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Cost Optimization of Queueing System with Working Vacation, Setup, Feedback, Reneging, and Retention of Reneged Customers

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Abstract

In this manuscript, Markovian queueing system with working vacation, Bernoulli schedule interruption, setup time under feedback, reneging of impatient customers, and retention of reneged customers are analyzed. The unsatisfied customers on service completion may either leave the system with probabilities β_1, β_2 or may rejoin the queue with complementary probabilities during working vacations and regular service, respectively. The waiting customers in the queue may lose patience due to vacations and decide to leave without getting the service with probability q. They may be retained in the system via some convincing mechanisms with probability (1-*q*). The mean system length, probability of server in various states, mean sojourn time are obtained using the probability generating function method. The MATLAB software is used for representing the observed behavior graphically.

Keywords: Feedback; Reneging; Retention; Working vacation; Queue.

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1. Introduction

Queueing systems with vacations are widely used to model mainly real-time systems such as manufacturing communication systems, telecommunication systems etc. Pioneer work in this field was performed by Levy and Yechiali [1]. Excellent surveys on such systems can be seen in Doshi [2], Tian and Zhang [3], Ke *et al*. [4], Chandrasekaran *et al*. [5], and references therein. On vacations, service is completely stopped, but a different class of vacations, i.e., working vacation, is studied first by Servi and Finn [6]. After that, many researchers worked on these models where instead of stopping service, the server provides service at a comparatively lower rate. For such queueing models, we refer the reader to [7-11] and references therein.

Queueing systems with vacations and customer impatience play a vital role in many congestion systems like call centers, industrial congestion systems, etc. The impatient customer behavior is a challenge in modeling queueing systems. In these systems,

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customers leave the system due to impatience before getting the service. There could be many reasons for the customer's impatience like delay in service, server vacation, server breakdown, slow service etc. Some of the related works can be seen in [12-16] and referenced therein. The transient solution of the queueing system provides a complete behavior of the system over time. Amar [17] analyzed the transient solution of a single server vacation queue with customer impatience. A multi-server queueing system with impatient customers under vacation is studied by Kadi *et al.* [18].

In this paper, feedback queueing system with working vacation, Bernoulli schedule interruption, reneging, retention of reneged customers, and setup time are analyzed. This model generalizes many of the existing queueing systems in the literature. Queueing model with setup time plays a very important role in meeting the demand for power-saving. The server still works on vacation but at a slower rate. On service completion, unsatisfied customers may rejoin the system with some probability. At the end of the vacation, the system enters a closed-down state if it is empty and only an arriving customer can activate set up time, and normal service resumes. To the best of the authors' knowledge, there is no research on feedback queueing systems with set up time under working vacations and the retention of impatient customers in queueing literature.

2. Model Description

The proposed model is considered under the following assumptions:

1. The customers arrive in the queueing system according to the Poisson process with a mean arrival rate of λ. The service is provided to customers on FCFS discipline. The service time in the active state follows an exponential distribution with the rate μ.
2. Whenever the system gets empty, the server goes on vacation, but instead of stopping service completely, it provides service at a slow rate μ_v. The service time in vacations and vacation time are both assumed to follow an exponential distribution.
3. The vacation will be interrupted with probability p or server remains in vacation state with complementary probability $\bar{p}$ (=1-p) depending on whether customers are waiting in the system at the moment of service completion in vacation state or not.
4. In any state of the server, at the instant of service completion, the customers may decide to leave the system, or an unsatisfied customer may rejoin it with probabilities $\overline{\beta_1}$ and $\overline{\beta_2}$ in working vacation and normal states, respectively.
5. The customers may become impatient due to server vacations and may leave the system without being served. But it is supposed that the impatient customers may be retained in the system via some convincing mechanisms with probability $\overline{q}$. The reneging time is assumed to follow an exponential distribution with rate ϕ.
6. If the queueing system is found to be empty at the end of the vacation, the system enters a closed-down state. When the arriving customer initiates a setup time, which is exponentially distributed with rate η, only then does regular service resume.

3. Steady-State Equations and Solution

Assuming *N(t)* represents the number of customers in the system at any time *t* and *J(t)* as the state of the server at instant t, {*N(t), J(t)*} is a continuous Markov chain.

Let $p_{n\,i}$ be the probability of n customers in the system in i[th] state of the server. The different possible states of the server are

$$\mathrm{J(t)} = \begin{cases} 0, server\ in\ active\ state \\ 1, server\ in\ working\ vacation \\ 2, server\ in\ closed\ down\ state \end{cases}$$

The equations governing the proposed model in steady-state, using the Markov process, are

$$\lambda p_{0\,2} = \theta p_{0\,1} \quad (1)$$

$$(\lambda+\eta)p_{n\,2} = \lambda p_{n-1\,2}, \quad n \geq 1 \quad (2)$$

$$(\lambda+\theta)p_{0\,1} = \mu\beta_2 p_{1\,0} + (\beta_1\mu_v + q\phi)p_{1\,1} \quad (3)$$

$$\left(\lambda+\eta q\phi+\theta+\beta_1\mu_v+\overline{\beta_1}p\mu_v\right)p_{n\,1} = (n+1)q\phi p_{n+1\,1} + \beta_1\mu_v\bar{p}p_{n+1\,1} + \lambda p_{n-1\,1}, n \geq 1 \quad (4)$$

$$(\lambda+\mu\beta_2)p_{1\,0} = \eta p_{1\,2} + \mu\beta_2 p_{2\,0} + \left(\theta+\overline{\beta_1}p\mu_v\right)p_{1\,1} + \beta_1 p\mu_v p_{2\,1} \quad (5)$$

$$(\lambda+\mu\beta_2)p_{n\,0} = \lambda p_{n-1\,0} + \eta p_{n\,2} + \mu\beta_2 p_{n+1\,0} + p\beta_1\mu_v p_{n+1\,1} + \left(\theta+p\overline{\beta_1}\mu_v\right)p_{n\,1}, n \geq 2 \quad (6)$$

Defining probability generating functions of a number of customers in the queueing system as

$$H_0(\mathrm{z}) = \sum_{n=1}^{\infty} P_{n\,0}z^n$$

$$H_1(\mathrm{z}) = \sum_{n=0}^{\infty} P_{n\,1}z^n$$

$$H_2(\mathrm{z}) = \sum_{n=0}^{\infty} P_{n\,2}z^n$$

Multiplying equation (2) with z^n and summing over all values of *n*, adding equation (1) we obtained

$$(\lambda+\eta-\lambda z)H_2(\mathrm{z}) = \theta\left(1+\frac{\eta}{\lambda}\right)P_{0\,1} \quad (7)$$

Using probability generating functions equation (3) and (4) yield

$$H_1'(z) + \left(-\frac{\lambda}{q\phi} + \frac{\beta_1\mu_v - p\mu_v\beta_1}{q\phi z} + \frac{p\mu_v+\theta}{q\phi(z-1)}\right)H_1(\mathrm{z}) = -\frac{1}{q\phi}\left(\frac{A_1}{1-z} - \frac{A_2}{z}\right) \quad (8)$$

Where

$A_1 = p\mu_v\beta_1 p_{1\,1} + \mu\beta_2 p_{1\,0} + p\mu_v p_{0\,1}$

$A_2 = \bar{p}\mu_v\beta_1 p_{0\,1}$

Solving differential equation (8)

$$I.F. = e^{\frac{-\lambda z}{q\phi}}(1-z)^{\frac{p\mu_v+\theta}{q\phi}} z^{\frac{\bar{p}\mu_v\beta_1}{q\phi}}$$

$$H_1(\mathrm{z}) = (-A_1B_1(z) + A_2B_2(z))\frac{1}{I.F.} \tag{9}$$

Where,

$$B_1(z) = \frac{1}{q\phi}\int_0^z e^{\frac{-\lambda z}{q\phi}}(1-z)^{\frac{p\mu_v+\theta}{q\phi}-1} z^{\frac{\bar{p}\mu_v\beta_1}{q\phi}}$$

$$B_2(z) = \frac{1}{q\phi}\int_0^z e^{\frac{-\lambda z}{q\phi}}(1-z)^{\frac{p\mu_v+\theta}{q\phi}} z^{\frac{\bar{p}\mu_v\beta_1}{q\phi}-1}$$

Multiplying equations (5) and (6) with the appropriate power of *z* and summing over *n*, we get

$$\begin{aligned}(1-z)(\lambda z-\mu\beta_2)H_0(\mathrm{z}) &= \eta z H_2(\mathrm{z}) + \left((\theta + p\mu_v\overline{\beta_1})z + p\mu_v\beta_1\right)H_1(\mathrm{z}) \\ &-(p\mu_v\beta_1 p_{1\,1} + \mu\beta_2 p_{1\,0} + p\mu_v p_{0\,1} + \theta p_{0\,1} + \eta p_{0\,2})z + p\mu_v\beta_1(z-1)p_{0\,1}\end{aligned} \tag{10}$$

Taking z = 1 in equation (10) we obtain

$$\eta H_2(1) + (\theta + p\mu_v)H_1(1) = (p\mu_v\beta_1 p_{1\,1} + \mu\beta_2 p_{1\,0} + (p\mu_v + \theta)p_{0\,1} + \eta p_{0\,2}) \tag{11}$$

Substituting in equation (10) we obtain

$$H_0(\mathrm{z}) = \frac{\eta z H_2(\mathrm{z}) + \left((\theta + p\mu_v\overline{\beta_1})z + p\mu_v\beta_1\right)H_1(\mathrm{z}) - z\left(\eta H_2(1) + (\theta + p\mu_v)H_1(1)\right)}{(1-z)(\lambda z - \mu\beta_2)} - \frac{p\mu_v\beta_1 p_{0\,1}}{(\lambda z - \mu\beta_2)} \tag{12}$$

Taking limit $z \to 1$ and using L' Hospital rule

$$H_0(1) = \frac{\eta H_2'(1) + (\theta + p\mu_v)H_1'(1) + p\mu_v\beta_1(p_{0\,1} - H_1(1))}{\mu\beta_2 - \lambda} \tag{13}$$

Rearranging the terms in equation (13) we get

$$\begin{aligned}(\theta + p\mu_v)H_1'(1) &= (\,p\mu_v\beta_1 - \mu\beta_2 + \lambda)H_1(1) - \left(\frac{(\lambda+\eta)\theta\mu\beta_2}{\lambda\eta} + p\mu_v\beta_1\right)p_{0\,1} \\ &+\mu\beta_2 - \lambda\end{aligned} \tag{14}$$

Now, adding equations (2), (4), (5), and (6), we obtained

$$\begin{aligned}&\lambda(p_{n\,2} + p_{n\,1} + p_{n\,0}) - \mu_v\beta_1 p_{n+1\,1} - (n+1)q\phi p_{n+1\,1} - \mu\beta_2 p_{n+1\,0} \\ &= \lambda(p_{n-1\,2} + p_{n-1\,1} + p_{n-1\,0}) - \mu_v\beta_1 p_{n\,1} - nq\phi p_{n\,1} - \mu\beta_2 p_{n\,0}\end{aligned} \tag{15}$$

Using recurrence in equation (15) together with equations (1) and (3)

$$\lambda(p_{n\,2} + p_{n\,1} + p_{n\,0}) - \mu_v\beta_1 p_{n+1\,1} - (n+1)q\phi p_{n+1\,1} - \mu\beta_2 p_{n+1\,0} = 0 \tag{16}$$

After some rearrangement of terms in equation (16) we get

$$\lambda(p_{n\,2} + p_{n\,1} + p_{n\,0}) = \mu_v\beta_1 p_{n+1\,1} + (n+1)q\phi p_{n+1\,1} + \mu\beta_2 p_{n+1\,0}$$

Summing over all possible values of *n* and using normalization condition

$$H_1'(1) = \frac{1}{q\phi}\left(\lambda - \mu\beta_2 + (\mu\beta_2 - \mu_v\beta_1)H_1(1) + \left(\mu_v\beta_1 + \frac{\theta\mu\beta_2(\lambda+\eta)}{\lambda\eta}\right)p_{0\,1}\right) \quad (17)$$

Taking limit $z \to 1$ in equation (9)

$$\frac{1}{I.F.} \to \infty, therefore\ A_2B_2(1) - A_1B_1(1) = 0 \quad (18)$$

From equation (11) we obtain

$$A_1 = (\theta + p\mu_v)H_1(1) \quad (19)$$

Using equations (14), (17), and (18) simultaneously, we get

$$p_{0\,1} = \left(\frac{\mu\beta_2-\lambda}{\theta+p\mu_v} + \frac{\mu\beta_2-\lambda}{q\phi}\right)\frac{B_1(1)}{k_1}(\theta+p\mu_v)^2\left(\frac{k_2}{k_1}(\theta+p\mu_v)^2B_1(1) - \bar{p}\beta_1\mu_vB_2(1)\right)^{-1} \quad (20)$$

Now, equation (3) and (19) gives

$$p\mu_v\beta_1p_{1\,1} + \mu\beta_2p_{1\,0} + p\mu_vp_{0\,1} = (\theta+p\mu_v)H_1(1) \quad (21)$$

$$\mu\beta_2p_{1\,0} = (\lambda+\theta)p_{0\,1} - (\mu_v\beta_1 + q\phi)p_{1\,1} \quad (22)$$

Using equations (21) and (22) together, we get

$$p_{1\,1} = \frac{(\theta+p\mu_v)^2}{(\mu_v\beta_1\bar{p} + q\phi)}\left[\left(\frac{\lambda+\theta+p\mu_v}{(\theta+p\mu_v)^2} - \frac{k_2}{k_1}\right)p_{0\,1} + \frac{\mu\beta_2-\lambda}{k_1(\theta+p\mu_v)} + \frac{\mu\beta_2-\lambda}{k_1q\phi}\right] \quad (23)$$

$p_{1\,0}$ can be obtained by using equations (20), (22), and (23) simultaneously.
Differentiating equation (12) taking limit $z \to 1$ and using L' Hospital's rule twice.

$$H_0'(1) = \frac{\left(2\eta H_2'(1) + \eta H_2''(1) + (\theta+p\mu_v)H_1''(1) + 2\left(\theta + \mu_vp\overline{\beta_1}\right)H_1'(1)\right)(\mu\beta_2-\lambda)}{2(\mu\beta_2-\lambda)^2}$$
$$+ \frac{\lambda(\eta H_2'(1) + (\theta+p\mu_v)H_1'(1) - \mu_vp\beta_1H_1(1) + \mu_vp\beta_1p_{0\,1})}{(\mu\beta_2-\lambda)^2} \quad (24)$$

Where

$$H_1''(1) = \frac{2}{(2q\phi - \theta - p\mu_v)}\left(\lambda H_1(1) + (\lambda + p\mu_v + \theta - \beta_1\mu_v\bar{p} - q\phi)H_1'(1)\right) \quad (25)$$

$H_1''(1)$ is obtained by differentiating equation (8) twice and taking limit $z \to 1$.

4. Performance Measures

$$\text{Expected system length } \mathrm{EL_S} = \sum_{i=1}^{2} Expected\ system\ length\ in\ i^{th}\ state$$
$$= \mathrm{H_0'(1) + H_1'(1) + H_2'(1)}$$

$$\text{Mean Sojourn time in the system } \mathrm{W_S} = \frac{\mathrm{EL_S}}{\lambda} \text{(By Little's formula)}$$

Probability of server in Working Vacation state $\mathrm{P_{WV}} = \mathrm{H_1(1)}$
Probability of server in normal state $\mathrm{P_N} = \mathrm{H_0(1)}$
Probability of server in close down $P_{CD} = \mathrm{H_2(1)}$

Rate of abandonment of impatient customers $R_a = q\phi\, H_1'(1)$

5. Graphical Illustrations

In this section, we represent the impact of various parameters on mean system size and different system states probabilities graphically. We have optimized the cost relative to the setup rate.

We have fixed the parameters as λ=1, μ=5, ϕ=3, θ=0.3, α=0.2, β=0.1, γ=0.7, ξ=3 in graphs unless they are varied as shown in the graphs.

(a) Sensitivity analysis

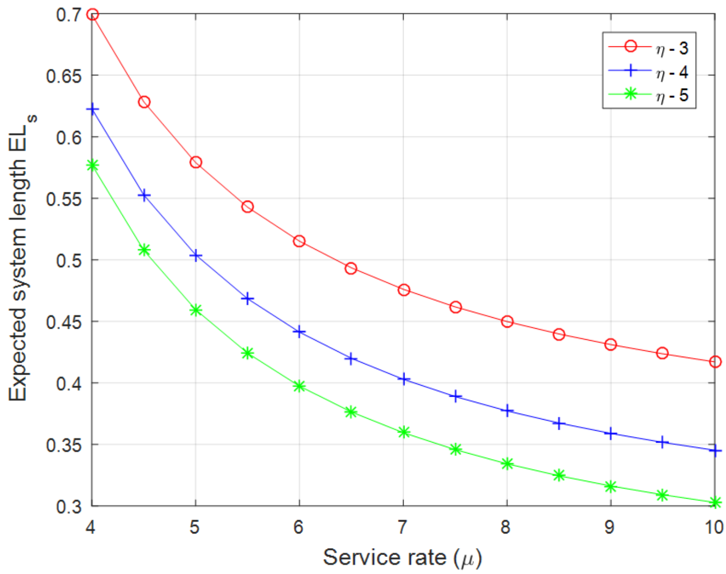

Fig. 1. Variation in Expected System Length with μ for different vales of η.

Fig. 1 depicts that as service rate μ increases, the expected system length EL_S decreases. This is because the service time, hence the expected system length, decreases with an increase in service rate. The decrease is more evident with an increase in set up rate η; this is because of the corresponding decrease in set up time.

Fig. 2 reveals the variation in mean system length with change in service rate μ for a different set of values of leaving probabilities β_1 and β_2. For fixed values of β_1 and β_2, the expected system length decreases with increase in service rate μ; this is due to reduction in service time with increase in service rate. The expected system length further decreases as β_1 and β_2 increase. This is expected since as leaving probabilities of customers increase, the mean system length will decrease.

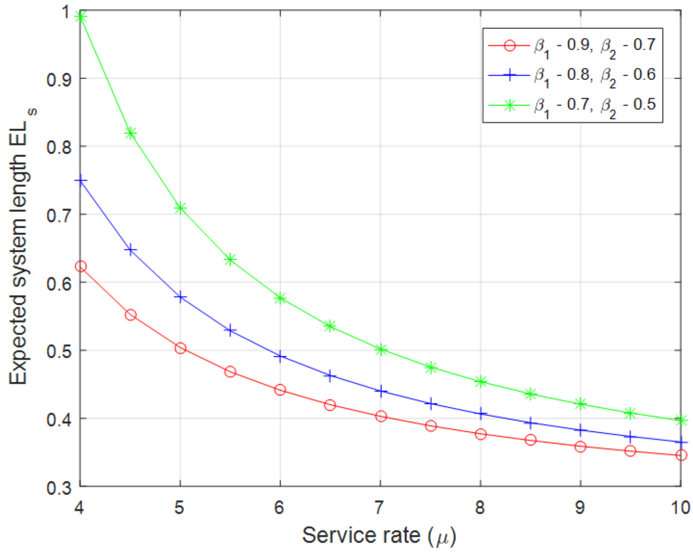

Fig. 2. Variation in Expected system length with μ for a different set of β_1 and β_2.

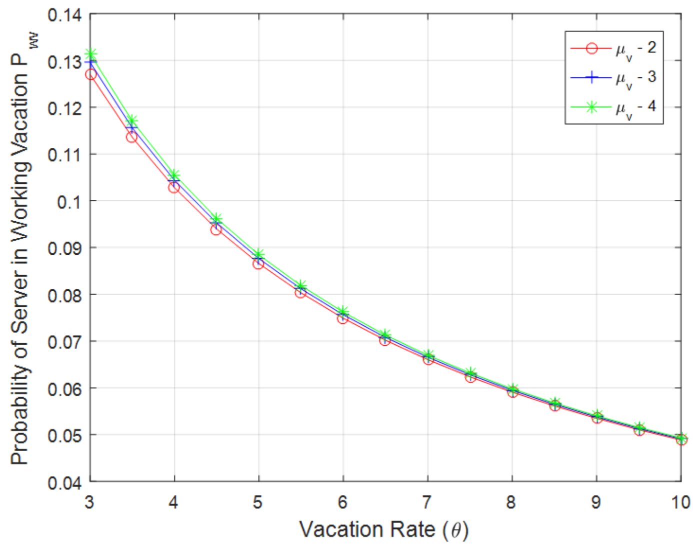

Fig. 3. Impact of vacation rate θ on the probability of server in a working vacation P_{WV} for different values of slow service rate μ_v.

As depicted in Fig. 3, the probability of the server being in a Working Vacation state decreases with an increase in vacation rate θ. The vacation time will decrease with an increase in θ. As the working vacation period decreases, the probability of the server being on working vacation will decrease. This probability further decreases with a decrease in slow service rate μ_v; the reason being the corresponding increase in service time in vacation.

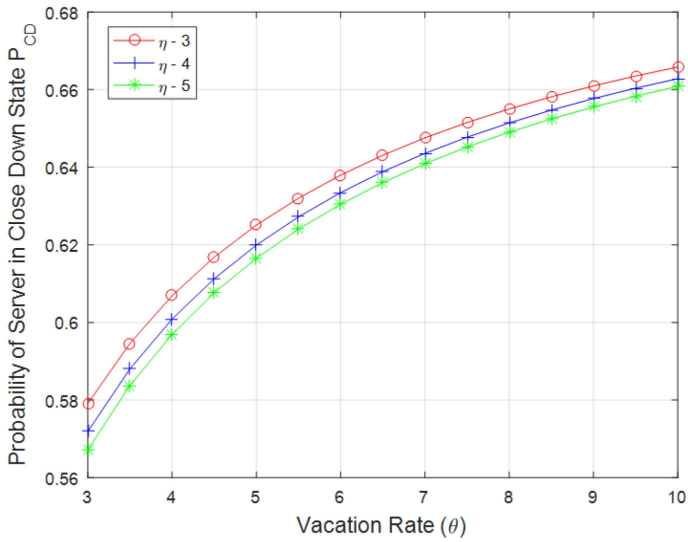

Fig. 4. Impact of vacation rate θ on the probability of server in Close down state P_{CD} for different values of set up rate η.

Fig. 4 reveals that the probability of the server being in close down state increases with an increase in vacation rate θ. This increase is due to a corresponding decrease in vacation time which increases the chances of the server being closed downstate. As shown in Fig, this increase becomes more obvious with a decrease in setup rate for a fixed value of θ. The reason is that with a decrease in set up rate, the setup time increases, increasing the probability of the server being in close downstate.

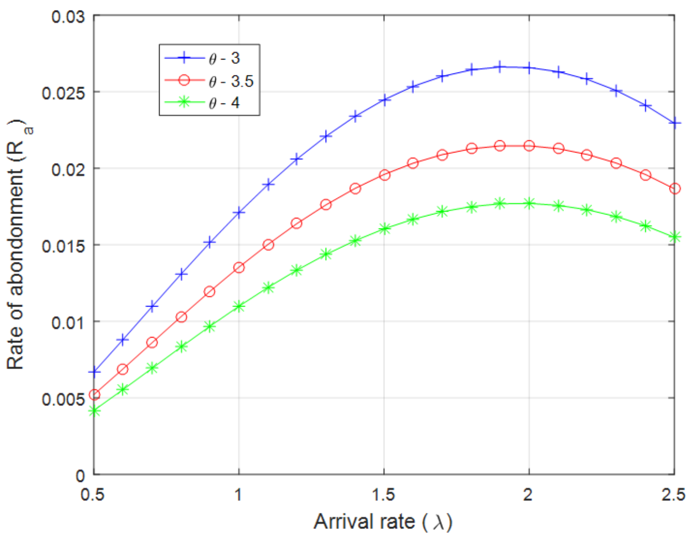

Fig. 5. Effect of arrival rate on the rate of abandonment for different values of vacation rate θ.

The rate of abandonment first increases with an increase in λ, and after reaching a maximum, it begins to decrease with a further increase in arrival rate, as we observe from Fig. 5. This abandonment rate further decreases with an increase in vacation rate θ. This observation is as intuitively expected; because vacation time will decrease with an increase in θ, and this will reduce the abandonment rate.

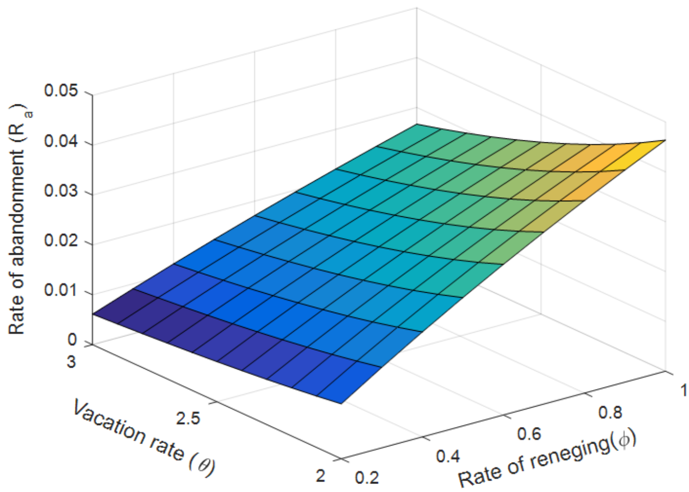

Fig. 6. Variation in Abandonment rate with θ and ϕ.

Fig. 6 reveals that. abandonment rate decreases with increase in θ for a fixed value of reneging rate ϕ, and it increases with increase in reneging rate ϕ for fixed vacation rate θ. These observations match our theoretical expectations.

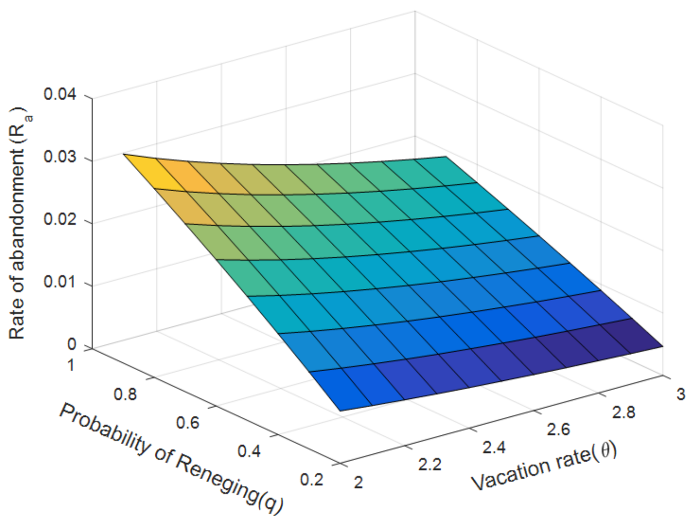

Fig. 7. Variation in Abandonment rate with q and θ.

Fig. 7. shows that the abandonment rate increases with an increase in q for the fixed value of vacation rate θ, decreasing with an increase in vacation rate θ for the fixed value of reneging probability q. These observations are again as expected.

(b) ***Cost model*:** In this subsection, we optimize the operating cost function with respect to set up rate η. For this purpose, we define some cost elements. Let C_{L_S} denotes cost per unit time for each customer present in the system; C_μ be the cost per unit time for service in a normal state; C_{μ_v} be the cost per unit time for service in working vacation state; C_θ as cost per unit time in vacation period; C_η denotes cost per unit time in setting up period.

Then cost function per unit time is defined as

$$F(\eta) = EL_S\, C_{L_S} + \mu C_\mu + \mu_v C_{\mu_v} + \theta C_\theta + \eta C_\eta$$

Cost elements are fixed as $C_L = 20$, $C_\mu = 25$, $C_{\mu_v} = 22$, $C_\theta = 18$, $C_\eta = 15$ to obtain the optimal value using the parabolic method. This method generates a quadratic function through the calculated points in each iteration. The point at which F(x) is optimum in three-point pattern $\{x_1, x_2, x_3\}$ is given by

$$x_L = \frac{0.5\left(F(x_1)(x_2{}^2 - x_3{}^2) + F(x_2)(x_3{}^2 - x_1{}^2) + F(x_3)(x_1{}^2 - x_2{}^2)\right)}{F(x_1)(x_2 - x_3) + F(x_2)(x_3 - x_1) + F(x_3)(x_1 - x_2)}$$

One of the three points is replaced by the new value obtained at every iteration to improve the current pattern. These steps are repeatedly used to obtain the result correct to the desired decimal places.

Table 1 shows that corresponding to η =1.12355, optimal cost =283.773478 is obtained with the permissible error of 10^{-5}. Fig. 8 verifies these observed results.

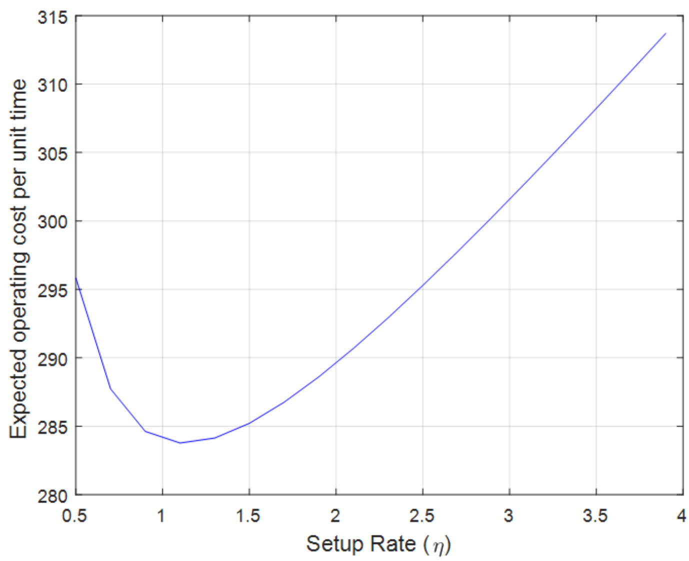

Fig. 8. Expected operating cost per unit time versus set up rate η.

Table 1. Cost optimization via quadratic fit approach.

#	η_1	η_2	η_3	$F(\eta_1)$	$F(\eta_2)$	$F(\eta_3)$	η
1	0.5	1.00	1.50	295.834059	284.008024	285.217461	1.20361
2	1.0	1.20	1.50	284.008024	283.855095	285.217461	1.13692
3	1.0	1.1369	1.2036	284.008024	283.775889	283.855095	1.12833
4	1.0	1.1283	1.1369	284.008024	283.773788	283.775889	1.12454
5	1.0	1.1245	1.1283	284.008024	283.773492	283.773788	1.12387
6	1.0	1.1239	1.1245	284.008024	283.773480	283.773492	1.12362
7	1.0	1.1236	1.1239	284.008024	283.773478	283.773480	1.12357
8	1.0	1.1236	1.1236	284.008024	283.773478	283.773478	1.12356
9	1.0	1.1236	1.1236	284.008024	283.773478	283.773478	1.12355

6. Conclusion

We have analysed a feedback queueing system with setup times under Bernoulli's schedule of working vacation, interruption, reneging, and retention of reneged customers. The expected system sizes in different states and probabilities of the server being in different states are obtained via probability generating functions. The operating cost of the system per unit time is minimized relative to the setup rate. The optimal value of the setup rate is also found using the parabolic method. With the aid of MATLAB software, the variation in Expected system length versus service rate, the probability of server in working vacation, and closed downstate versus vacation rate are studied graphically. The graphical variations are as expected intuitively. The model can be further extended to batch arrival or batch service.

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