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Hybrid Projective Synchronization of 4-D Hyperchaotic Systems via Adaptive Control

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Abstract

This paper designs a procedure for investigating the hybrid projective synchronization (HPS) scheme between two identical 4-D hyperchaotic systems. Based on Lyapunov stability theory (LST), an adaptive control technique (ACT) has been designed to achieve the desired HPS scheme. The suggested technique determines globally the asymptotic stability and identification of parameters simultaneously using HPS scheme. It is noted that complete , hybrid and anti-synchronization turns into particular cases of HPS scheme. Numerical simulations are presented to validate the effectivity and feasibleness of the considered technique by using MATLAB. Remarkably, the theoretical and computational outcomes are in complete agreement. Also, the considered HPS scheme is very efficient as it has numerous applications in encryption and secure communication.

Keywords: Hyperchaotic system, Adaptive control, Lyapunov stability, Hybrid projective synchronization, MATLAB**.**

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1. Introduction

Chaos theory is a field of applied mathematics which describes the behavior of extremely complex nonlinear dynamical systems possessing the property of high dependency on the initial conditions. This property is described as butterfly effect in the available literature which is observed by Lorenz [1] in 1963 while studying a simplified model of convection for weather predictions. Chaos theory has a wide range of applications in numerous areas of engineering and applied sciences, for example, jerk systems [2], robotics [3], finance models [4], weather models [5], ecological models [6], chemical reactions [7], circuits [8], oscillations [9], encryption [10], neural networks [11], biomedical engineering [12] etc. Consequently, chaos synchronization and control of nonlinear chaotic systems attracts researchers as well as academicians from various scientific fields in the recent years.

Poincare [13], a French mathematician and physicist, firstly discovered chaos in the late 19th century while dealing with three body problem which contains sun, moon and

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earth in order to study stability of the solar system. But the first introduction to chaos was given by Lorenz [1]. Because of the undesirable immensely complex characteristics of chaotic systems, the chaos control and synchronization of chaotic systems becomes the most prominent concern for the researchers over the past few years.

Pecora and Carroll [14] put forward the idea of chaos synchronization for the first time in 1990 using master-slave configuration with distinct initial conditions. Afterwards, several researchers continued the pioneered work established by them. Chaos synchronization is a process in which state trajectories of two or more chaotic systems with different initial conditions converge and synchronization error tends to zero asymptotically as time approaches infinity.

Till date, an enormous variety of synchronization techniques have been developed in synchronization theory, for instance, complete synchronization[15], anti-synchronization [16], hybrid synchronization [17], hybrid projective synchronization [18], function projective synchronization [19], lag synchronization [20], phase synchronization [21], projective synchronization [22], function projective synchronization [19], modified projective synchronization [23] etc.

Up to now, a variety of control methods to achieve chaos control have been introduced in control theory such as active control [24], adaptive control [25], backstepping design [26], feedback control [27], sliding mode control [28], impulsive control [29] etc.

A hyperchaotic system is known as a chaotic system possessing at least two positive Lyapunov exponents. Rossler [30] introduced the first classical hyperchaotic system in 1979. During the past decades, many classical hyperchaotic systems have been discovered, for example, Lorenz system, Cai system, Chen system, Liu system, Nikolov system and so on.

Hubler [31], in 1989, firstly introduced ACT to synchronize chaotic systems. Mainieri and Rehacek [32] proposed the idea of projective synchronization to achieve chaos synchronization among chaotic systems in 1999. In [33], the synchronization theory of chaotic systems such as Chua's circuit and Rossler-like system have been developed separately using ACT and also it has been shown via simulation results that it has applications in secure communications. Also, Yassen studied synchronization of a modified Chua's circuit system using ACT [34]. Further, projective synchronization between chaotic systems is discussed [35]. Also, Li *et al.* studied adaptive backstepping scheme for synchronizing nonlinear chaotic systems [36]. Moreover, Wu and his team discussed complex projective chaos synchronization among complex chaotic systems [37]. Various control techniques have been analyzed in detail for newly constructed hyperchaotic systems [38-40].

Influenced from the above stated discussions, this paper aims to study a hybrid projective synchronization (HPS) between two identical 4-D hyperchaotic systems by using ACT. ACT is very useful in estimating the parameters among master and slave systems. Therefore, by using this approach, not much information is necessary for synchronizing the master and slave systems. Moreover, we investigate in detail an adaptive control laws along with an estimated parameter update laws using LST.

The current paper is categorized as follows: In Section 2, basic preliminaries having some notations and essential terminology used within this paper has been elaborated. Section 3, contains the basic structured properties of the system. Section 4 examines the ACT along with controller laws and estimated parameter updating laws to stabilize globally and asymptotically the given hyperchaotic systems. In Section 5, numerical simulation results are illustrated to show the accuracy and feasibleness of the proposed HPS approach. Finally, Section 6 consists of concluding remarks and discussions.

3. Preliminaries

In the present section, we formally introduce few notations and terminology and mention some basic results to be used in the subsequent sections of the paper. Consider the master system and the corresponding slave system as:

$$\dot{x}_1 = f(x_1) \tag{1}$$

$$\dot{y}_1 = g(y_1) + u \tag{2}$$

where $x_1 = (x_{11}, x_{12}, \dots, x_{1n})^T$, $y_1 = (y_{11}, y_{12}, \dots, y_{1n})^T$ are the state vectors of (1) and (2) respectively, $f, g: R^n \to R^n$ are two nonlinear continuous vector functions and $u = (u_{11}, u_{12}, \dots, u_{1n}) \in R^n$ is the suitable controller to be constructed.

Definition 1: The systems (1) and (2) are said to be in hybrid projective synchronization (HPS) if

$$\lim_{t\to\infty} \| e \| = \lim_{t\to\infty} \| y_1 - l x_1 \| = 0 \tag{3}$$

for some $l = \mathrm{diag}(l_1, l_2, \dots, l_n)$ and $\|.\|$ denotes vector norm.

Remark 2.1: For $l_1 = l_2 = \dots = l_n = 1$, complete synchronization is obtained.

Remark 2.2: For $l_1 = l_2 = \dots = l_n = -1$, anti-synchronization is achieved.

Remark 2.3: If l_i's are not all zeros and $l_i \neq l_j$ for some i and j, then modified projective synchronization is obtained.

3. System Description

Proposed by Zhang *et al.* [41], the investigated hyperchaotic system is given as:

$$\begin{aligned}
\dot{x}_{11} &= -a_1 x_{11} + b_1 y_{11} z_{11} \\
\dot{y}_{11} &= -c_1 y_{11}^3 + d_1 x_{11} z_{11} + x_{11} w_{11} \\
\dot{z}_{11} &= d_1 z_{11} - x_{11} y_{11} \\
\dot{w}_{11} &= k_1 w_{11} + z_{11},
\end{aligned} \tag{4}$$

where $(x_{11}, y_{11}, z_{11}, w_{11})^T \in R^4$ is the state vector and a_1, b_1, c_1, d_1 and k_1 are positive parameters. When $a_1 = 2.6$, $b_1 = 10$, $c_1 = 7$, $d_1 = 3$ and $k_1 = 0.05$ the system (4) displays hyperchaotic behavior. Further, Fig. 1(a-f) exhibits the phase portraits of (4).

4. Illustrative Example

Here, we examine HPS scheme to describe the laws in order to estimate parameters along with adaptive controllers in such a way that each of the state variables x_{11}, y_{11}, z_{11} and w_{11} tend to equilibrium points for t approaching infinity.

Conveniently, the system (4) is taken as the master system and the corresponding slave system may be given as:

$$\begin{aligned}
\dot{x}_{21} &= -a_1 x_{21} + b_1 y_{21} z_{21} + u_{11} \\
\dot{y}_{21} &= -c_1 y_{21}^3 + d_1 x_{21} z_{21} + x_{21} w_{21} + u_{12} \\
\dot{z}_{21} &= d_1 z_{21} - x_{21} y_{21} + u_{13} \\
\dot{w}_{21} &= k_1 w_{21} + z_{21} + u_{14},
\end{aligned} \quad (5)$$

where u_{11},u_{12},u_{13} and u_{14} are adaptive control inputs to be determined so that the HPS scheme among two identical hyperchaotic systems will be attained.

We describe the state errors as

$$\begin{aligned}
e_{11} &= x_{21} - l_1 x_{11} \\
e_{12} &= y_{21} - l_2 y_{11} \\
e_{13} &= z_{21} - l_3 z_{11} \\
e_{14} &= w_{21} - l_4 w_{11}
\end{aligned} \quad (6)$$

The prime focus of this paper is to propose controllers u_{1i},
$(i = 1,2,3,4)$ so that the state errors described in (6) must satisfy
$\lim_{t\to\infty} e_{1i}(t) = 0$ for (i = 1,2,3,4).

The consequent error dynamics simplifies to

$$\begin{aligned}
\dot{e}_{11} &= -a_1 e_{11} + b_1 (y_{21} z_{21} - l_1 y_{11} z_{11}) + u_{11} \\
\dot{e}_{12} &= -c_1 (y_{21}^3 - l_2 y_{11}^3) + d_1 (x_{21} z_{21} - l_2 x_{11} z_{11}) + x_{21} w_{21} - l_2 x_{11} w_{11}) + u_{12} \\
\dot{e}_{13} &= d_1 e_{13} - x_{21} y_{21} + l_3 x_{11} y_{11} + u_{13} \\
\dot{e}_{14} &= k_1 e_{14} + z_{21} - l_4 z_{11} + u_{14}
\end{aligned} \quad (7)$$

Next, we define the adaptive control inputs as:

$$\begin{aligned}
u_{11} &= \hat{a}_1 e_{11} - \hat{b}_1 (y_{21} z_{21} - l_1 y_{11} z_{11}) - K_1 e_{11} \\
u_{12} &= \hat{c}_1 (y_{21}^3 - l_2 y_{11}^3) - \hat{d}_1 (x_{21} z_{21} - l_2 x_{12} z_{12}) - x_{21} w_{21} + l_2 x_{12} w_{12} - K_2 e_{12} \\
u_{13} &= -\hat{d}_1 e_{13} + x_{21} y_{21} - l_3 x_{11} y_{11} - K_3 e_{13} \\
u_{14} &= -\hat{k}_1 e_{14} - z_{21} + l_4 z_{11} - K_4 e_{14}
\end{aligned} \quad (8)$$

where $K_{1,}$, K_2, K_3 and K_4 are positive gain constants.

By putting the values of control inputs (8) in error dynamics (7), we obtain

$$\begin{aligned}
\dot{e}_{11} &= -(a_1 - \hat{a}_1) e_{11} + (b_1 - \hat{b}_1)(y_{21} z_{21} - l_1 y_{11} z_{11}) - K_1 e_{11} \\
\dot{e}_{12} &= -(c_1 - \hat{c}_1)(y_{21}^3 - l_2 y_{11}^3) + (d_1 - \hat{d}_1)(x_{21} z_{21} - l_2 x_{11} z_{11}) - K_2 e_{12} \\
\dot{e}_{13} &= (d_1 - \hat{d}_1) e_{13} - K_3 e_{13} \\
\dot{e}_{14} &= (k_1 - \hat{k}_1) e_{14} - K_4 e_{14}
\end{aligned} \quad (9)$$

where $\hat{a}_1$, $\hat{b}_1$, $\hat{c}_1$, $\hat{d}_1$, $\hat{h}_1$, $\hat{k}_1$ are estimated values of unknown parameter a_1, b_1, c_1, d_1, h_1, k_1 respectively.

Now, the parameter estimation error is defined as:

$$\tilde{a}_1 = a_1 - \hat{a}_1, \tilde{b}_1 = b_1 - \hat{b}_1, \tilde{c}_1 = c_1 - \hat{c}_1, \tilde{d}_1 = d_1 - \hat{d}_1, \tilde{k}_1 = k_1 - \hat{k}_1 \quad (10)$$

Applying (10), the error dynamics (9) becomes:

$$\begin{aligned}
\dot{e}_{11} &= -\tilde{a}_1 e_{11} + \tilde{b}_1 (y_{21} z_{21} - l_1 y_{11} z_{11}) - K_1 e_{11} \\
\dot{e}_{12} &= -\tilde{c}_1 (y_{21}^3 - l_2 y_{11}^3) + \tilde{d}_1 (x_{21} z_{21} - l_2 x_{11} z_{11}) - K_2 e_{12} \\
\dot{e}_{13} &= \tilde{d}_1 e_{13} - K_3 e_{13} \\
\dot{e}_{14} &= \tilde{k}_1 e_{14} - K_4 e_{14}
\end{aligned} \quad (11)$$

The derivative of parameter estimation error (10) is given by

$$\dot{\tilde{a}}_1 = -\dot{\hat{a}}_1, \dot{\tilde{b}}_1 = -\dot{\hat{b}}_1, \dot{\tilde{c}}_1 = -\dot{\hat{c}}_1, \dot{\tilde{d}}_1 = -\dot{\hat{d}}_1, \dot{\tilde{k}}_1 = -\dot{\hat{k}}_1 \quad (12)$$

Defining the Lyapunov function as

$$V = \frac{1}{2}\left[e_{11}^2 + e_{12}^2 + e_{13}^2 + e_{14}^2 + \tilde{a}_1{}^2 + \tilde{b}_1{}^2 + \tilde{c}_1{}^2 + \tilde{d}_1{}^2 + +\tilde{k}_1{}^2\right] \quad (13)$$

which shows that V is positive definite.

Derivative of V is obtained as:

$$\dot{V} = e_{11}\dot{e}_{11} + e_{12}\dot{e}_{12} + e_{13}\dot{e}_{13} + e_{14}\dot{e}_{14} - \tilde{a}_1 \dot{\hat{a}}_1$$

$$-\tilde{b}_1\dot{\hat{b}}_1 - \tilde{c}_1\dot{\hat{c}}_1 - \tilde{d}_1\dot{\hat{d}}_1 - \tilde{k}_1\dot{\hat{k}}_1 \quad (14)$$

Considering (14), we define the parameter estimation laws as :

$$\begin{aligned}
\dot{\hat{a}}_1 &= -e_{11}^2 + K_5(a_1 - \hat{a}_1) \\
\dot{\hat{b}}_1 &= (y_{21}z_{21} - l_1 y_{11}z_{11})e_{11} + K_6(b_1 - \hat{b}_1) \\
\dot{\hat{c}}_1 &= -(y_{21}^3 - l_2 y_{11}^3)e_{12} + K_7(c_1 - \hat{c}_1) \\
\dot{\hat{d}}_1 &= (x_{21}z_{21} - l_2 x_{11}z_{11})e_{12} + e_{13}^2 + K_8(d_1 - \hat{d}_1) \\
\dot{\hat{k}}_1 &= e_{14}^2 + K_9(k_1 - \hat{k}_1),
\end{aligned} \quad (15)$$

where K_5, K_6, K_7, K_8 and K_9 are positive gain constants.

Theorem 1: The hyperchaotic systems (4)-(5) are hybrid projective synchronized globally and asymptotically for every initial states $(x_{11}(0), y_{11}(0), z_{11}(0), w_{11}(0)) \in R^4$ by the adaptive control inputs (8) and the parameter update law (15).

Proof. The Lyapunov function V as described in (13) is a positive definite function. On simplifying equations (14) and (15), we have

$$\dot{V} = -K_1e_{11}^2 - K_2e_{12}^2 - K_3e_{13}^2 - K_4e_{14}^2 - K_5\tilde{a}_1^{\,2} - K_6\tilde{b}_1^{\,2} - K_7\tilde{c}_1^{\,2} - K_8\tilde{d}_1^{\,2} - K_9\tilde{k}_1^{\,2}$$
$$< 0$$

which implies that $\dot{V}$ is negative definite .

Hence, by LST, we deduce that the HPS error $e(t) \to 0$ exponentially as $t \to \infty$ for every initial conditions $e(0) \in R^4$. This finishes the proof.

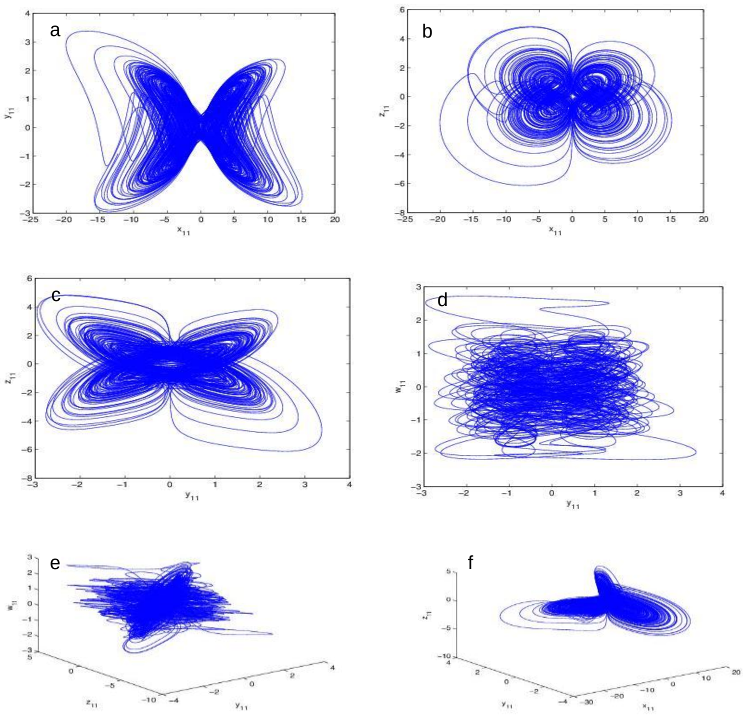

Fig. 1. Phase portraits of 4-D hyperchaotic system in (a) $x_{11} - y_{11}$ plane, (b)$x_{11} - z_{11}$ plane, (c) $y_{11} - z_{11}$ plane ,(d) $y_{11} - w_{11}$ plane, (e) $y_{11} - z_{11} - w_{11}$ space , (f) $x_{11} - y_{11} - z_{11}$ space.

5. Numerical Simulations

This section provides some simulation results to demonstrate the efficiency and feasibleness of the investigated HPS technique using ACT. Here, we simply use the fourth-order Runge-Kutta method to solve system of differential equations. For the given system, the parameters are taken as $a_1 = 2.6$, $b_1 = 10$, $c_1 = 7$, $d_1 = 3$ and $k_1 = 0.05$ to make sure that the system depicts chaotic behavior in the absence of control inputs. The initial states of the master and slave systems are $(x_{11}(0) = 0.4, y_{11}(0) = -0.5, z_{11}(0) = -0.1, w_{11}(0) = 0.7)$ and $(x_{21}(0) = 3, y_{21}(0) = 5, z_{21}(0) = 3, w_{21}(0) = 4)$ respectively. The control gains are chosen as $K_i = 10$ for $i = 1,2,\ldots,9$.

We achieve HPS scheme between master and slave systems by selecting the scaling matrix l with $l_1 = 2, l_2 = -2, l_3 = 3, l_4 = -3$. The numerical simulations are shown in Fig. 4(a-d) which depicts the trajectories of master and slave systems. The synchronization errors $(e_{11}, e_{12}, e_{13}, e_{14}) = (2.2, 4, 3.3, 6.1)$ tend to zero for t tending to infinity in Fig. 4(e). Also, Fig. 4(f) exhibits that estimated values $(\hat{a}_1, \hat{b}_1, \hat{c}_1, \hat{d}_1, \hat{k}_1)$ of uncertain parameters converging to their original values asymptotically and globally with time. Therefore, the considered HPS scheme between master and slave systems is justified computationally.

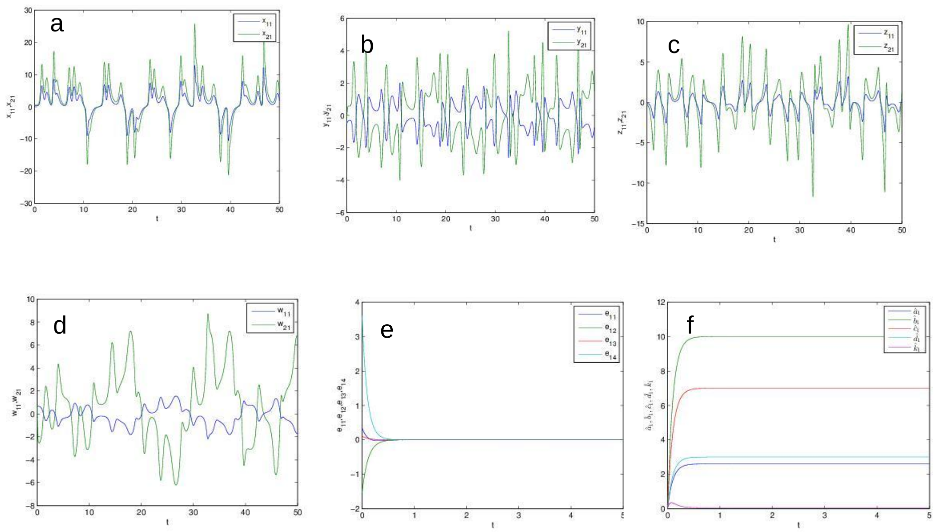

Fig. 2. Phase portraits of the trajectories of master and slave systems under HPS scheme (a) between $x_{11}(t) - x_{21}(t)$, (b) between $y_{11}(t) - y_{21}(t)$, (c) between $z_{11}(t) - z_{21}(t)$, (d) between $w_{11}(t) - w_{21}(t)$, (e) synchronization error, (f) Parameter estimation.

6. Concluding Remarks and Discussion

In this research work, we have explored the proposed HPS scheme among identical hyperchaotic systems using ACT. By designing proper control inputs according to LST, the proposed HPS scheme is achieved. It is observed that the anti-synchronization, complete synchronization, and hybrid synchronization are specific cases of HPS scheme. The effectivity and feasibleness of the analytical results are justified by performing simulations using MATLAB. Significantly, the theoretical work and the numerical results both are in excellent agreement. Also, the investigated HPS scheme is very effective as it has several applications in encryption and secure communication. In this work, the time taken by the synchronization error converges to zero is less in comparison with earlier related published work. We noticed that our proposed methodology is basic yet theoretically rigorous. Moreover, we understand that the considered HPS scheme can be generalized by using other control schemes.

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