Publications

Available Online

JOURNAL OF
SCIENTIFIC RESEARCH

J. Sci. Res. **12** (3), 251-257 (2020)

www.banglajol.info/index.php/JSR

A Magnetized Dark Energy Type R/W Model with Polytropic Equation of State in Brans-Dicke Theory

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Received 28 September 2019, accepted in final revised form 8 February 2020

Abstract

In this paper, we study the spatially homogeneous Robertson-Walker cosmological models with magnetized isotropic dark energy like fluid in the scalar-tensor theory of gravitation proposed by Brans-Dicke. Variable cosmological constant Λ and Polytropic equation of state have been used to find exact solutions of the models with volumetric expansion and power-law relation. The Physical and dynamical behaviors of the models have been discussed using some physical quantities like energy density, pressure, and coefficient of bulk viscosity.

Keywords: Dark energy; Brans–Dicke theory; Polytropic EOS; Bulk viscosity.

doi: http://dx.doi.org/10.3329/jsr.v12i3.43313 J. Sci. Res. **12** (3), 251-257 (2020)

1. Introduction

The accelerated expansion of the universe is one of the major discoveries of the twentieth century [1,2]. A mysterious force dark energy (DE) was suggested to be behind this accelerated expansion. The universe is composed of 73% DE, 23% dark matter, and 4% baryonic matter [3] as suggested by the WMAP experiment. Planck Collaboration [4] results of cosmological parameters show different types of dark energy like ΛCDM and SCDM. In Brans-Dicke (B-D) theory [5] context many works [6-13] on cosmological models are going on for a long time and the results show the supportive evidence with observational data. Different researchers [14-17] have also worked a lot on cosmological models with electromagnetic fields. Different Polytropic gas models are investigated by some of the relativists [18-24]. Researchers [25-29] are some of the many who studied cosmological models with variable cosmological constant Λ term. In 2014, Katore *et al.* [30] discussed Kaluza-Klein magnetized anisotropic fluid cosmological model within the Brans-Dicke scalar-tensor theory of gravitation. In 2018, Sharif *et al.* [31] studied about cosmic evolution of the Bianchi type I universe by using a new holographic dark energy model in the context of the Brans-Dicke theory for both non-interacting and interacting cases between dark energy and dark matter. In 2019, Bhoyar *et al.* [32] investigated about

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Kantowaski-Sachs cosmological model with bulk viscous and cosmic string in the framework of Teleparallel Gravity. The solutions obtained by using variable Λ term, models with a magnetic field and other models with a Polytropic equation of state studied by different researchers are the motivation behind this paper. The solutions are new and the cosmological parameters conform with experimental data obtained from different collaboration works of researchers. In this paper, a cosmological model with electromagnetic field and Brans-Dicke field has been discussed using the Polytropic equation of state.

2. Metric and Field Equations

The spherically symmetric Robertson-Walker metric is

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta\, d\varphi^2)\right] \tag{1}$$

where k is the curvature index which can take values -1,0, 1.

The B-D theory of gravity is described by the action (in units $h = c = 8\pi G = 1$)

$$S = \int \sqrt{|g|}\left[\frac{1}{16\pi}\left(\phi R - \frac{\omega}{\phi} g^{sl}\phi_{,l}\phi_{,s}\right) + L_m\right] d^4x \tag{2}$$

where R represents the curvature scalar; g is the determinant of g_{ij}; ϕ is a scalar field; ω is a dimensionless coupling constant; L_m is the Lagrangian of the ordinary matter component.

The Einstein field equations in the most general form are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -\frac{\kappa}{\phi}T_{ij} - \frac{\omega}{\phi^2}[\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi^{,s}\phi_{,s}] - \frac{1}{\phi}(\phi_{ij} - g_{ij}\phi_{;s}{}^{,s}) \tag{3}$$

where

$$(3 + 2\omega)\phi_{;s}{}^{,s} = 8\pi T, \tag{4}$$

where T is the trace of T_{ij}, Λ is the cosmological constant, R_{ij} is Ricci-tensor, g_{ij} is metric tensor and $\phi_{,i}$ is the partial differentiation with respect to x^icoordinate.

The energy-momentum tensor is

$$T_{ij} = (\bar{p} + \rho)u_i u_j - p g_{ij} + E_{ij} \tag{5}$$

where $\bar{p} = \mathrm{p} - \xi \mathrm{u}^{\mathrm{i}}{}_{;\mathrm{i}}$ (6)

and

$$E_{ij} = -F_{il}F_j{}^l + \frac{1}{4}g_{ij}F_{lm}F^{lm} \tag{7}$$

where $u^i = (0,0,0,1)$ is the matter 4-velocity vectorsatisfying $g^{ij}u_i u_j = 1$, $\bar{p}$ is the viscosity pressure, p is the pressure, ρ is the energy density, ξ is the coefficient of viscosity, $\mathrm{u}^{\mathrm{i}}{}_{;\mathrm{i}}$ is the scalar expansion and F_{il} is the electromagnetic field component.

The non-vanishing electromagnetic energy-momentum tensor $E_j{}^i$ are

$$E_1{}^1 = -E_2{}^2 = -E_3{}^3 = E_4{}^4 = -\frac{1}{2}g^{11}g^{44}F_{14}{}^2 = \frac{1}{2}\frac{1-kr^2}{R^2}F^2{}_{14} \tag{8}$$

Shear scalar is defined as

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i{}^2 - 3H^2\right) \tag{9}$$

The average anisotropy parameter Δ is defined as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{10}$$

where $H_i, i = 1,2,3$ represent the directional Hubble parameters in x, y, z directions respectively.
Gravitational variable [33] is defined as

$$G = \frac{1}{\phi}\left(\frac{4+2\omega}{3+2\omega}\right) \tag{11}$$

The deceleration parameter is defined as

$$q = -\frac{R\ddot{R}}{\dot{R}^2} \tag{12}$$

3. Solutions of Field Equations

Assuming Brans-Dicke scalar field ϕ to be a function of time t only, the metric (1) along with field equations (3)-(5) gives

$$\frac{k}{R^2} + \frac{\dot{R}^2}{R^2} + 2\frac{\ddot{R}}{R} - \Lambda = -\frac{8\pi\,\bar{p}}{\phi} - \frac{4\pi}{\phi}\frac{1-kr^2}{R^2}F^2{}_{14} - \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - 2\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} - \frac{\ddot{\phi}}{\phi} \tag{13}$$

$$3\left(\frac{k}{R^2} + \frac{\dot{R}^2}{R^2}\right) - \Lambda = \frac{8\pi\rho}{\phi} + \frac{4\pi}{\phi}\frac{1-kr^2}{R^2}F^2{}_{14} + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} \tag{14}$$

$$(3+2\omega)[3\frac{\dot{R}\dot{\phi}}{R} + \ddot{\phi}] = 8\pi(\rho - 3\bar{p}), \tag{15}$$

From eq. (13), (14) and (15), we get

$$6(\frac{k}{R^2} + \frac{\dot{R}^2}{R^2} + \frac{\ddot{R}}{R}) - 4\Lambda = -\frac{8\pi}{\phi}\frac{1-kr^2}{R^2}F^2{}_{14} + \omega\,[6\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} + 2\frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}^2}{\phi^2}] \tag{16}$$

Here, we consider

where $\phi = \phi_0 R^{\frac{1}{\omega}}$ (17)

ϕ_0 is a constant.
Using eq. (17), (16) becomes

$$F^2{}_{14} = \frac{\phi_0}{8\pi}\frac{R^{\frac{1+2\omega}{\omega}}}{1-kr^2}\left[\left(\frac{1-2\omega}{\omega}\right)\frac{\dot{R}^2}{R^2} - 4\frac{\ddot{R}}{R} - \frac{6k}{R^2} + 4\Lambda\right] \tag{18}$$

We assume the scale factor as

$$R(t) = c_2(2\sqrt{t} - c_1)^2 e^{\sqrt{t}} \tag{19}$$

From eq. (19) and (12), we get

$$q = -1 + \frac{1}{\sqrt{t}}\left[1 + \frac{4(c_1-4)}{(2\sqrt{t}+4-c_1)^2}\right] \tag{20}$$

Brans-Dicke scalar field is

$$\phi = \phi_0[c_2(2\sqrt{t} - c_1)^2 e^{\sqrt{t}}]^{\frac{1}{\omega}} \tag{21}$$

The Gravitational variable is

$$G = \left(\frac{4+2\omega}{3+2\omega}\right)\phi_0{}^{-1}[c_2(2\sqrt{t} - c_1)^2 e^{\sqrt{t}}]^{-\frac{1}{\omega}} \tag{22}$$

Spatial volume, Hubble's parameter and Scalar expansion are given by

$$V = [c_2(2\sqrt{t} - c_1)^2 e^{\sqrt{t}}]^3 \tag{23}$$

$$H = \frac{2\sqrt{t}+4-c_1}{2\sqrt{t}(2\sqrt{t}-c_1)} \tag{24}$$

$$\Theta = 3.\frac{2\sqrt{t}+4-c_1}{2\sqrt{t}(2\sqrt{t}-c_1)} \tag{25}$$

The directional Hubble's parameter on the x, y, z axes is

$$H_x = H_y = H_z = \frac{2\sqrt{t}+4-c_1}{2\sqrt{t}(2\sqrt{t}-c_1)} \tag{26}$$

3.1. *Case I: Flat model* $k = 0, \Lambda = a\left(\frac{\dot{R}}{R}\right)^2 + b\frac{\ddot{R}}{R}$

Using eq. (19), eq. (18) becomes

$$F^2{}_{14} = \frac{\phi_0[c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}]^{\frac{1+2\omega}{\omega}}}{8\pi}\left[B_1\left(\frac{2\sqrt{t}+4-c_1}{2\sqrt{t}(2\sqrt{t}-c_1)}\right)^2 + B_2\left\{\frac{c_1-4}{4t^{\frac{3}{2}}(2\sqrt{t}-c_1)} - \frac{2\sqrt{t}+4-c_1}{2t(2\sqrt{t}-c_1)^2}\right\}\right] \quad (27)$$

where $B_1 = \left\{\frac{1}{\omega} + 4a + 4b - 6\right\}, \quad B_2 = 4(b-1)$

Using (17)-(19), eq. (13) and (14) gives

$$\bar{p} = -\frac{\phi_0[c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}]^{\frac{1}{\omega}}}{8\pi}\left[B_3\left(\frac{2\sqrt{t}+4-c_1}{2\sqrt{t}(2\sqrt{t}-c_1)}\right)^2 + B_4\left\{\frac{c_1-4}{4t^{\frac{3}{2}}(2\sqrt{t}-c_1)} - \frac{2\sqrt{t}+4-c_1}{2t(2\sqrt{t}-c_1)^2}\right\}\right] \quad (28)$$

$$\rho = \frac{\phi_0[c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}]^{\frac{1}{\omega}}}{8\pi}\left[B_5\left(\frac{2\sqrt{t}+4-c_1}{2\sqrt{t}(2\sqrt{t}-c_1)}\right)^2 + B_6\left\{\frac{c_1-4}{4t^{\frac{3}{2}}(2\sqrt{t}-c_1)} - \frac{2\sqrt{t}+4-c_1}{2t(2\sqrt{t}-c_1)^2}\right\}\right] \quad (29)$$

Where $B_3 = \left(\frac{a\omega^2+3\omega+b\omega^2+1}{\omega^2}\right), B_4 = \left(\frac{b\omega+1}{\omega}\right), B_5 = \left\{\frac{(8-6a-3b)\omega+4}{2\omega}\right\}, B_6 = \left(\frac{4-3b}{2}\right)$

Now, restricting the distribution by considering Polytropic equation of state as $p = \alpha\rho^n$ where α and n are polytropic constant and index respectively.

Using eq. (6), we obtain the explicit form of physical quantities p and ξ as

$$p = \alpha\left\{\frac{\phi_0[c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}]^{\frac{1}{\omega}}}{8\pi}\right\}^n\left[B_5\left(\frac{2\sqrt{t}+4-c_1}{2\sqrt{t}(2\sqrt{t}-c_1)}\right)^2 + B_6\left\{\frac{c_1-4}{4t^{\frac{3}{2}}(2\sqrt{t}-c_1)} - \frac{2\sqrt{t}+4-c_1}{2t(2\sqrt{t}-c_1)^2}\right\}\right]^n \quad (30)$$

$$\xi = \alpha\left\{\frac{\phi_0\left[c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}\right]^{\frac{1}{\omega}}}{8\pi}\right\}^n\left[B_5\left\{\frac{2\sqrt{t}+4-c_1}{2\sqrt{t(2\sqrt{t}-c_1)}}\right\}^2 + B_6\left\{\frac{c_1-4}{4t^{\frac{3}{2}}(2\sqrt{t}-c_1)} - \frac{2\sqrt{t}+4-c_1}{2t(2\sqrt{t}-c_1)^2}\right\}\right]^n \frac{2\sqrt{t}(2\sqrt{t}-c_1)}{3(2\sqrt{t}+4-c_1)} +$$
$$\frac{\phi_0[c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}]^{\frac{1}{\omega}}}{8\pi}\left[B_3\left\{\frac{2\sqrt{t}+4-c_1}{6\sqrt{t}(2\sqrt{t}-c_1)}\right\} + B_4\left\{\frac{c_1-4}{6t(2\sqrt{t}-c_1)(2\sqrt{t}+4-c_1)} - \frac{1}{3\sqrt{t}(2\sqrt{t}-c_1)}\right\}\right] \quad (31)$$

3.2. *Case II: Open model* $k = -1$ *and* $\Lambda = a\left(\frac{\dot{R}}{R}\right)^2 + b\frac{\ddot{R}}{R}$

Using eq. (19), eq. (18) becomes

$$F^2{}_{14} = \frac{\phi_0[c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}]^{\frac{1+2\omega}{\omega}}}{8\pi(1+r^2)}\left[B_1\left(\frac{2\sqrt{t}+4-c_1}{2\sqrt{t}(2\sqrt{t}-c_1)}\right)^2 + B_2\left\{\frac{c_1-4}{4t^{\frac{3}{2}}(2\sqrt{t}-c_1)} - \frac{2\sqrt{t}+4-c_1}{2t(2\sqrt{t}-c_1)^2}\right\} + \frac{6}{\left\{c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}\right\}^2}\right] \quad (32)$$

Using (17)-(19), eq. (13) and (14) gives

$$\bar{p} = -\frac{\phi_0[c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}]^{\frac{1}{\omega}}}{8\pi}\left[B_3\left(\frac{2\sqrt{t}+4-c_1}{2\sqrt{t}(2\sqrt{t}-c_1)}\right)^2 + B_4\left\{\frac{c_1-4}{4t^{\frac{3}{2}}(2\sqrt{t}-c_1)} - \frac{2\sqrt{t}+4-c_1}{2t(2\sqrt{t}-c_1)^2}\right\} + \frac{2}{\{c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}\}^2}\right] \tag{33}$$

$$\rho = \frac{\phi_0[c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}]^{\frac{1}{\omega}}}{8\pi}\left[B_5\left(\frac{2\sqrt{t}+4-c_1}{2\sqrt{t}(2\sqrt{t}-c_1)}\right)^2 + B_6\left\{\frac{c_1-4}{4t^{\frac{3}{2}}(2\sqrt{t}-c_1)} - \frac{2\sqrt{t}+4-c_1}{2t(2\sqrt{t}-c_1)^2}\right\} - \frac{6}{\{c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}\}^2}\right] \tag{34}$$

Again, restricting the distribution by considering Polytropic equation of state as $p = \alpha\rho^n$ and using eq. (6), we obtain the explicit form of physical quantities p and ξ as

$$p = \alpha\left\{\frac{\phi_0[c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}]^{\frac{1}{\omega}}}{8\pi}\right\}^n\left[B_5\left(\frac{2\sqrt{t}+4-c_1}{2\sqrt{t}(2\sqrt{t}-c_1)}\right)^2 + B_6\left\{\frac{c_1-4}{4t^{\frac{3}{2}}(2\sqrt{t}-c_1)} - \frac{2\sqrt{t}+4-c_1}{2t(2\sqrt{t}-c_1)^2}\right\} - \frac{6}{\{c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}\}^2}\right]^n \tag{35}$$

$$\xi = \alpha\left\{\frac{\phi_0[c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}]^{\frac{1}{\omega}}}{8\pi}\right\}^n\left[B_5\left(\frac{2\sqrt{t}+4-c_1}{2\sqrt{t}(2\sqrt{t}-c_1)}\right)^2 + B_6\left\{\frac{c_1-4}{4t^{\frac{3}{2}}(2\sqrt{t}-c_1)} - \frac{2\sqrt{t}+4-c_1}{2t(2\sqrt{t}-c_1)^2}\right\} - \frac{6}{\{c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}\}^2}\right]^n \frac{2\sqrt{t}(2\sqrt{t}-c_1)}{3(2\sqrt{t}+4-c_1)} + \frac{\phi_0[c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}]^{\frac{1}{\omega}}}{8\pi}\left[B_3\left\{\frac{2\sqrt{t}+4-c_1}{6\sqrt{t}(2\sqrt{t}-c_1)}\right\} + B_4\left\{\frac{c_1-4}{6t(2\sqrt{t}-c_1)(2\sqrt{t}+4-c_1)} - \frac{1}{3\sqrt{t}(2\sqrt{t}-c_1)}\right\} + \frac{4\sqrt{t}}{3{c_2}^2(2\sqrt{t}-c_1)^3 e^{2\sqrt{t}}(2\sqrt{t}+4-c_1)}\right] \tag{36}$$

Cosmological constant takes the form

$$\Lambda = (a+b)\left\{\frac{2\sqrt{t}+4-c_1}{2\sqrt{t}(2\sqrt{t}-c_1)}\right\}^2 + b\left\{\frac{c_1-4}{4t^{\frac{3}{2}}(2\sqrt{t}-c_1)} - \frac{2\sqrt{t}+4-c_1}{2t(2\sqrt{t}-c_1)^2}\right\} \tag{37}$$

where $0.5 \le a \le 1$, $0.5 \le b \le 1$ and $1.5 \le a+b \le 2$.

4. Conclusion

In this paper, we have assumed a scale factor $R(t) = c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}$. So, spatial volume becomes an exponential function of time and tends to infinity as $t \to \infty$, so the model universes are expanding. The deceleration parameter is time-dependent and as time tends to infinity $-1 \le q < 0$. For all the cases accelerated expansion can be found for large values of ω. For both the models, F_{14} decreases as time increases. Here the fluid density is positive and decreases as time increases. We observe that the scalar field $\phi \to \infty$ as $t \to \infty$. The gravitational variable G decreases as time passes and for $t \to \infty$, $G \to 0$. This helps in expanding the model universe. For $\alpha = 0$, we have $p = 0$ (for all n in the equation of state), so we can say that for dust-filled Universe, there is no distinction between Barotropic and Polytropic equations of state. For non-dust cases we get dark energy models as phantom energy ($\alpha < -1$) or quintessence ($-1 < \alpha < 0$) or vacuum fluid ($\alpha = -1$). So, using the Polytropic equation of state it has been possible to show that non-dust cases admit the presence of a driving force behind inflation in the form of either quintessence or vacuum fluid or phantom energy and in the dust cases there is no distinction between different equations of states. The Λ term decreases with time from a large value at an initial stage to a small positive value at the late time of evolution. Also, model universes are found with a fluid having bulk viscosity

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