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Two Interacting Fluids with Quadratic EOS in Five-Dimensional Bianchi Model

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Abstract

In this paper, a five-dimensional Bianchi type cosmological model that has two interacting ideal fluids that are related to dark energy and dark matter, together with a quadratic EOS with time-dependent parameters $\gamma(t)$ and $\Lambda(t)$ is obtained. In this work, the Einstein field equations have been solved. Also found equation of state parameter $\gamma(t)$ and $\Lambda(t)$ as $t \to \infty$. Our outstanding results shows that the resultant universe depends on the parameter $\gamma(t)$ and $\Lambda(t)$ for interacting dark energy. Also studied LR and PR behaviors. We discuss the two cases of LR model for the defined Hubble parameter $H(t) = H_0 e^{\lambda t}$ and $H(t) = (t^n/\tau^n) - 1$ and then discuss the physical parameters for $t \to 0$ & $t \to \pm\infty$. In PR model we consider Hubble parameter $H = H_0 - H_1 e^{-\lambda t^{(n+1)}}$ and discuss the density for the dark matter for $t \to 0$ & $t \to \pm\infty$. It is observed that, the universe is expanding faster than usual due to influence of coupling between pair DE and DM.

Keywords: Bianchi type I metric; LR; PR; Dark Energy-Dark Matter; Quadratic EOS.

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1. Introduction

The universe is expanding at an accelerated rate, according to new data from type I supernovae in distant galaxies, CMBR anisotropies, mass-energy density estimations from galaxy clusters, weak lensing, and largescale structure [1,2]. An unusual element characterized by negative pressure and called "dark energy" and the ability to produce repulsive gravitational force, has been implicated in this expansion. The Wilkinson Microwave Anisotropy Probe (WMAP) measurements in the CMBR and the Large-Scale Structure (LSS) observations have indirectly confirmed the evidence for Dark Energy (DE).

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According to WMAP, baryon matter only accounts for 4 % of the universe's total energy, while DE makes up around 73 % of it and dark matter (DM) takes up 23 %. Consequently, the fast expansion of the universe now requires a cosmological model. The equation of state (EOS) $\gamma = {}^{p}/_{\rho}$, where ρ is the density, p is the pressure, and γ need not be time independent, is used to model the dark energy as a cosmic fluid. Because of its weird repulsive gravity, which $\gamma = -1$, many cosmologists believe that the Einstein cosmological constant is the most straightforward explanation for DE. The dark energy issue is a result of the peculiarities of this exotic matter source.

The literature contains a variety of solutions for the DE issue. These ideas can be divided into two groups: those that modify the Einstein-Hilbert action to create so-called modified or alternative theories of gravity, allowing the origin of the acceleration to match the gravitational theory [3-7] and those that introduce an energy momentum tensor to explain the acceleration phases within the context of Einstein's general relativity [8-12]. The universes filled with an abnormal fluid were presented by [13-16]. Several cosmological scenarios, including the big rip [17-19], the little rip (LR) [20-27], the pseudo rip (PR) [28], and the quasi rip [29], have been proposed for the progression of the universe. The incidence of the LR and PR models was examined by Brevik *et al*. [30] in relation to the impact of the interaction between the DE and DM in the inhomogeneous EOS $p_d = \gamma(t)\rho_d + \Lambda(t)$ and $p_m = \tilde{\gamma}(t)\rho_m$ for DE and DM, respectively. Recently some researchers studied with quadratic EOS [31-34].

The cosmological model observed by Shelote and Khadekar [35] showed that the dark fluid model having EOS in the quadratic form with parameter depends upon time *t*:

$$p_d = [1 + \gamma(t)]\rho_d^2 + \Lambda(t), \quad (1)$$

The Bianchi models fall within the category of non-standard, anisotropic, spatially homogenous cosmological models. These could potentially be considered as a generalization of the popular Friedman-Lemaître-Robertson-Walker (FLRW) cosmological models. These models were first developed by Bianchi [36,37], who categorized them based on how they created homogeneous space-time surfaces. These models are quite intriguing from a cosmological perspective because they offer a mechanism to explore the anisotropy early in the history of the expansion of our universe [38]. Scientists have demonstrated how a single geometric structure may unite the forces of gravity and electromagnetic in this new concept. A higher-dimensional cosmological model developed by Chodos and Detweiler [39] in which the extra dimension contracts and illustrates the effects of this cosmological evolution. In contrast to the traditional inflationary situation, extra dimensions give huge amounts of entropy at the time of contraction, offering a solution to the flatness and horizon difficulties [40,41]. In order to examine various matter fields and present their findings, a number of scholars [42-46] analyzed cosmological models in higher dimensions. According to certain research, the fourth-dimensional space-time expands when the fifth dimension shrinks or stays constant. Recently some researchers [47-49] studied the nature and properties of DE and DM, which together constitute the majority of the universe’s mass-energy content.

In this deal, dark energy models are investigated, where LR and PR conducts have been seen, with parameters $\gamma(t)$ and $\Lambda(t)$ depends upon time t with quadratic EOS as well as inhomogeneous EOS for DM. In this result, the parameters $\gamma(t)$ and $\Lambda(t)$ are used to analyse the results of the relationship between dark energy and special form dark matter. It is observed that in the context of coupled dark energy models, equation of state plays a significant role for both LR and PR occurrences.

2. Model and Field Equations

In this paper, 5D Kaluza-Klein model of the given form is assumed

$$ds^2 = -dt^2 + {S_1}^2(dx^2 + dy^2 + dz^2 + d\psi^2), \tag{2}$$

Where S_1 depends on time t.

Einstein's field equations are

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R = -8\pi G T_{ij} \tag{3}$$

The energy-momentum tensor with two interacting ideal fluids is as follows:

$$T_{ij} = (\rho_d + \rho_m + p_d + p_m)\mu_i \mu_j + (p_d + p_m) g_{ij},$$

Where $p_d,\ p_m$ and ρ_d, ρ_m correspond to the pressure of dark energy, dark matter and energy density of dark energy, dark matter.

For the 5D model equation (2), the Einstein Field equation (3) results in

$$3H^2 = \frac{1}{2} C^2(\rho_d + \rho_m), \tag{4}$$

$$3\dot{H} = -C^2(p_d + \rho_d + p_m + \rho_m), \tag{5}$$

Where $C^2 = 8\pi G$, G stands for the Newtonian constant of gravitation, H stands for the Hubble parameter, and (.) is used to differentiate with respect to the proper period t.

For DE and DM, the conservation equation leads to

$$\dot{\rho}_d + 4H(p_d + \rho_d) = -N_0 \tag{6}$$

and

$$\dot{\rho}_m + 4H(p_m + \rho_m) = N_0 \tag{7}$$

Where N_0 is the interaction rate between DE and DM.

If $N_0 > 0$, Dark energy decays into DM according to the energy transfer from DE to DM, whereas if $N_0 < 0$, Energy flow is in the opposite direction, and dark matter becomes DE.

In this work, equation (1) for DE mentions that the universe consists of two interacting perfect fluids with quadratic EOS and time-dependent parameters $\gamma(t)$ and $\Lambda(t)$.

By Nojiri *et al.* [14], we have

$$p_m = \tilde{\gamma}(t)\rho_m \tag{8}$$

Using Equation (8) in the Equation (7), it gives

$$\dot{\rho}_m + 4H(1 + \tilde{\gamma})\rho_m = N_0 \tag{9}$$

With the help of Equation (1) and (4), equation (6) takes the form for dark energy as follows:

$$\frac{12H\dot{H}}{C^2} - \dot{\rho}_m + 4H\left[\left(1 + \left(1 + \gamma(t)\right)\rho_d\right)\rho_d + \Lambda(t)\right] = -N_0 \tag{10}$$

The changes in the cosmos are noticed in the current study at a time $t = 0$, which is initially unknown save for the fact that it relates to an initial instance in the very early universe. Hence here taken into account the LR and PR models. Now two cosmological models LR and PR are analyzed, where an interaction between DE and DM components are assumed with quadratic EOS with time-dependent parameter $\gamma(t)$ and $\Lambda(t)$.

Case I: LR Model

As it's known that, it takes an infinite amount of time to achieve the singularity in the LR cosmology, which is characterized by an asymptotic increase in energy density with time t. It implies that EOS parameter $\gamma < -1$ but $\gamma \to -1$ asymptotically which is slightly different approach of future singularity.

Here the following two cases of LR model with defined Hubble parameter H is discussed.

Case (i): $H(t) = H_0 e^{\lambda t}$ (Following [22])

Case (ii): $(t) = (t^n/\tau^n) - 1$.

It follows that as $t \to \infty, H \to \infty$.

Case (i): $H(t) = H_0 e^{\lambda t}, \lambda > 0, H_0 > 0$.

In case (i), it is observed $H = H_0$ as $t = 0$, where H_0 is the present-time Hubble parameter.

Let's assume the thermodynamic parameter $\tilde{\gamma}(t)$ for dark matter is as follows in order to solve Equation (9) [30].

$$\tilde{\gamma}(t) = e^{-\lambda t} - 1 \tag{11}$$

Here, considering N_0 i.e. an interaction rate between DE and DM in the quadratic form as given below

$$N_0(t) = N_1\, t^2 + N_2 \tag{12}$$

Where N_1 and N_2 are constants.

By following the references Nojiri and Odinston [50,51]. Here, selecting equation (12), which depends on a physical consideration.

Adding value of (11) and the value of N_0 from equation (12) to equation (9), equation (9) gives the following expression of dark matter

$$\rho_m = \frac{1}{4H_0}\left[t^2 N_1 - \frac{2N_1}{4H_0} t + \frac{2N_1}{16H_0^2} + N_2\right] + M_1 . e^{-4H_0 t} \tag{13}$$

Where M_1 is the integrating constant.

In the above equation (13) it is observed that $\rho_m > 0$ i.e. energy density for dark matter will always be positive for the value $N_1 > 0, N_2 > 0$ at $t > 0$.

But at $t = 0$, Equations (13) reduces to

$$\rho_m(0) = \frac{1}{4H_0}\left[\frac{2N_1}{16H_0^2} + N_2\right] + M_1.$$

By considering $M_1 = -\frac{1}{4H_0}\left[\frac{2N_1}{16H_0^2} + N_2\right]$, ρ_m is zero, which states that dark matter starts to appear in the universe at $t = 0$ and when $t \to \infty$ it implies $\rho_m \to \infty$. Also accelerating expansion observed for $t \to -\infty$.

Following Brevik *et al.* [30]

$$\gamma(t) = -1 - \frac{1}{4C^2H^2} \tag{14}$$

In equation (14), it can interpret that γ is always less than -1 for $t < 0$ and $t > 0$.

In the limit $t \to \pm\infty$, $\gamma(t) \to -1$. Subsequently for $t \to -\infty$, there is an acceleration in expansion, which may be related to the early universe's inflation.

Equation (10), with the value of $\gamma(t)$ from equation (14), yields

$$\Lambda(t) = \frac{3}{C^2}\left(H^2\left(\frac{3}{C^4} - 2\right) - \lambda H\right) - \rho_m\left(\tilde{\gamma}(t) + \frac{3}{C^4} - \frac{\rho_m}{4C^2H^2}\right) \quad (15)$$

From (15) it is clear that, when $t \to \infty$ it gives $\Lambda(t) \to \infty$.

When $t = 0$, then equations (14) and (15) reduces to

$$\gamma(0) = -1 - \frac{1}{4C^2{H_0}^2} \text{ and } \Lambda(0) = \frac{3}{C^2}\left({H_0}^2\left(\frac{3}{C^4} - 2\right) - \lambda H_0\right) \quad (16)$$

From equation (10), the value of $\Lambda = \Lambda(t)$ gives

$$\gamma(t) = \frac{-1}{\rho_m^2\left[\left(\frac{6H^2}{\rho_m C^2}\right) - 1\right]^2}\left[\frac{3(\lambda + 2H)H}{C^2} + \tilde{\gamma}(t).\rho_m + \Lambda(t)\right] - 1 \quad (17)$$

From equation (17), it is observed that

$\gamma < -1$ at given time t which gives LR cosmology in the form of EOS parameter $\Lambda(t)$ and $\gamma(t)$ for the coupled fluid.

Case (ii): $H(t) = (t^n/\tau^n) - 1$, where τ is constant and $n > 0$

In case (ii), if $t \to \infty$, $\mathrm{H}(t) \to \infty$ and for $t = 0$, $\mathrm{H}(t) \to -1$.

Now for the value of scale factor $S_1(t)$, LR model gives $S_1(t) = e^{f(t)}$,

where $f(t) = \left[\frac{t^{n+1}}{\tau^n(n+1)} - t + c\right]$.

$f(t)$ is a non-singular function i.e. it satisfied $\frac{d^2f}{dt^2} > 0$.

As per Frampton *et al*. [21], The equation of the form $S_1(t) = e^{f(t)}$ are use to described LR models $\left(with\ \frac{d^2f}{dt^2} > 0\right)$ and physically, at any given point in time, neither the scale factor $S_1(t)$ nor the density tend to infinity in LR.

For the thermodynamic dark matter $\tilde{\gamma}$, consider the EOS as given below:

$$\tilde{\gamma}(t) = t^n - 1 \quad (18)$$

Equation (18) implies that for $t = 0$, $\tilde{\gamma} \to -1$ and then increases as time increases.

Take into account $N_0(t)$'s value in the following form:

$$N_0(t) = (e^{4t^{n+1}/(n+1)})\frac{4t^{2n}}{\tau^n} \quad (19)$$

With the help of (18) and (19), the density for dark matter is obtained from (9) as given below:

$$\rho_m(t) = \left(e^{4t^{n+1}/(n+1)}\right) + M_2 \exp 4\left[\frac{t^{n+1}}{n+1} - \frac{t^{2n+1}}{\tau^n(2n+1)}\right] \quad (20)$$

where M_2 is constant of integration.

For $t = 0$, equation (20) just produces.

$$\rho_m(0) = 1 + M_2 \quad (21)$$

By suitable value of M_2 i.e. $M_2 = -1$ in equation (21), $\rho_m = 0$ is obtained.

It is also noticed that in the far future for $t \to \infty, \rho_m(\infty) \to \infty$.

From equation (10), getting the value of $\Lambda(t)$as:

$$\Lambda(t) = \frac{1}{C^2}\left[3H^2\left(\frac{3}{C^4} - 2\right) - \frac{3nt^{n-1}}{\tau^n}\right] - \rho_m\left(\gamma(t) + \frac{3}{C^4} - \frac{\rho_m}{4C^2H^2}\right) \quad (22)$$

Case (ii) at $t = 0$, equation (14) and (22) becomes

$$\gamma(0) = -1 - \frac{1}{4C^2} \text{ And } \Lambda(0) = \frac{1}{C^2}\left[3\left(\frac{3}{C^4} - 2\right)\right] - (1 + M_2)\left(\frac{3}{C^4} - 1 - \frac{1+M_2}{4C^2}\right) \quad (23)$$

For $H(t) = (t^n/\tau^n) - 1$ and the value of $\Lambda = \Lambda(t)$ from equation (10), yields the value of $\gamma(t)$ as

$$\gamma(t) = \frac{-1}{\rho_m^2\left[\left(\frac{6H^2}{\rho_m C^2}\right)-1\right]^2}\left[Z_0 + \tilde{\gamma}(t).\rho_m + \Lambda(t)\right] - 1 \tag{24}$$

Where $Z_0 = \frac{3nt^{n-1}}{\tau^n C^2} + \frac{6}{C^2}\left[(t^n/\tau^n) - 1\right]^2$

So, till equation (24), two cases of little rip cosmology for time-dependent parameters in EOS, by taking into account the interaction rate N(t) between DE and DM are obtained.

Case II: PR Model

This section examines a model in which the Hubble parameter grows towards a constant in the far future, which implies that the universe approaches asymptotically to a de-sitter space. Now the Hubble parameter with different behavior is considered in PR model which has the following form:

$$H = H_0 - H_1 e^{-\lambda t^{(n+1)}}, \tag{25}$$

Where H_0, H_1, λ, and n are positive constants and for $t > 0$, H_0 is greater than H_1

Consider the structure of the parameter $\tilde{\gamma}(t)$ as in equation (18), in order to solve equation (9) for dark matter in the PR model

Let us consider $N_0(t)$ as

$$N_0(t) = 4H_0 t^n \left[exp\left(\frac{-4H_1}{\lambda(n+1)}.exp(-\lambda t^{(n+1)})\right)\right] \tag{26}$$

and get the solution of equation (9) for DM by considering the above equation is as

$$\rho_m = e^{((-4H_1/\lambda(n+1))e^{-\lambda t^{(n+1)}}}\left[1 + M_3 e^{-4H_0 t^{(n+1)}/(n+1)}\right] \tag{27}$$

Where M_3 is the integration constant.

From equation (27), the energy density of dark matter is calculated

At $t \to -\infty$, $\rho_m \to 0$

At $t \to 0$, $\rho_m \to e^{((-4H_1/\lambda(n+1))}(1 + M_3)$

At $t \to \infty$, $\rho_m \to 1$

For the PR model, by taking the reference of above expressions, it is observed that, there is no dark matter in the past time. With increasing time, it will start to exist in the universe. The following formula for the cosmological constant from equation (10) is obtained, if the thermodynamic parameter $\gamma(t)$ for the DE takes the form as in equation (14):

$$\Lambda(t) = \frac{1}{C^2}\left[3H^2\left(\frac{3}{C^4} - 2\right) - 3(n+1)H_1\lambda e^{-\lambda t^{(n+1)}} t^n\right] - \rho_m\left(\tilde{\gamma}(t) + \frac{3}{C^4} - \frac{\rho_m}{4C^2H^2}\right) \tag{28}$$

At $t = 0$ for PR model from equation (14) and (28) gives

$\gamma(0) = -1 - \frac{1}{4C^2(H_0 - H_1)^2}$ and

$$\Lambda(0) = \frac{1}{C^2}\left[3(H_0 - H_1)^2\left(\frac{3}{C^4} - 2\right)\right] - e^{-4H_1/\lambda(n+1)}(1 + M_3)\left(\tilde{\gamma}(t) + \frac{3}{C^4} - \frac{e^{-4H_1/\lambda(n+1)}(1+c_2)}{4C^2(H_0 - H_1)^2}\right) \tag{29}$$

For PR model equation (25), for the value of $\Lambda = \Lambda(t)$ in equation (10), the value of $\gamma(t)$ is obtained as follows:

$$\gamma(t) = \frac{-1}{\rho_m^2\left[\left(\frac{6H^2}{\rho_m C^2}\right)-1\right]^2}\left[Z + \tilde{\gamma}(t).\rho_m + \Lambda(t)\right] - 1 \tag{30}$$

Where, $Z = 3\left[\frac{(n+1)H_1\lambda e^{-\lambda t^{(n+1)}}t^n}{c^2} + \frac{2H^2}{c^2}\right]$.

From the equation (30), it is observed that the PR model yield $\gamma(t) \to -1$ for any value of Λ.

3. Conclusion

Even after several scientific experiments, the interpretation of Dark Energy and Dark Matter is still difficult today. In this article, we have studied Bianchi type I universe filled with DE and DM by considering EOS which is quadratic in the form with time dependent parameter $\gamma(t)$ and $\Lambda(t)$ in the framework of general relativity. Here it is studied that LR and PR cosmology changes exponentially with respect to $\Lambda(t)$. The dark energy, dark matter density, and time-dependent parameters $\gamma(t)$ and $\Lambda(t)$, in which the behavior of LR and PR is observed, are all expressed in this article.

Case (i): It is seen from the equation that the initial time t is equals to zero, Dark matter becomes constant in the universe. By choosing suitable values of the constants, we obtain $\rho_m(0) = 0$. It means at $t \to 0$, there exist p and ρ , where as ρ_m & p_m becomes zero which shows that there might be a short interval in the resultant universe when the dark matter disappears.

Case (ii): The PR model and LR model case (ii) operate similarly. The energy density for dark matter is also zero when we choose appropriate constant values for $t = 0$, we observed that $\rho_m \to 1$ for $t \to \infty$ and $\rho_m \to 0$ for $t \to -\infty$. Hence it is clear that there was no dark matter in the past time and as the time t increases, it will manifest in the universe. As a result, it must state that the energy density of dark matter ranges from 0 to 1.

Finally, it is investigated that the value of $\Lambda(t)$ is an increasing function of t or decreasing function of t or constant value for different expressions and we always get a value of $\gamma(t) \to -\infty$ for both cases of LR model and PR model. It has been demonstrated that the universe is expanding faster than usual due to the influence of coupling between pair DE and DM. In addition, it was discovered how LR and PR behaved for coupled dark energy with quadratic EOS with time-dependent parameters $\gamma(t)$ and $\Lambda(t)$.

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