nuclear sensitivities $\xi^{(0\nu)}$ and $\xi^{(0N)}$ for the $(\varepsilon\beta^{+})_{0\nu}$ and $(\beta^{+}\beta^{+})_{0\nu}$ modes.

Nuclei	$T_{1/2}^{0\nu}$ $(\langle m_\nu\rangle = 50 meV)$		$\langle M_N\rangle$(GeV)	$\xi^{(0\nu)}$		$\xi^{(0N)}$	
	$\varepsilon\beta^{+}$	$\beta^{+}\beta^{+}$	$\varepsilon\beta^{+}/\beta^{+}\beta^{+}$	$\varepsilon\beta^{+}$	$\beta^{+}\beta^{+}$	$\varepsilon\beta^{+}$	$\beta^{+}\beta^{+}$
^{96}Ru	8.48×10^{29}	9.66×10^{30}	2.79×10^{8}	0.71	0.21	20.53	6.09
^{106}Cd	8.11×10^{29}	1.09×10^{31}	5.92×10^{8}	1.45	0.39	44.56	12.12
^{124}Xe	5.52×10^{29}	9.55×10^{30}	2.99×10^{8}	0.61	0.18	27.23	6.55
^{130}Ba	8.33×10^{29}	5.70×10^{31}	3.06×10^{8}	0.53	0.16	22.75	2.75

In the absence of stringent limits on the effective neutrino masses $\langle m_\nu \rangle$ and $\langle M_N \rangle$, we can calculate the nuclear sensitivity, defined as [37]

$$\xi^{(K)} = 10^8 \sqrt{G_{01}} \left| M^{(K)} \right| \tag{24}$$

where K stands for 0ν or $0N$ mode, and an arbitrary normalization factor 10^8 is introduced to make the nuclear sensitivity of the order of unity. It is observed that, in general, nuclear sensitivities for $(\varepsilon\beta^+)_{0\nu}$ mode are larger than those for $(\beta^+\beta^+)_{0\nu}$ mode. Further, in case of light neutrinos, the nuclear sensitivities for ^{106}Cd, ^{96}Ru, ^{124}Xe, and ^{130}Ba isotopes are in decreasing order of magnitude for $(\varepsilon\beta^+)_{0\nu}$ and $(\beta^+\beta^+)_{0\nu}$ modes. In case of heavy neutrinos, the nuclear sensitivities show decreasing behavior for ^{106}Cd, ^{124}Xe, ^{130}Ba, and ^{96}Ru isotopes respectively for $(\varepsilon\beta^+)_{0\nu}$ mode while for $(\beta^+\beta^+)_{0\nu}$ mode, similar behavior is observed except ^{96}Ru isotope having nuclear sensitivity greater than ^{130}Ba isotope.

3.2. *Effect of phase space factors on effective light and heavy Majorana neutrino mass limits*

In order to observe the effect of phase space factors on effective light $\langle m_\nu \rangle$ and heavy $\langle M_N \rangle$ Majorana neutrino mass limits, we have presented the phase factors (PSF) for $(\varepsilon\beta^+)_{0\nu}$ and $(\beta^+\beta^+)_{0\nu}$ modes in units of yr^{-1} calculated by Kotila and Iachello [32] and Doi and Kotani [38] in Table 3.

Table 3. Phase space factors for $(\varepsilon\beta^+)_{0\nu}$and $(\beta^+\beta^+)_{0\nu}$ modes in units of yr^{-1} taken from Kotila and Iachello [32], Doi and Kotani [38], and Boehm and Vogel [39] by removing g_A^4.

Nuclei	Phase space factors					
	$(\varepsilon\beta^+)_{0\nu}$ (10^{-18} yr^{-1})			$(\beta^+\beta^+)_{0\nu}$(10^{-20} yr^{-1})		
	KI	DK	BV	KI	DK	BV
^{96}Ru	9.62	10.8	-	84.5	90.7	12.2
^{106}Cd	13.0	14.7	-	96.2	102	14.5
^{124}Xe	19.7	22.9	-	114	123	18.1
^{130}Ba	17.6	19.8	-	25.7	21.0	1.67

Kotila and Iachello have calculated PSF using exact Dirac wave functions with finite nuclear size and electron screening, while Doi and Kotani obtained the results by using approximate electron wave functions. For comparison, we have also included the PSF calculated by Boehm and Vogel [39], but their results are not available for $(\varepsilon\beta^+)_{0\nu}$ mode. Using these PSF, we have obtained the limits on effective masses of light $\langle m_\nu \rangle$ and heavy $\langle M_N \rangle$ Majorana neutrinos for the $(\beta^+\beta^+)_{0\nu}$ and $(\varepsilon\beta^+)_{0\nu}$ modes of ^{96}Ru, ^{106}Cd, ^{124}Xe, and ^{130}Ba isotopes and are shown in Table 4 for comparison.

It is observed that the mass limits $\langle m_\nu \rangle$ on light neutrinos obtained by PSF of Kotila and Iachello (KI) and Doi and Kotani (DK) are nearly same but increase by a factor of 2.6 approximately in case of $(\beta^+\beta^+)_{0\nu}$ mode for ^{96}Ru, ^{106}Cd, ^{124}Xe isotopes and by a factor of 3.5 nearly for ^{130}Ba isotope for PSF of Boehm and Vogel (BV). In the case of $(\varepsilon\beta^+)_{0\nu}$ mode, the mass limits do not change much for PSF of KI and DK, while results of BV are not present for this mode. In the case of heavy Majorana neutrinos, once again, the mass limits $\langle M_N \rangle$ are approximately the same for PSF of KI and DK for the two modes, while for $(\beta^+\beta^+)_{0\nu}$ mode, the limits decrease by the same factors as in the case of light neutrinos for ^{96}Ru, ^{106}Cd, ^{124}Xe, and ^{130}Ba isotopes.

Table 4. Effect of phase space factor values on the effective mass limits of light $\langle m_\nu \rangle$ and heavy $\langle M_N \rangle$ Majorana neutrinos for the $(\beta^+\beta^+)_{0\nu}$ and $(\varepsilon\beta^+)_{0\nu}$ modes of ^{96}Ru, ^{106}Cd, ^{124}Xe, and ^{130}Ba isotopes.

Nuclei	$\langle m_\nu \rangle (10^3$ eV)					$\langle M_N \rangle$ (10^3 GeV)				
	$(\beta^+\beta^+)_{0\nu}$			$(\varepsilon\beta^+)_{0\nu}$		$(\beta^+\beta^+)_{0\nu}$			$(\varepsilon\beta^+)_{0\nu}$	
	KI	DK	BV	KI	DK	KI	DK	BV	KI	DK
^{96}Ru	14	13	36	5.2	4.9	1.0	1.1	0.39	2.7	2.8
^{106}Cd	1.5	1.5	3.9	0.58	0.55	9.8	10	3.8	26	27
^{124}Xe	240	230	590	34	32	0.06	0.07	0.03	0.44	0.47
^{130}Ba	5.9	6.6	23	0.72	0.68	2.6	2.3	0.67	21	23

Thus it is concluded that the limits obtained on effective masses of light $\langle m_\nu \rangle$ and heavy $\langle M_N \rangle$ Majorana neutrinos for the $(\beta^+\beta^+)_{0\nu}$ and $(\varepsilon\beta^+)_{0\nu}$ modes of ^{96}Ru, ^{106}Cd, ^{124}Xe and ^{130}Ba isotopes using the PSF of KI and DK are nearly the same while results differ by a factor of 2.6 and 3.5 approximately for PSF of BV in case of ^{96}Ru, ^{106}Cd, ^{124}Xe, and ^{130}Ba isotopes respectively.

4. Conclusion

We have calculated the limits on the effective mass of light $\langle m_\nu \rangle$ and heavy $\langle M_N \rangle$ Majorana neutrinos for the $(\varepsilon\beta^+)_{0\nu}$ and $(\beta^+\beta^+)_{0\nu}$ modes of ^{96}Ru, ^{106}Cd, ^{124}Xe and ^{130}Ba isotopes using matrix elements calculated in PHFB model. The best limits are obtained for ^{106}Cd isotope. Also, the half-lives of the isotopes mentioned above for $\langle m_\nu \rangle = 50$meV is calculated in case of $(\varepsilon\beta^+)_{0\nu}$ and $(\beta^+\beta^+)_{0\nu}$ modes and corresponding limits on heavy neutrino mass, $\langle M_N \rangle$ are extracted. The highest limit is obtained for ^{106}Cd isotope corresponding to $(\varepsilon\beta^+)_{0\nu}$ and $(\beta^+\beta^+)_{0\nu}$ modes. Further, the nuclear sensitivities are calculated, and in general, the sensitivities for $(\varepsilon\beta^+)_{0\nu}$ mode are found larger than those of $(\beta^+\beta^+)_{0\nu}$ mode. The three different phase space factors are used to see the effect on mass limits $\langle m_\nu \rangle$ and $\langle M_N \rangle$ of light and heavy Majorana neutrinos and it is observed that the mass limits due to two PSF using exact and approximate electrons wave functions by KI and DK respectively do not change much but differ by a large factor due to PSF of BV.

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