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MHD Flow of a Viscous Fluid with Equal Kinematic and Magnetic Viscosity Pasta Perfectly Conducting Porous Plate

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Abstract

The effect of MHD flow of viscous, incompressible fluid with small electrical conductivity past an infinitely long perfectly conducting porous plate started impulsively from rest in the presence of a constant transverse magnetic field fixed relative to the fluid is investigated. We also imposed a small uniform suction or injection at the plate.Skin friction at the plate, non-dimensional forms of the velocity, and the induced magnetic field have been obtained. The graphs of velocity and magnetic field are also drawn for various values of suction, and magnetic parameters are clearly explained. From the investigations, it is found that the fluid velocity decreases due to the increasing value of kinematic viscosity and constant suction velocity. It also finds that the induced magnetic field increases due to constant suction velocity.

Keywords:Kinematic viscosity; Magnetic viscosity; Conducting plate; Skin-frictions.

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1. Introduction

The branch of science that deals with the interaction of the magnetic field with electrically conducting fluid is termed MHD. Saltwater, liquid metals, plasmas, mercury, electrolytes, and molten metals are some common examples of such fluids. Noted Swiss scientist Hannes Alfven, Noble Prize winner of Physics, initiated the field of MHD. However substantial contributions in MHD made by Cowling, Shercliff, Ferraro, Plumpton, Crammer, and Pai [1-5] have attracted the works towards physical relevance.

There are several applications of MHD in modern technologies. In the presence of a strong electric field, the electrical conductivity is affectedby magnetic fields. Geophysical and astrophysical applications of MHD are nicely elaborated by Dormy and Nunez [6]. Dynamo, motor, fusion reactors, dispersion of metals, and metallurgical industries have wide applications of MHD. Aeronautical applications of MHD were studied exclusively by Li *et al*. [7]. Farrokhi *et al.* [8] focused on biomedical applications of MHD. Rana *et al.* [9] investigated how microbes swim in the blood flow of nano-bioconvective Williamson fluid.

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The porous medium contains holes or voids that are filled with solid particles, which permit the fluid to pass through these. The mechanism of porous flow finds its applications in inkjet printing, nuclear waste disposal, electrochemistry, combustion technology, etc. Dwivedi *et al*. [10] studied MHD flow through the vertical channel in a porous medium, while Raju *et al*. [11] observed the MHD flow through a horizontal channel, taking viscous dissipation and Joule heating into account. Kuiry and Bahadur [12] studied the unsteady MHD flow of a dusty viscoelastic fluid between parallel plates with an exponentially Decreasing pressure gradient in an inclined magnetic field. Ali *et al*. [13] investigated the Hall effects on steady MHD Heat and mass transfer free convection flow along a stretching sheet with suction and heat generation. Islam *et al*. [14] studied unsteady Heat and mass transfer slip flow over an Exponentially permeable stretching sheet. Kuiry and Vishwakarma [15] discussed the Hydromagnetic flow of a viscous fluid of equal kinematic and magnetic viscosity past a non-conducting porous plate.

In this paper, the work done by the present authors [15] is extended by introducing a perfectly conducting porous plate with equal kinematic and magnetic viscosity under the influence of a transverse magnetic field. A small uniform suction or injection at the plate is imposed. The solutions to the problem under a constant transverse magnetic field fixed relative to the fluid are discussed through graphs.

2. Mathematical Formulation and Solutions of the Problem

The unsteady MHD flow of an incompressible viscous fluid through a perfectly conducting porous plate under the influence of a constant transverse magnetic field is considered.. Let the x-axis and y-axis be taken in the direction of fluid flow and perpendicular to the conducting wall, respectively. The z-axis is perpendicular to both the x-axis and y-axis.

Let the imposed magnetic field. $\overrightarrow{H_0}$ be perpendicular to the conducting wall and v_s the suction velocity at the plate. Then, the equation of continuity $\frac{\partial v}{\partial y} = 0$at y = 0 gives $v = v_s$ everywhere over the electrically conducting infinite porous flat plate.

On account of motion, let a magnetic field H_x will be induced in the flowdirection. From the symmetry of the problem considered, all flow variables are functions of the coordinate y and time t only.

Let the plate be started impulsively from rest with a constant velocity U. Then the following differential equations will govern the fluid flow [16,17] as:

$$\rho \left(\frac{\partial u}{\partial t} + v_s \frac{\partial u}{\partial y}\right) = \mu \overrightarrow{H_0} \frac{\partial H_x}{\partial y} + \eta \frac{\partial^2 u}{\partial y^2} \qquad (1)$$

and

$$\frac{\partial H_x}{\partial t} + v_s \frac{\partial H_x}{\partial y} = \overrightarrow{H_0} \frac{\partial u}{\partial y} + \nu \frac{\partial^2 H_x}{\partial y^2}\,, \qquad (2)$$

where p, the fluid pressure assumed to be a constant.

Boundary and initial conditions for a non-conducting plate are :

at t = 0, y > 0, u = H_x= 0 ,
at y = 0, t > 0, u = U, H_x = 0, (3)
at y → ∞, u = 0, H_x = 0.

The considered problem is to be solved after deriving first the boundary conditions to be satisfied at the wall for an arbitrary conducting wall.Then, the same is deduced for a perfectly conducting wall. From Ampere's law and Ohm's law, after taking MHD approximations into account, we have:

$$\vec{J} = \nabla \times \vec{H}$$

and

$$\vec{J} = \sigma \left[\vec{E} + \vec{V} \times \vec{B}\right]$$

Consequently, we have,

$$\vec{E} = \frac{\nabla \times \vec{H}}{\sigma} - \mu \vec{V} \times \vec{H}$$

where $\vec{H} = \left(H_x, H_y, 0\right) = (H_x, H_0, 0)$
and $\vec{V} = (u, v_s, 0)$
Therefore, we have

$$\vec{E} = \left(0,0, \frac{1}{\sigma}\frac{\partial H_x}{\partial y} - \mu H_0 u + \mu v_s H_x\right). \quad (4)$$

Let us assume that the magnetic permeability μ of the medium and the induced magnetic field. H_x in the flow direction isthe same for both the fluid and the wall. Then equation (4) yields that the plate started impulsively from rest with velocity U:

$$\overrightarrow{E_w} = \left(0,0, \frac{1}{\sigma_w}\frac{\partial H_{x_w}}{\partial y} - \mu H_0 U + \mu v_s H_{x_w}\right). \quad (5)$$

Since the tangential component of the electric field is continuous at an interface, it yields from (4) and (5), the boundary conditions at the wall for y = 0.

$$\frac{1}{\sigma}\frac{\partial H_x}{\partial y} = \frac{1}{\sigma_w}\frac{\partial H_{x_w}}{\partial y} + \mu v_s H_{x_w} - \mu v_s H_x \quad (6)$$

Since the wall is perfectly conducting, we have $\sigma_w = \infty$ and hence we have the condition:

$$\frac{\partial H_x}{\partial y} = 0. \quad (7)$$

The other boundary and initial conditions are the same as given in (3).
Hence, equations (1) and (2) are to be solved subject to the boundary and initial conditions given in (3) and (7). Using equations (1) and (2), we obtained respectively as:

$$\frac{\partial u_1}{\partial t} + v_s \frac{\partial u_1}{\partial y} = \theta H_0 \left(\frac{\partial u}{\partial y} + \frac{\mu}{\theta \rho}\frac{\partial H_x}{\partial y}\right) + \nu \frac{\partial^2 u_1}{\partial y^2} \quad (8)$$

and

$$\frac{\partial u_2}{\partial t} + v_s \frac{\partial u_2}{\partial y} = -\theta H_0 \left(\frac{\partial u}{\partial y} - \frac{\mu}{\theta \rho} \frac{\partial H_x}{\partial y} \right) + \nu \frac{\partial^2 u_2}{\partial y^2}, \tag{9}$$

where $u_1, u_2 = u \pm \theta H_x$.

Let us choose $\theta = \sqrt{\frac{\mu}{\rho}}$, which reduces equations (8) and (9) respectively to:

$$\frac{\partial u_1}{\partial t} + v_s \frac{\partial u_1}{\partial y} = \psi \frac{\partial u_1}{\partial y} + \nu \frac{\partial^2 u}{\partial y^2} \tag{10}$$

and

$$\frac{\partial u_2}{\partial t} + v_s \frac{\partial u_2}{\partial y} = -\psi \frac{\partial u_2}{\partial y} + \nu \frac{\partial^2 u_2}{\partial y^2}, \tag{11}$$

where $\psi = \sqrt{\frac{\mu}{\rho}} . H_0$.

The steady-state solutions of equations (10) and (11) are respectively given as under:

$$u_1 = A' + B' e^{-(\psi - v_s)\frac{y}{\nu}},$$

$$u_2 = C' + D' e^{(\psi + v_s)\frac{y}{\nu}},$$

where A', B', C' and D' are arbitrary constants, and consequently, the solutions of equations (10) and (11) are respectively of the forms:

$$u_1 = A\, erf_c \left(\frac{y + (\psi - v_s)t}{2\sqrt{\nu t}} \right) + B e^{-(\psi - v_s)\frac{y}{\nu}} erf_c \left[\frac{y - (\psi - v_s)t}{2\sqrt{\nu t}} \right] \tag{12}$$

and

$$u_2 = C\, erf_c \left(\frac{y - (\psi + v_s)t}{2\sqrt{\nu t}} \right) + D e^{(\psi + v_s)\frac{y}{\nu}} erf_c \left[\frac{y + (\psi + v_s)t}{2\sqrt{\nu t}} \right] \tag{13}$$

Therefore the solutions of the problem will be given by

$u = \frac{1}{2}(u_1 + u_2)$ and

$$H_x = \frac{1}{2}\sqrt{\frac{\rho}{\mu}}(u_1 - u_2),$$

The constants A, B, C, and D are to be determined from (3) and (7) because the first and third conditions of (3) are automatically satisfied byu_1 and u_2. The second condition of (3) yields as:

$$U = \frac{1}{2}\left[Aerf_c\left\{\frac{(\psi - v_s)t}{2\sqrt{\nu t}}\right\} + Berf_c\left\{\frac{-(\psi - v_s)t}{2\sqrt{\nu t}}\right\} + Cerf_c\left\{\frac{-(\psi + v_s)t}{2\sqrt{\nu t}}\right\} + Derf_c\left\{\frac{(\psi + v_s)t}{2\sqrt{\nu t}}\right\}\right]$$

$$= \frac{1}{2}\left[(A+B) - \frac{2}{\sqrt{\pi}}(A-B)\int_0^{\frac{(\psi - v_S)t}{2\sqrt{\nu t}}} e^{-\xi^2}\, d\xi + (C+D) + \frac{2}{\sqrt{\pi}}(C-D)\int_0^{\frac{(\psi + v_S)t}{2\sqrt{\nu t}}} e^{-\xi^2}\, d\xi\right]$$

Which will be satisfied if we have

$$\frac{1}{2}[(A+B) + (C+D)] = U$$

and

$$A - B = C - D$$

or,

$$A + D = B + C$$

and therefore, $B + C = U$ (14)

and from the condition (7), it yields

$$\left(\frac{\partial H_x}{\partial y}\right)_{y=0} = 0,$$

i.e. $\frac{1}{2}\left[-\frac{A}{\sqrt{\pi\nu t}}e^{-(\psi - v_s)^2\frac{t}{4\nu}} - B\,\frac{(\psi - v_s)}{\nu}erf_c\left\{\frac{-(\psi - v_s)t}{2\sqrt{\nu t}}\right\} - \frac{B}{\sqrt{\pi\nu t}}e^{-(\psi - v_s)^2\frac{t}{4\nu}} + \frac{C}{\sqrt{\pi\nu t}}e^{-(\psi - v_s)^2\frac{t}{4\nu}} - \frac{D}{\nu}(\psi + v_s)erf_c\left\{\frac{(\psi + v_s)t}{2\sqrt{\nu t}}\right\} + \frac{D}{\sqrt{\pi\nu t}}e^{-(\psi + v_s)^2\frac{t}{4\nu}}\right] = 0$

which will be satisfied if we have

$$A + B = C + D$$

and

$B = D = 0, A = C$ (15)

and from (14), $C = U$, since $B = 0$, and from (15), $A = C = U$.

Hence the required solutions are given respectively by

$$u = \frac{U}{2}\left[erf_c\left\{\frac{y+(\psi - v_S)t}{2\sqrt{\nu t}}\right\} + erf_c\left\{\frac{y-(\psi + v_S)t}{2\sqrt{\nu t}}\right\}\right] \quad (16)$$

and

$$H_x = \frac{U}{2}\sqrt{\frac{\rho}{u}}\left[erf_c\left\{\frac{y+(\psi - v_S)t}{2\sqrt{\nu t}}\right\} - erf_c\left\{\frac{y-(\psi + v_S)t}{2\sqrt{\nu t}}\right\}\right]. \quad (17)$$

The absence of the exponential factor in the above solutions indicates that for a perfectly conducting wall, there is no Hartmann layer in the ultimate state.

3. Results and Discussion

From equations (16) and (17), results can be deduced below:
i. When $\psi \neq 0, v_s = 0$, it yields as:

$$u = \frac{U}{2}\left[erf_c\left(\frac{y+\psi t}{2\sqrt{\nu t}}\right) + \ erf_c\left(\frac{y-\psi t}{2\sqrt{\nu t}}\right)\right]$$

and

$$H_x = \frac{U}{2}\sqrt{\frac{\rho}{\mu}}\left[erf_c\left(\frac{y+\psi t}{2\sqrt{\nu t}}\right) - \ erf_c\left(\frac{y-\psi t}{2\sqrt{\nu t}}\right)\right]$$

which coincide with the solutions obtained by Lord Rayleigh [2].
ii. When $\psi = 0, v_s = 0$, it yields as:

$u = U\ erf_c\left(\frac{y}{2\sqrt{\nu t}}\right)$ and

$H_x = 0$
which coincides with the results obtained in ordinary Hydrodynamic flow [17].
iii. When $\psi = 0,\ v_s \neq 0$, it yields as:

$u = U\ erf_c\left(\frac{y-v_s t}{2\sqrt{\nu t}}\right)$ and

$H_x = 0$
which coincides with the ordinary hydrodynamic results [17] in the absence of suction and the total absence of a transverse magnetic field.

3.1. *Skin-Frictions*

From (16), the skin-friction τ_0 at the surface of the plate can be derived as:

$$\tau_0 = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$= -\frac{U}{2\sqrt{\pi \nu t}}\left[e^{-\frac{(\psi-v_s)^2 t}{4\nu}} + \ e^{-\frac{(\psi+v_s)^2 t}{4\nu}}\right]$$

and skin-frictions for other cases are:

$\tau_0 = -\frac{U}{\sqrt{\pi \nu \mathrm{t}}}e^{-\frac{\psi^2 t}{4\nu}}$, when $\psi \neq 0, v_s = 0,$

$\tau_0 = -\frac{U}{\sqrt{\pi \nu t}}$, when $\psi = 0, v_s = 0$

and

$\tau_0 = -\frac{U}{\sqrt{\pi \nu t}} e^{-\frac{v_s^2 t}{4\nu}}$, when $\psi = 0,\ v_s \neq 0.$

3.1.1. *Non-dimensional forms*

The non-dimensional forms of the velocity and the induced magnetic field distribution can be derived respectively by the following forms:

$$\frac{u}{U} = \frac{1}{2}\left[erf_c\left\{\frac{y + (\psi - v_s)t}{2\sqrt{\nu t}}\right\} + erf_c\left\{\frac{y - (\psi + v_s)t}{2\sqrt{\nu t}}\right\}\right]$$

and

$$\frac{H_x}{U}\sqrt{\frac{\mu}{\rho}} = \frac{1}{2}\left[erf_c\left\{\frac{y + (\psi - v_s)t}{2\sqrt{\nu t}}\right\} - erf_c\left\{\frac{y - (\psi + v_s)t}{2\sqrt{\nu t}}\right\}\right]$$

and the non-dimensional forms for other subcases are obtained as:

$\frac{u}{U} = \frac{1}{2}\left[erf_c\left\{\frac{y+\psi t}{2\sqrt{\nu t}}\right\} + erf_c\left\{\frac{y-\psi t}{2\sqrt{\nu t}}\right\}\right]$,

$H_x\sqrt{\frac{\mu}{\rho}} = \frac{1}{2}\left[erf_c\left\{\frac{y+\psi t}{2\sqrt{\nu t}}\right\} - erf_c\left\{\frac{y-\psi t}{2\sqrt{\nu t}}\right\}\right]$,

$\frac{u}{U} = erf_c\left(\frac{y}{2\sqrt{\nu t}}\right)$, and

$\frac{u}{U} = erf_c\left(\frac{y - v_s t}{2\sqrt{\nu t}}\right)$.

The non-dimensional forms of the skin-frictions are evaluated in the forms below:

$\frac{\tau_0}{U} = -\frac{1}{\sqrt{\pi \nu t}}\left[e^{-\frac{(\psi - v_s)^2 t}{4\nu}} + e^{-\frac{(\psi + v_s)^2 t}{4\nu}}\right]$,

$\frac{\tau_0}{U} = -\frac{1}{\sqrt{\pi \nu t}} e^{-\frac{\psi^2 t}{4\nu}}$,

$\frac{\tau_0}{U} = -\frac{1}{\sqrt{\pi \nu t}}$ and

$\frac{\tau_0}{U} = -\frac{1}{\sqrt{\pi \nu t}} e^{-\frac{v_s^2 t}{4\nu}}$.

The results obtained on the velocity of the fluid due to variation in the parameters.η_1, Ψ_1andφ_1 under the influence of the transverse magnetic field are shown in Figs. 1 through 3.

In Fig. 1, taking $\lambda_1 = -1, \Psi_1 = 1$; $\lambda_1 = -2, \Psi_1 = 1$; $\lambda_1 = 0, \Psi_1 = 1$and $\lambda_1 = 0, \Psi_1 = 2$, the velocity of fluid is graphically represented, and we observed that the velocity of fluid decreases as η_1 increases.

In Fig. 2, taking $\lambda_1 = 0, \Psi_1 = 1$; $\lambda_1 = 0, \Psi_1 = 2$, we notice that the induced magnetic field increases when there is no suction velocity and increases of η_1.

In Fig. 3 taking $\Psi_1 = 0, \lambda_1 = -0.5$; $\Psi_1 = 0, \lambda_1 = -1, \Psi_1 = 0$; $\lambda_1 = -1.5$ and $\Psi_1 = 0, \lambda_1 = -2$ the fluid velocity decreases as the suction velocity decreases.

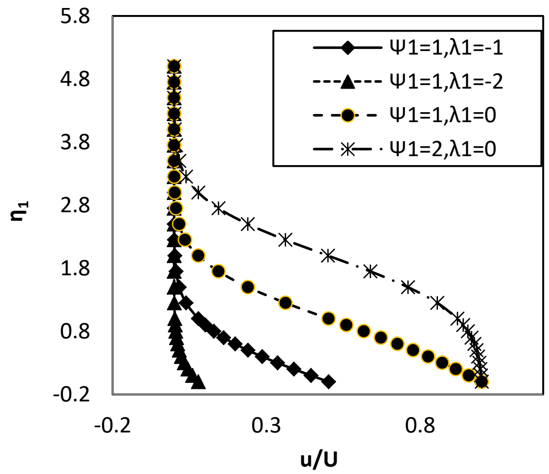

Fig. 1. Velocity profile against η_1.

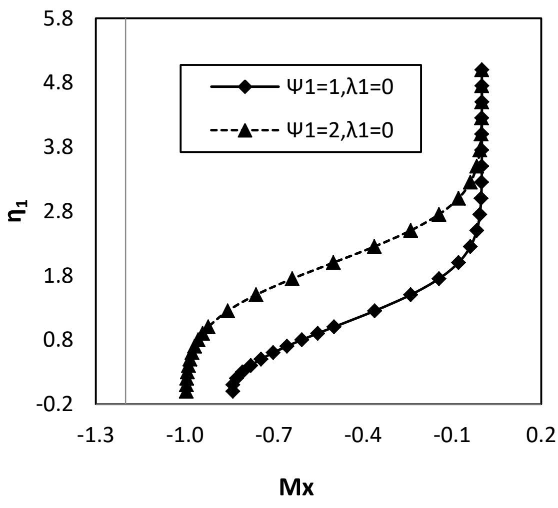

Fig. 2. Magnetic field with no section.

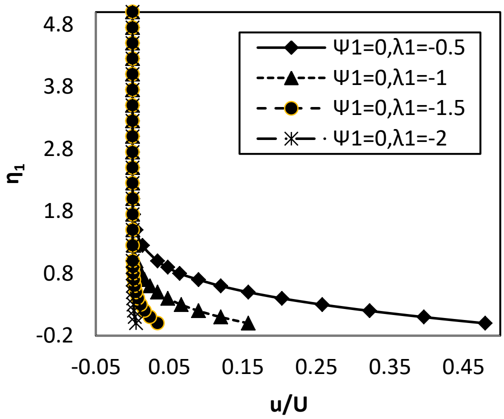

Fig. 3. Velocity profile for Ψ_1=0.

4. Conclusion

The focus of the present work is to study mainly the effect of a transverse magnetic field on a viscous, incompressible fluid with equal kinematic and magnetic viscosity past through a conducting porous plate. The behavioral study of flow under the action of different parameters was carried out with the aid of graphs. The important outcomes of the present work are as follows:

The magnitude of velocity decreases due to increasing values of kinematic viscosity and constant suction velocity.The induced magnetic field also increases due to constant suction velocity.The conclusions made from the above analysis are that the problem is amenable to an exact solution for the perfectly conducting plate having electrical conductivity $\sigma = \infty$ and when the fluid has equal kinematic and magnetic viscosity.

Nomenclature

MHD	Magnetohydrodynamics.
u	Velocity component in x-direction.
μ	Magnetic viscosity of the fluid.
η	Co-efficient of viscosity of the fluid.
ρ	Density of the fluid.
p	Pressure of the fluid.
ν	Kinematic viscosity, $\nu = \frac{\eta}{\rho}$.
$\vec{H_0}$	Imposed magnetic field.
σ	Electrical conductivity.
$\vec{E}$	Electric field in the fluid.
H_x	Induced magnetic field.
$\vec{J}$	Density of the current induced.

$$\eta_1 = \frac{y}{2\sqrt{\nu t}}$$

$$\psi_1 = \frac{\psi t}{2\sqrt{\nu t}}$$

$$\lambda_1 = \frac{v_s t}{2\sqrt{\nu t}}$$

References

1. H. Alven, Nature **150**, 405 (1942). https://doi.org/10.1038/150405d0
2. T. G. Cowling and R. B. Lindsay, Phys. Today **10**, 40 (1957). https://doi.org/10.1063/1.3060498
3. J. A. Shercliff, A text Book of Magntohydrodynamics, (Pergamon press, Oxford, 1965).
4. V. C. A. Ferraro and C. Plumpton, An Introduction to Magneto Fluid Mechanics, 2nd Edition (Oxford Clarendon Press, Oxford, 1966).
5. K. R. Crammer and S. I. Pai, Magneto Fluid Dynamics for Engineers and Applied Physicists (McGraw-Hill Book Company, New York, 1973).
6. E. Dormy and M. Nunez, Geophys. Astrophys. Fluid Dyn. **101**, 169 (2007). https://doi.org/10.1080/03091920701523287
7. Y. Li, B. Zhang, L. Xiao, Y. Wang, and G. He, Adv. Mech. **47**, 452 (2017).
8. H. Farrokhi, D.O. Otuya, A. Khimchenko, and J. Dong, Nanofluid Flow Porous Med. Intech Open, Massachusetts, USA, 2019.
9. B. M. J. Rana, S. M. Arifuzzaman, S. Islam, S. K. Reza-E-Rabbi, A. Al-mamun et al., Therm. Sci. Eng. Prog. **25**, ID 101018 (2021). https://doi.org/10.1016/j.tsep.2021.101018
10. K. Dwivedi, R. K. Khare, and A. Paul, Int. J. Appl. Eng. Res. **13**, 11923 (2018).
11. K. V. S. Raju, T. S. Reddy, M. C. Raju, P. V. S. Narayana, and S. Venkataramana, Ain Shams Eng. J. **5**, 543 (2014). https://doi.org/10.1016/j.asej.2013.10.007
12. D. R. Kuiry and S. Bahadur, J. Sci. Res. **8**, 149 (2016). https://doi.org/10.3329/jsr.v8i2.25745
13. M. Ali and M. S. Alam, J. Sci. Res. **6**, 457 (2014). https://doi.org/10.3329/jsr.v6i3.16903
14. M. A. Islam, M. Y. Ali, and S. M. O. Gani, J. Sci. **14**, 443 (2022). https://doi.org/10.3329/jsr.v14i2.55577
15. D. R. Kuiry and D. K. Vishwakarma, J. Calcutta Math. Soc. **19**, 117 (2023).
16. P. A. Davidson, An Introduction to MHD, 1st Edition (Cambridge University press, UK, 2001).
17. H. Schlichting, Boundary Layer Theory (McGraw – Hill Book Co., New York, 1986).