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Qualitative Behavior of Cosmological Model with Cosmic Strings and Minimally Interacting Dark Energy

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Abstract

This article is devoted to the study of Kaluza-Klein space-time filled with cosmic string and attractive massive scalar field in the presence of minimally interacting dark energy in general relativity. Some physically significant conditions have been utilized to obtain a deterministic solution of the field equations. The behavior of cosmological parameters such as volume, scalar field, deceleration parameter, equation of state (EoS) parameter, statefinder parameter, Om diagnostic are discussed. It is also worth noting that the conclusions of the cosmological parameter are consistent with modern observational data.

Keywords: Kaluza-Klein space-time; Dark energy model; Cosmic string; Massive scalar field.

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1. Introduction

The structure formation of our universe is the biggest puzzle of this century. Symmetry breaking in the early universe after the Big Bang explosion resulted in the formation of topological defects such as cosmic strings, domain walls, and monopoles. Amongst them, it is assumed that cosmic strings are significant and play a crucial role in the early evolution of the universe in the formation of galaxies. Letelier [1,2] launched the general relativistic study of strings. Cosmological models with massive scalar fields have been discussed by several authors in general and in modified theories of gravitation. In the presence of an attractive massive scalar field, Naidu *et al*. [3] developed an anisotropic and spatially homogeneous Bianchi type-V dark energy cosmological model in general relativity. Poonia *et al*. [4] analyzed the Bianchi type-VI inflationary cosmological model with massive string source in general relativity. The homogenous, anisotropic L.R.S. Bianchi type I universe with holographic dark energy and interacting dark matter has been investigated by Mete *et al*. [5]. Mete *et al*. [6] , have examined the magnetized Bianchi type-IX bulk viscous string cosmological model in general relativity. The Kaluza-Klein string cosmological model was investigated by Pawar *et al*. [7] within the framework of

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$f(R,T)$ theory of gravity. Five-dimensional plane symmetric Bianchi type-I string cosmological model with bulk viscosity was presented by Mete *et al.* [8].

According to supernova 1a experimental observations, the modern cosmological scenario reveals that our present universe is accelerating, and the reason for this is supposed to be an exotic force with negative pressure known as "dark energy" (DE) [9,10]. Subsequently, numerous research studies have sustained the study of DE in the presence of many physical sources and in various alternative theories of gravitation. Mete *et al.* [11] have presented interacting dark fluid in the Kaluza-Klein universe with variable deceleration parameters in general relativity. Aditya *et al.* [12] examined Kaluza-Klein dark energy models in Lyra manifolds with massive scalar fields. Naidu [13] explored Bianchi type-II modified holographic Ricci dark cosmological model in the presence of the massive scalar field. Based on a mixture of a cosmic string cloud and anisotropic dark energy fluid as the source of gravitation, Locally rotationally symmetric Bianchi type-II cosmological models have been constructed by Rao *et al.* [14]. A spatially homogenous and anisotropic Bianchi type -VI0 cosmological model with dark energy fluid and massive scalar field was studied by Aditya *et al.* [15]. Recently, an attractive massive scalar field with Bianchi type-I in Lyra manifolds with perfect fluid and attractive scalar field has been discussed by Naidu *et al.* [16], Aditya *et al.* [17] investigated plane-symmetric dark energy cosmological model in the presence of an attractive massive scalar field by using some physically relevant conditions. In the context of Lyra Geometry, Nimkar *et al.* [18] examined N-dimensional Kaluza-Klein space-time in the presence of cosmic string. Aditya, Y. *et al.* [19] have investigated the dark energy phenomena by examining the Tsallis holographic dark energy within the context of the Brans-Dicke (BD) scalar-tensor theory of gravity. A spatially homogeneous, anisotropic Kantowski-Sachs universe with anisotropic ghost dark energy fluid was described by Sultana *et al.* [20]. Vijaya Santhi *et al.*[21] have analyzed Bianchi type-II, VIII, and IX spatially homogeneous and anisotropic space-times in the background of the Brans–Dick theory of gravity within the framework of viscous holographic dark energy, Bianchi type VI_0 space-time with generalized Ghost Pilgrim dark energy was investigated by Bharali *et al.* [22] in general relativity. A Locally Rotationally Symmetric (LRS.) Bianchi-II cosmological model with matter and Holographic Dark Energy (HDE) was studied by Shaikh *et al.* [23] within the framework of the $f(R)$ theory of gravity. In the presence of a cosmic string and quintessence matter, Hossain *et al.* [24] analyzed the Hawking radiation of massless and massive charged particles via tunneling method from the Schwarzschild and Reissner-Nordstrom black holes, Bhoyar *et al.* [25] have examined Kantowaski-Sachs cosmological model with bulk viscous and cosmic string in the context of $f(T)$ gravity, Das *et al.* [26] established a correspondence between the New Agegraphic Dark Energy (NADE) models and the Polytropic Gas model of DE and reconstructed the potential and the dynamics for the scalar field of the Polytropic model. Brahma *et al.* [27] examined the Bianchi type-V dark energy cosmological model with the electromagnetic field in Lyra based on $f(R,T)$ gravity. Recently, Mete *et al.* [28] investigate a higher-dimensional bulk viscous fluid cosmology model with a one-dimensional cosmic string in the presence of an interacting zero mass scalar field in Lyra geometry using Bianchi type-III space-time. The Bianchi type VI_0 space-time in the presence of a string of clouds coupled with perfect fluid in the framework of *f*(*R*,*T*) gravity has been presented by Pawar *et al*. [29]. Ugale *et al*. [30] introduced a modified holographic Ricci dark energy model within the context of *f*(*R*,*T*) gravity theory, employing Bianchi type VI_0 space-time. Diamary *et al*. [31] examined the spatially homogeneous and anisotropic five-dimensional Bianchi type-V cosmological model in the presence of bulk viscous fluid with one-dimensional cosmic strings in Saez and Ballester's scalar-tensor theory of gravitation,

Inspired by the above works in cosmology, the investigation of the Kaluza-Klein dark energy model with cosmic string in the presence of an attractive massive scalar field is presented. This paper is organized as follows: Section 2 is devoted to metrics and the derivation of field equations. In Section 3, the field equations are solved using some significant physical conditions to obtain a deterministic model. The dynamic parameters of the model are evaluated, and corresponding physical significance is presented in sections 4 and 5,, respectively. The last section includes concluding remarks.

2. Metric and Field Equations

Kaluza-Klein space-time is considered in the form.

$$ds^2 = dt^2 - A^2(dx^2 + dy^2 + dz^2) - B^2 d\psi^2, \quad (1)$$

where A and B are functions of cosmic time t and fifth coordinate ψ is space-like.
Einstein field equations in the presence of anisotropic dark DE and cosmic string coupled with a massive scalar field can be written as

$$R_{ij} - \frac{1}{2} R g_{ij} = -\left(T_{ij}{}^{(\Lambda)} + T_{ij}{}^{(cs)} + T_{ij}{}^{(\phi)}\right), \quad (2)$$

Where R_{ij} is the Ricci tensor and$R$ is the Ricci scalar.
Here

$$T_{ij}{}^{(\Lambda)} = (\rho_\Lambda + p_\Lambda) - p_\Lambda g_{ij} \ , \quad (3)$$

is the energy-momentum tensor of DE fluid can also be written as

$$T_i^{j\,(\Lambda)} = diag[T_0^0, T_1^1, T_2^2, T_3^3, T_4^4] \quad (4)$$

which can be parameterized as follows:

$$T_i^{j\,(\Lambda)} = diag\left[\rho, -p_x, -p_y, p_z, -p_\psi\right]$$
$$= diag\left[1, -\omega_x, -\omega_y, \omega_z, -\omega_\psi\right]\rho_\Lambda$$
$$= diag[1, -\omega_\Lambda, -\omega_\Lambda, -\omega_\Lambda, -(\omega_\Lambda + \delta)]\rho_\Lambda, \quad (5)$$

where ρ_Λ and p_Λ being the energy density and pressure of DE, respectively.
δ is the deviation from ω_Λ on ψ axis.ω_Λ is the equation of state (EoS) parameter of DE is given as

$$\omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda}. \tag{6}$$

The energy-momentum tensor for cosmic string is given by

$$T_{ij}{}^{(cs)} = \rho u_i u_j - \lambda x_i x_j \,, \tag{7}$$

where, ρ is the energy density in the string, λ is the string tension density, u^i is the four-velocity and x_i , the string direction, which is taken along z-axis.
Here,

$$u^i u_i = -x^i x_i = 1, \quad u^i x_i = 0 \tag{8}$$

and

$$\rho = \rho_p + \lambda, \tag{9}$$

where ρ_p is the rest of the energy density of the particles attached to the string.
The energy-momentum tensor for the massive scalar field is given by

$$T_{ij}{}^{(\phi)} = \phi_{,i}\phi_{,j} - \frac{1}{2}\left(\phi_{,k}\phi^{,k} - M^2\phi^2\right), \tag{10}$$

Where M is the mass of the scalar field ϕ, which satisfies the Klein-Gordan equation

$$g^{ij}\phi_{;ij} + M^2\phi = 0 \tag{11}$$

here a comma (,) and semicolon (;) denote ordinary and covariant differentiation respectively and $\phi = \phi(t)$.
The conservation equations can be written as

$$T^{ij}_{;j} = \left(T^{ij(\Lambda)} + T^{ij(cs)}\right)_{;j} = 0. \tag{12}$$

Here, minimally interacting fields are considered so that from equation (12), the following is obtained.

$$T^{ij(\Lambda)}_{;j} = 0 \tag{13}$$

and

$$T^{ij(cs)}_{;j} = 0. \tag{14}$$

The following cosmological parameters are useful to solve the field equations:
The spatial volume V of the universe is defined as,

$$V = a^4(t) = A^3 B, \tag{15}$$

where a is the mean scale factor.
The average Hubble's parameter H for Kaluza-Klein space-time is given by

$$H = \frac{1}{4}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right), \tag{16}$$

where overhead dot (˙) denotes derivatives with respect to time t. The directional Hubble parameters in the direction of x, y, z and ψ are

$$H_x = H_y = H_z = \frac{\dot{A}}{A} \quad \text{and} \quad H_\psi = \frac{\dot{B}}{B}. \tag{17}$$

The expansion scalar θ is given as

$$\theta = 4H = 3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}. \tag{18}$$

The shear scalar σ and the mean anisotropy parameter Δ are defined as

$$\sigma^2 = \frac{1}{2}\left[\sum_{i=1}^{4} H_i^2 - 4H^2\right], \tag{19}$$

$$\Delta = \frac{1}{4}\left(\frac{H_i - H}{H}\right)^2, \ (i = 1,2,3,4). \tag{20}$$

The deceleration parameter q is given by

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right). \tag{21}$$

Using the co-moving coordinates, the field equations for the metric (1) are

$$3\left(\frac{\dot{A}}{A}\right)^2 + 3\frac{\dot{A}\dot{B}}{AB} = (\rho_\Lambda + \rho) + \frac{\dot{\phi}^2}{2} + \frac{M^2\phi^2}{2}, \tag{22}$$

$$\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} + 2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} = -\omega_\Lambda\rho_\Lambda - \frac{\dot{\phi}^2}{2} + \frac{M^2\phi^2}{2}, \tag{23}$$

$$\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} + 2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} = \lambda - \omega_\Lambda\rho_\Lambda - \frac{\dot{\phi}^2}{2} + \frac{M^2\phi^2}{2}, \tag{24}$$

$$3\left(\frac{\dot{A}}{A}\right)^2 + 3\frac{\ddot{A}}{A} = -(\omega_\Lambda + \delta)\rho_\Lambda - \frac{\dot{\phi}^2}{2} + \frac{M^2\phi^2}{2}, \tag{25}$$

$$\ddot{\phi} + \dot{\phi}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) + \frac{M^2\phi^2}{2} = 0 \tag{26}$$

and using equations (13) and (14), the conservation equations are

$$\dot{\rho}_\Lambda + \rho_\Lambda\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)(1 + \omega_\Lambda) = 0, \tag{27}$$

$$\dot{\rho} + \rho\ \left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) - \lambda\frac{\dot{B}}{B} = 0, \tag{28}$$

3. Solution of the Field Equations

Now, equations (22)-(26) constitute a set of five nonlinear equations with eight unknowns $A, B, \rho, \rho_\Lambda, \lambda, \omega, \delta$ and ϕ. To solve the above system of highly nonlinear differential equations, the following physically relevant conditions are used.

(i) The expansion scalar θ of the model is proportional to the shear scalar σ^2, which leads to

$$A = B^n, \tag{29}$$

where $n \neq 1$ is a positive constant that retains the anisotropy of the space-time. [32,33]

(ii) $\rho + \lambda = 0$ (Reddy string) (30)

i.e., the sum of rest energy density and string tension density for a cloud of string vanishes. The motivation behind considering this relation is explained in literature (Rachel [34], Kibble [35], Letelier [2], Reddy [36,37], Reddy and Rao [38,39], Singh [40]).
(iii) To reduce the mathematical complexity of the system, here the following assumption is considered

$$(3n+1)\frac{\dot{B}}{B} = -\frac{\dot{\phi}}{\phi}\,. \tag{31}$$

This is a consequence of the relation between the scalar field and the average scale factor. Many authors have used this relation to construct scalar field cosmological models. Naidu *et al.* [3], Singh [41], Singh and Rani [42], and Aditya *et al.* [12,43] have investigated Bianchi-type cosmological models with massive scalar fields using the above relation
From equations (26), (29), and (31), the scalar field ϕ if obtained as

$$\phi = exp\left(\phi_0 t - \frac{M^2t^2}{2} + \phi_1\right), \tag{32}$$

where ϕ_0 and ϕ_1 are constants of integration.
Now, using equations (29), (31) and (32),

$$A = exp\, n\left(\frac{\frac{M^2t^2}{2} - \phi_0 t - \phi_1}{3n+1}\right), \tag{33}$$

$$B = exp\left(\frac{\frac{M^2t^2}{2} - \phi_0 t - \phi_1}{3n+1}\right). \tag{34}$$

With the help of equations (33) and (34) in equation (1), the Kaluza-Klein model is obtained as

$$ds^2 = dt^2 - exp\quad 2n\left(\frac{\frac{M^2t^2}{2} - \phi_0 t - \phi_1}{3n+1}\right)(dx^2 + dy^2 + dz^2) - exp\, 2\left(\frac{\frac{M^2t^2}{2} - \phi_0 t - \phi_1}{3n+1}\right)d\psi^2 \tag{35}$$

4. Dynamical Parameters of the Model

In this section, we obtained the following physical and geometrical parameters for the model (35), which are important in the discussion of cosmology. The average Hubble parameter H, the expansion scalar θ, the spatial volume V, The shear scalar σ^2, The average anisotropy parameter Δ and the deceleration parameter q are obtained as

$$H = \frac{1}{4}(M^2 t - \phi_0), \tag{36}$$

$$\theta = (M^2 t - \phi_0), \tag{37}$$

$$V = exp\left(\frac{M^2t^2}{2} - \phi_0 t - \phi_1\right), \tag{38}$$

$$\sigma^2 = \frac{3}{8}\left(\frac{n-1}{3n+1}\right)^2 (M^2 t - \phi_0)^2, \tag{39}$$

$$\Delta = 3\left(\frac{n-1}{3n+1}\right)^2, \tag{40}$$

and

$$q = -\left[1 + \frac{4M^2}{(M^2 t - \phi_0)^2}\right]. \tag{41}$$

From equation (28), the energy density of the string is given as

$$\rho = k\, exp\left\{-\frac{4n+1}{3n+1}\left(\frac{M^2 t^2}{2} - \phi_0 t - \phi_1\right)\right\}. \tag{42}$$

With the help of equation (22), the energy density of DE is obtained as

$$\rho_\Lambda = \frac{3n(n+1)(M^2 t - \phi_0)^2}{(3n+1)^2} - k\, exp\left(-\frac{4n+1}{3n+1}\right)\left(\frac{M^2 t^2}{2} - \phi_0 t - \phi_1\right) - \left[\frac{(\phi_0 - M^2 t)^2 + M^2}{2}\right] exp(2\phi_0 t - M^2 t^2 + 2\phi_1) \tag{43}$$

From equation (6), the EoS parameter of DE as

$$\omega_\Lambda = -\frac{1}{\rho_\Lambda}\left\{\begin{array}{l}\frac{[(n+1)^2 + 2n^2](M^2 t - \phi_0)^2}{(3n+1)^2} + \frac{(2n+1)M^2}{(3n+1)} + \\ \left[\frac{(\phi_0 - M^2 t)^2 - M^2}{2}\right] exp(2\phi_0 t - M^2 t^2 + 2\phi_1)\end{array}\right\} \tag{44}$$

Equations (30) and (42) together yield

$$\rho = -\lambda = ke\left(-\frac{4n+1}{3n+1}\left(\frac{M^2 t^2}{2} - \phi_0 t - \phi_1\right)\right). \tag{45}$$

The skewness parameter δ is

$$\delta = \frac{1}{\rho_\Lambda}\left\{\frac{(1-n)}{(3n+1)}\left[(M^2 t - \phi_0)^2 + M^2\right]\right\}. \tag{46}$$

5. Physical Discussion of the Model

The spatial volume V: Fig.1 depicts the behavior of the volume V versus cosmic time t; it has been found that the spatial volume increases exponentially and attains infinite value as time t increases.

The Scalar field: Fig. 2 describes the behavior of scalar field ϕ versus time t. It can be seen that the scalar field is a positive and decreasing function of cosmic time t. The behavior of the scalar field of our model is quite similar to the scalar field shown in the model investigated by [14,44].

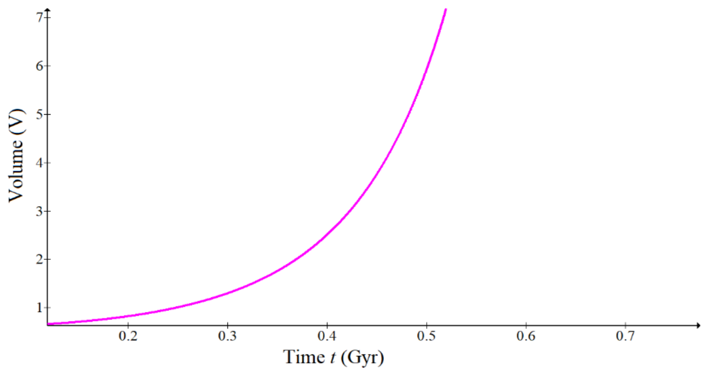

Fig. 1. Plot of volume V versus time t for $\phi_0 = \phi_1 = 0.5$, $n = 0.9$ and $M = 4.5$.

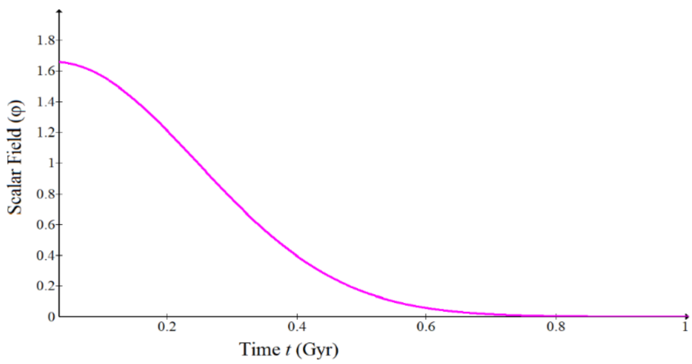

Fig. 2. Plot of scalar field ϕ versus time t for $\phi_0 = \phi_1 = 0.5$, $n = 0.9$and $M = 4.5$.

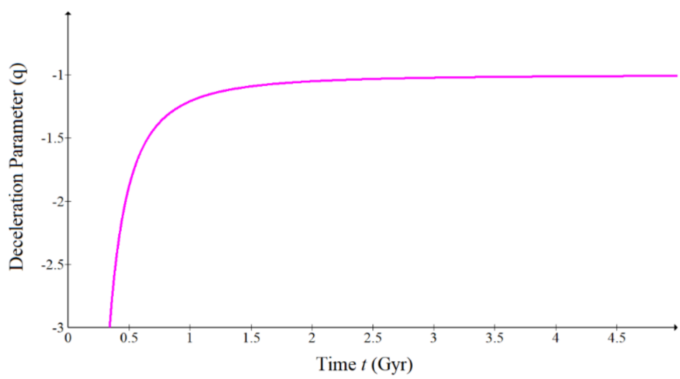

Fig. 3. Plot of deceleration parameter q versus time t for $\phi_0 = 0.5$ and $M = 4.5$.

The deceleration parameter q: In Fig. 3, it is depicted the behavior of deceleration parameter q versus cosmic time t. One of the important physical quantities is the deceleration parameter, which shows whether the universe is accelerating or decelerating. For the model (35), it is observed that initially $q < -1$; hence, a universe with the super-exponential expansion is obtained, and it approaches $q = -1$, which indicates the exponential expansion of the universe.

The EoS parameter ω_Λ: The relationship between pressure p_Λ and energy density ρ_Λ of DE is defined by EoS parameter whose expression is given by$\omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda}$. The EoS parameter is used to classify the decelerated and accelerated expansion of the universe, and it categorizes various epochs as follows: when $\omega_\Lambda = 1$, it represents stiff fluid, if $\omega_\Lambda = \frac{1}{3}$, the model shows a radiation-dominated phase, and when $\omega_\Lambda = 0$, it represents the dominated phase. Also, in DE dominated accelerated phase, $-1 < \omega_\Lambda < -\frac{1}{3}$ shows the quintessence phase and $\omega_\Lambda = -1$ shows the cosmological constant i.e., ΛCDM model and $\omega_\Lambda < -1$ indicates the phantom era. From Fig.4, it is observed that the model starts from the aggressive phantom region. $\omega_\Lambda < -1$ and approaches to $\omega_\Lambda = -1$ for $\phi_0 = 0.5$.

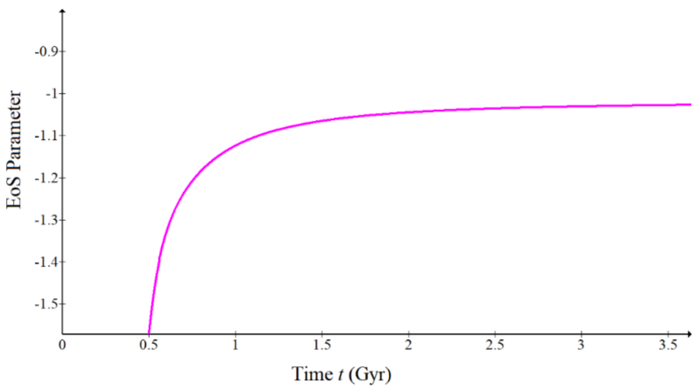

Fig. 4. Plot of EoS parameter ω versus time t for $\phi_1 = 0.5$, $n = 0.9$ and $M = 4.5$.

The Statefinder and Om diagnostic: To differentiate various DE models, Sahni *et al.* [45] have introduced two new dimensionless parameters known as state-finders (r, s) defined as follows:

$$r = \frac{\ddot{a}}{aH^3}, \quad s = \frac{r-1}{3\left(q-\frac{1}{2}\right)}. \tag{47}$$

These state-finders define the well-known regions as For $(r, s) = (1,0)$ and $(r, s) = (1, 1)$, which defines ΛCDM and standard cold dark matter (CDM), respectively. However, $s > 0$ and $r < 1$ constitute the DE regions such as phantom and quintessence, $s < 0$ and $r > 1$ give the Chaplygin gas model. Recently, many authors have investigated the state-finders analysis with different geometries [46-52].

For our model, the parameters above are reduced to

$$r = 1 + \frac{12M^2}{(M^2t-\phi_0)^2}, \tag{48}$$

$$s = \frac{-8M^2}{3(M^2t-\phi_0)^2+8M^2}. \tag{49}$$

Fig. 5 shows the statefinder parameter from which it is observed that our model approaches ΛCDM model. To distinguish different phases of the universe, Sahni *et al.* [53] have introduced another tool named Om-diagnostic. It is also used to distinguish the ΛCDM for non-minimally coupled scalar field, quintessence model, and phantom field through trajectories of the curves. The phantom DE era corresponds to the positive trajectory, whereas the negative trajectory means that DE constitutes quintessence.

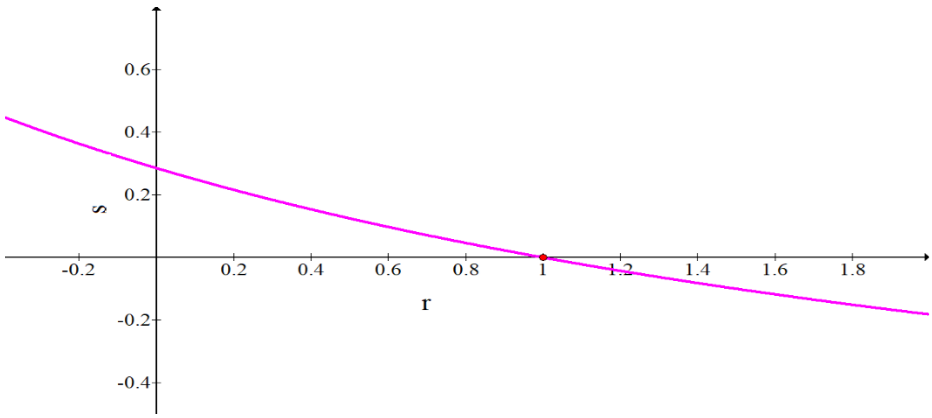

Fig. 5. Plot of statefinder parameter.

$$Om(x) = \frac{h(x)^2-1}{x^3-1}, \tag{50}$$

where $h(x) = \frac{H(x)}{H_0}$, $x = (1+z)$ here $z$is the red shift parameter and,H_0 is the present value of the Hubble parameter [54].

For our model (35), we obtain

$$Om(x) = \frac{(M^2x-\phi_0)^2-16{H_0}^2}{16{H_0}^2(x^3-1)}. \tag{51}$$

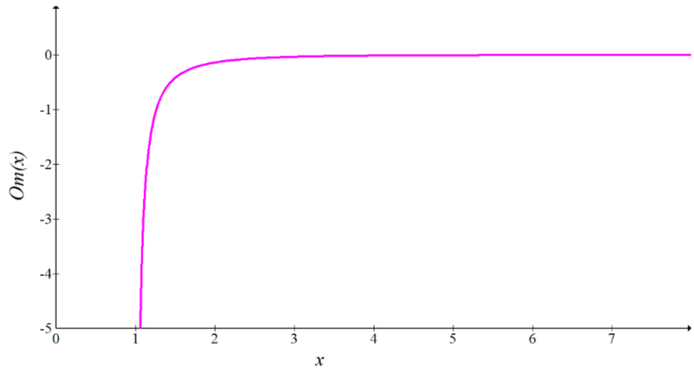

Fig. 6. Plot of $Om(x)$ diagnostic versus x.

From Fig. 6, it is observed that the slope of the curve is negative and remains constant $x \geq 3$. Thus, our model represents the quintessence behavior of the universe. This type of behavior is consistent with recent observational data.

6. Conclusion

In this paper, we have investigated the Kaluza-Klein universe when the source of gravitation is the mixture of cosmic string coupled with an attractive massive scalar field and minimally interacting anisotropic DE fluid in the general theory of relativity. The main outcomes of the constructed model are summarized below:

- In the Kaluza-Klein model, energy density is an increasing function of cosmic time t and is always positive throughout the evolution. The string tension density is always negative throughout the evolution and is a decreasing function of cosmic time t.
- At an initial epoch, the spatial volume of our model admits constant value and then increases exponentially as time increases. Thus, the model expands.
- Anisotropy parameter Δ is constant; thus, the model is uniform throughout and homogeneous. Also, when $n = 1$, the model becomes isotropic (since $\Delta = 0$) and shear-free.
- The massive scalar field of the model is positive throughout the evolution of the universe and increases rapidly at the present epoch.
- The physical parameters like H, θ and σ^2 diverges as $t \to \infty$, and they all reduce to constant values at $t = 0$.
- At an initial epoch, the model exhibits super-exponential expansion, and after some finite time, it approaches exponential expansion.
- In the model, the EoS parameter behaves like a phantom dark energy.
- Statefinder diagnostic and Om diagnostic are applied in the model in order to distinguish our dark energy model from other existing dark energy models.

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