

Thermodynamic Free Energy Calculation with One Loop Correction at Finite Chemical Potential

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Received 25 February 2026, accepted in final revised form 12 July 2026

Abstract

The thermodynamic free energy of quark gluon plasma (QGP) with one loop correction is calculated in presence of finite chemical potential to create bubble/droplet formation of QGP. The equilibrium/ stable and unstable droplets are formed at parametrized values of quarks and gluons at the range of $\gamma_q = \frac{1}{8}$ and $22\gamma_q \leq \gamma_g \leq 30\gamma_q$. However, the energy equilibration/stable of the droplet is found at the range of quark and gluon parameterization as $\gamma_q = \frac{1}{8}$ and $26\gamma_q \leq \gamma_g \leq 28\gamma_q$. It means that the phenomenological parameters introduced, boost in forming the droplet of QGP. However, the effect of the stable droplet size after the incorporation of chemical potential at one loop correction factor is visible with smaller size.

Keywords: Quark-gluon plasma; Quantum chromodynamics; Free energy evolution; Chemical potential.

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doi: <https://dx.doi.org/10.3329/jsr.v18i3.88032>

J. Sci. Res. **18** (3), 501-510 (2026)

1. Introduction

The theory of strong interactions of coloured quarks and gluons is explained by quantum chromodynamics (QCD). These coloured particles of quark and gluon are described by fields of gauge theory. The early universe phase transition is happened between the phases of coloured quarks, gluons and uncoloured particle phases of hadrons. The universe's phase transition is also defined by the strong interaction theory of QCD [1,2]. So, the change of phases can be occurred in two different conditions say at a very high nuclear density which is dominated by rich chemical potential and another one can be achieved at a very excessive energy scale or soaring temperature [3]. However, it is believed that the state of free quarks and gluons is very short life period and study about this short life is a difficult phenomenon [4-6]. Moreover, it is our duty to find and investigate what and how it is formed. So, such dark and complicated situation has kept our minds busy for its investigation. After the investigation about the existing short period of phases, it is found that the matter body is formed of the unbound quarks and gluons known as quark-gluon plasma (QGP) [7-9]. The matter is still believed to be an ideological concepts and research on the short life matter is

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never-ending till today. Obviously, it is nature of humankind that research on the multiplex nature is the most attractive and fascinating issue. So, it pulls a lot of people towards the topic as it has a lot of interesting works and lot of attempts have been considered caring with working out to investigate the transformation of quark-hadron phase transition. Therefore, few known experimental opportunities are set up around the world like relativistic heavy-ion collision (RHIC) at Brookhaven National Laboratory (BNL) and large hadron collider (LHC) at CERN for investigation and research on this peculiar topic [10-13]. In addition to these two famous experiments, chemical potential-based experiments like FAIR at Darmstadt and NICA at Dubna are built up particularly focused on chemical potential [14,15]. These two laboratory studies are known as dense baryonic matter or compact baryon matter (CBM). The Nuclotron (BM@N) experiments use beams of ions extracted from modernized Nuclotron and collide to produce particular facts about the presence of large number of particles in the early universe formation. However it will also provide the existence of early universe phase transition, short life phase of QGP droplet formation and phase diagram of QCD [16-19]. So studying QGP in Ultra Relativistic Heavy-Ion Collision is an thrilling field in the current contexts of heavy ion collider physics and just because of the fascinating knowledge, theory and experimental groups focus towards the subject named “quark gluon plasma” formation under the initialisation of chemical potential as a final target of their studies [20,21]. However, there is information that the convergence of chemical potential and temperature at a particular point says about the existence of mixed phase [22].

In this short article, the calculation of free energy formation is checked with the changes of the free energy vs the size of droplet taking different flow parameters of quark and gluon. The calculation is done by considering the chemical potential probably in the scale of quantum chromodynamic limits. It is taken into account by the existence of huge number of charge particles in the system. Therefore, the presence of chemical potential is accountable in the current contexts of QGP formation. Experimentally too it has solved that the accelerating particles carry charge particles. On the basis of this information, they have set up the laboratory like CBM in Darmstadt, Germany for the detailed study about it. Thus, the initialisation of chemical potential in the current calculation is so suitable approach for the choice for a phenomenological purpose of the study. To find the free energy, a thermodynamic partition function correlating the Gibbs free energy of the system with the inclusion of chemical potential is developed. To incorporate all these parameters with one loop correction in the mean field of the free energy, the density of state in the construction of the free energy is modified. It is also found through the flow parameters of quark and gluon. Therefore, there are modifications in the free energy expansion of QGP fireball, and due to the modifications, the free energy with the equilibrium/stable droplet formation is also varied with the change of dynamical quark and gluon flow parameters [23]. Therefore, the flow parameters of quark and gluon have the important role in the evolution of free energy with the equilibrium droplets/bubbles with respect of the change of temperature. Again, there are number of probable signals of formation of QGP like photon/dilepton productions, strangeness enhancement and hydrodynamical models [24,25]. One of the

most successful models to explain about the description of QGP formation is the relativistic hydrodynamics which provides the quantitative pictures of heavy-ion collision dynamics [26-28] However, the hydrodynamic picture is tending to produce the next generation approximation in terms of improvement in model data agreement [29,30].

Briefly, the thermodynamic free energy through the density of state at finite chemical potential incorporating one loop correction is studied and evaluated [31,32]. In the evaluation, the temperature is chosen as a parameter for looking QGP-phase transition and bubble/droplet formation varying temperature at the range of normally observed values of (150-270) MeV. Due to it, we have found the bubble/droplet formation within these ranges of temperature with a slight expansion of temperature, probably found as a mixed phase of quark and hadrons. However, the chemical potential is considered on a random choice with the scale of the temperature. Now the changes in the free energy produced by all these parameters are incorporated in this article.

Therefore, the article is organized as: In section 1, the introduction of QGP is highlighted. In section 2 the density of state in presence of chemical potential with one loop correction is discussed. Then in section 3, the thermodynamic free energy at finite chemical potential is established with one loop correction. Section 4 gives the numerical results and at last in section 5 it is summarized as concluded.

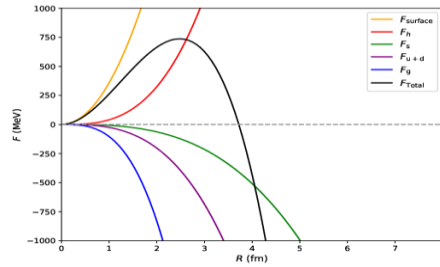


Fig. 1. Free energy graph of individual particles vs. R at $\gamma_q = \frac{1}{8}$, $\gamma_g = 12\gamma_q$ for particular temperature $T = 170 \text{ MeV}$ and chemical potential $\mu = 300 \text{ MeV}$

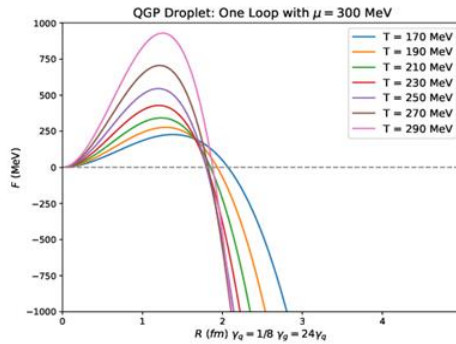


Fig. 2. Total free energy vs. R the size of the droplet at $\gamma_q = \frac{1}{8}$, $\gamma_g = 24\gamma_q$ and $\mu = 300 \text{ MeV}$ for different values of temperature.

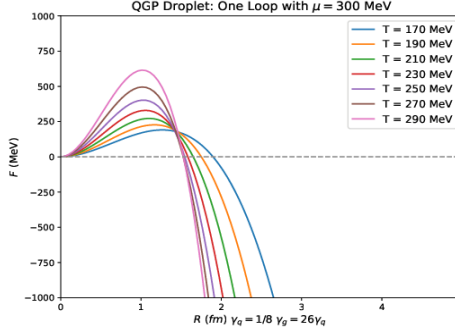


Fig. 3. Total free energy vs R the size of the droplet at $\gamma_q = \frac{1}{8}$, $\gamma_g = 26\gamma_q$ and $\mu = 300 \text{ MeV}$ for different values of temperature.

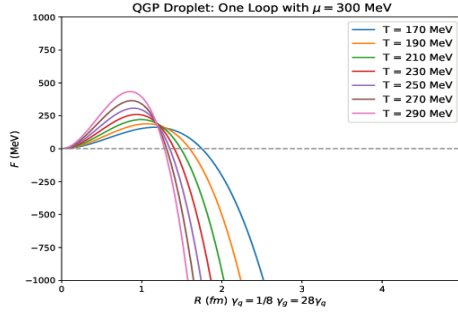


Fig. 4. Total free energy vs. R the size of the droplet at $\gamma_q = \frac{1}{8}$, $\gamma_g = 28\gamma_q$ and $\mu = 300 \text{ MeV}$ for different values of temperature.

2. Density of State at Finite Chemical Potential with One Loop Correction

In current calculation of density of state (DOS), the potential model of the Thomas-Fermi electronic configuration is used as the quarks are having a similar fermionic state of electrons. The interacting potential defined at this finite chemical potential is derived by extending the earlier one loop correction of potential with inclusion of chemical potential [33,34]. The one loop potential is obtained through the heavy quark interaction and here, we assume that the dynamical quarks are modified due to thermal excitation and its mass goes up with the temperature. So, the potential produced by heavy quarks is similar to the potential of modified quark mass, known as quasi mass as they give much larger value within the range of temperature considered herewith. It, therefore, fits to explain the free energy evolution of QGP. Now the modified potential through the quasi mass is given as:

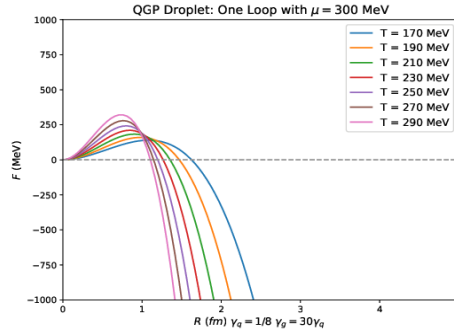


Fig. 5. Total free energy vs. R the size of the droplet at $\gamma_q = \frac{1}{8}$, $\gamma_g = 30\gamma_q$ and $\mu = 300 \text{ MeV}$ for various values of temperature.

$$V(k) = \frac{32\pi^2\gamma}{27k} \frac{T'^2}{\ln\left[1 + \frac{k^2 + \mu^2}{\Lambda^2}\right]} \left[1 + \frac{\alpha_s(k)}{4\pi} a_1\right] \quad (1)$$

where $T'^2 = \left(T^2 + \frac{\mu^2}{\pi^2}\right)$, γ is the parameter given by root mean square value of the quark and gluon flow parameters say $\gamma_q = \frac{1}{8}$ and $\gamma_g = (22 - 30)\gamma_q$. $\alpha_s(k)$ is defined as done in earlier papers. It is:

$$\alpha_s(k) = \frac{4\pi}{(33 - 2n_f) \ln\left[1 + (k^2 + \mu^2)/\Lambda^2\right]}$$

in which Λ is known as quantum chromodynamics parameter normally taken as 150 MeV . μ is finite chemical potential. a_1 is the one loop correction factor defined as

$$a_1 = 2.5833 - 0.2778n_l \quad (2)$$

with n_l as the number of light quarks [35,36]. So, using the potential we obtain the density of state defined in phase space after the inclusion of one loop correction in the interaction potential at finite chemical potential μ as:

$$\rho(k) = \frac{v}{\pi^2} \left[\frac{\gamma^2 T'^2}{2}\right]^3 g^6(k) C. \quad (3)$$

In the expression, v is the volume covered by QGP drop; $g^2(k) = 4\pi\alpha_s(k)$ and C is constant obtained by simplification through the potential in above Eq. (1) after the induction of one loop correction. It is finally obtained as:

$$C = \left\{1 + \frac{\alpha_s(k)a_1}{\pi}\right\}^2 \left[\frac{\left(1 + \frac{\alpha_s(k)a_1}{\pi}\right)}{k^4} + \frac{2\left(1 + \frac{2\alpha_s(k)a_1}{\pi}\right)}{k^2(K^2 + \Lambda^2) \ln\left(1 + \frac{k^2 + \mu^2}{\Lambda^2}\right)} \right]. \quad (4)$$

3. Thermodynamic Free Energy at Finite Chemical Potential

The thermodynamic canonical ensemble could be developed by understanding thermodynamic parameters like the temperature, the chemical potential and the Hamiltonian of the system. So, the partition function of the system is derived in most of literature as [37]

$$Z(T, \mu, V) = \text{Tr} \{ \exp[-\beta(H - \mu N)] \} \quad (5)$$

where μ represents chemical potential, N represents quark number and $\beta = \frac{1}{T}$, inverse of temperature. Correlating the Gibbs free energy using the partition function, it is found in this way as [38-40]:

$$F_i = T \ln Z(T, \mu, V) \quad (6)$$

The free energy is now calculated using the above density of states where the Fermi-Dirac and the Bose-Einstein distribution functions for the corresponding particles are used. It is expressed in the following relation as:

$$F_i = \pm T g_i \int \rho_{q,g}(k) \ln[1 \mp \exp - (\sqrt{m^2 + k^2} - \mu)/T] dk \quad (7)$$

where all the symbols are respectively defined and they have their usual meanings as used in earlier literature; g_i is degree of freedom for colour and particle antiparticle degeneracy for quarks and gluons; $\rho_{q,g}$ is the density of states of quarks and gluons in phase space after the inclusion of one loop correction in the interacting potential at chemical potential μ .

Incorporating all these factors in the Eq. (7) the free energies of fermions and bosons are calculated. Besides the free energies, an inter-facial energy which take care of the hydrodynamic effects and the Bag energy constant is given below. In this inter facial energy the gamma factor is used [41,42].

$$F_{interface} = \frac{\gamma T R^2}{4} \int k^2 \delta(k - T) dk \quad (8)$$

in which γ is the root mean square value of quark γ_q , gluon flow parameter γ_g and R is QGP droplet size. However, there are some light mesonic free energies with the corresponding degeneracy factor of g given as [43,44]:

$$F_h = (gT/2\pi^2) \nu \int_0^\infty k^2 \ln \left[1 - e^{-\left(\sqrt{m_h^2 + k^2} - \mu/T\right)} \right] dk \quad (9)$$

where m_h is the corresponding mass of mesonic particles. These hadronic particles are considered in the peripheral phases of the QGP formation because of an assumption of probable existence of a few light hadrons at the temperatures (150-350) MeV obtained after subsequent cooling process. Due to the contributions of light hadrons, there may be some effects in the evolution of free energy which bring the model suitable pictures in the calculation of thermodynamic free energies and its parameters. Finally, all these contributed free energies are summed up and then compute the total modified free energy F_{total} as:

$$F_{total} = \sum_i F_i + F_{interface} + F_h \quad (10)$$

where i represents for u , d and s quark and gluon. So, the dynamics of QGP flow are determined and look into the details about the enhancing properties of phase transition to hadrons.

4. Results

The numerical solutions of the free energy evaluated through density of states obtained through the potential model are shown in the figures. The Fig. 1 directs the change of the free energy of the individual participants say quark, gluon, inter-facial and the most dominant contribution pion among the hadrons, with the size of droplets of these individual particles. The individual particles show the corresponding energies carried by them. The figure shows the traces at the chemical potential $\mu = 300 \text{ MeV}$, and temperature $T = 170$

MeV at one random quark and gluon flow parameter $\gamma_q = \frac{1}{8}$ and $\gamma_g = 12\gamma_q$. In the figure, the combined free energy of all these particles and contribution of few light element particles of hadrons having less masses than $1000 MeV$, taken into consideration in the calculation of free energy is plotted.

In Fig. 2 the plot of the combined free energies is shown at different temperatures at the chemical potential $\mu = 300 MeV$. The droplet formation of different sizes according to the different temperatures at quark and gluon parameters $\gamma_q = \frac{1}{8}$ and $\gamma_g = 24\gamma_q$ is observed. At this particular parameter the droplets are formed without stability /equilibrium in the droplet of QGP. It only shows that there is formation of QGP droplet. With the increasing temperature, the size of the droplet decreases at the finite chemical potential with the flow parameter $= \frac{1}{8}$ and $\gamma_g = 24\gamma_q$.

In Fig. 3 the plot is the free energy versus droplet size with the quark flow parameter fixed at $\gamma_q = \frac{1}{8}$ and increased gluon flow parameter to $\gamma_g = 26\gamma_q$. At such parameter the droplets/bubbles are visible and the droplets are also stable droplet at the chemical potential $\mu = 300 MeV$ with the smaller sizes, indicating the improvement over the droplet stability formation from the earlier droplets at $\gamma_q = \frac{1}{8}$ and $\gamma_g = 24\gamma_q$.

It means that the droplet size is still observable obtaining with smaller droplet by the increased gluon flow parameter and fixed quark flow parameter. From this analysis, the stable droplets/bubbles exist in the dynamics of QGP fluid depending on the value of quark and gluon parametrization.

In Fig. 4 it is observed that the stable droplet is found with the size of the droplet around $9 fm$ at $\gamma_q = \frac{1}{8}$ and $\gamma_g = 28\gamma_q$. It indicates that there is stable drop of QGP if the choice of the gluon flow parameter is made suitable. It is a similar situation with the Fig. 3 slightly or marginally changed in the size of the droplets. It shows that the droplet is more stable with the increase of temperature with the increased gluon parameter.

In Fig. 5, the gluon parameter is further increased and look at the possible modification in the stability of the droplet. The droplets are not much stable with modification in the gluon flow parameter. The result only gives the idea of forming the droplet. However, there is no stability in the QGP droplet beyond the value of the gluon parameter $\gamma_g = 28\gamma_q$. It says that the equilibrium free energy or stable droplet is obtained in a particular range of quark and gluon parameters say $\gamma_q = \frac{1}{8}$ and $26\gamma_q \leq \gamma_g \leq 28\gamma_q$ with larger amplitude in free energy with the consideration of modified quark mass. So, the dynamics of the QGP droplet is turbulent at any temperature and chemical potential in this model when the parameters are considered at $\gamma_g < 26\gamma_q$ and $\gamma_g > 28\gamma_q$. However, the droplet of the QGP fluid can be returned when the quark and gluon flow parameter are in the range of $22\gamma_q \leq \gamma_g \leq 30\gamma_q$ with stable and unstable droplets. So, such effects are visible due to presence the chemical potential in the system of one loop correction in the interacting potential and therefore, the free energy amplitude is affected by the modified quark mass.

So, it is concluded that the extensive work of the earlier calculation of one loop correction with the incorporation of chemical potential can still put forward the stable droplet in the flow of QGP dynamics and enhance free energy amplitude obtained with quasi model quark mass.

5. Conclusion

It is summarized that the numerical results due to the initialisation of chemical potential and one loop correction of the mean field potential are analysed. The stability and the non-stability of droplets are found to be observed in the range of quark and gluon flow parameters as $\gamma_q = \frac{1}{8}$ and $22\gamma_q \leq \gamma_g \leq 30\gamma_q$. However, the stability is observed in the range $26\gamma_q \leq \gamma_g \leq 28\gamma_q$ with the free energy amplitude enhanced. The decrease of the droplet size is also observed with the inclusion of chemical potential in this one loop correction. So, it is concluded that QGP fluid movements can be safeguarded by some sorts of hydrodynamical parameter called quark and gluon flow parameters to get the stable droplets of the fluid. The stability conditions of the droplet enhance the further studies about other thermodynamic properties like quark number density, susceptibility and Taylor coefficients of QGP droplets.

Acknowledgments

Some of the colleagues are very much thankful for their useful discussions in preparing manuscript and critical reading of the manuscript. I thank Institution of Eminence, University of Delhi, Delhi for providing faculty research program grant Ref. No./IOE/2025-26/12/FRP.

List of abbreviations: BNL: Brookhaven National Laboratory; CBM: Compact Baryon Matter; CERN: European Organization for Nuclear Research; DOS: density of state; FAIR: Facility for Antiproton and Ion Research; LHC: Large Hadron Collider; NICA: NuclotronBased Ion Collision Facility; QCD: Quantum Chromodynamics; QGP: Quark Gluon Plasma, RHIC: Relativistic Heavy-Ion Collision

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