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A Bayesian Approach for Estimating Parameter of Rayleigh Distribution

J. Mahanta*, M. B. A. Talukdar

Department of Statistics, University of Chittagong, Chittagong-4331, Bangladesh

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Abstract

This paper is concerned with estimating the parameter of Rayleigh distribution (special case of two parameters Weibull distribution) by adopting Bayesian approach under squared error (SE), LINEX, MLINEX loss function. The performances of the obtained estimators for different types of loss functions are then compared. Better result is found in Bayesian approach under MLINEX loss function. Bayes risk of the estimators are also computed and presented in graphs.

Keywords: Linear exponential (LINEX) loss function; Modified linear exponential (MLINEX) loss function; Squared error loss function (SE).

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1. Introduction

Rayleigh distribution is named after the British Nobel prize winner physicist Lord Rayleigh (1842-1919). This distribution has got remarkable attention in the field of reliability theory and survival analysis, probability theory and operations research. Apart from this, in communication theory to model multiple paths of dense scattered signals reaching a receiver and in the physical sciences to model wind speed, wave heights [8], sound/light radiation, radio signals and wind power, the application of this distribution is noticeable. In addition to, in engineering, to measure the lifetime of an object, where the lifetime depends on the object's age. For example, resistors, transformers, and capacitors in aircraft radar sets.

Rayleigh distribution is also used in mixture models. Most of the researchers worked on the classical and Bayesian analysis of two and three components mixture models. Saleem and Aslam [9] discussed the use of the informative and the non-informative priors for Bayesian analysis of the two component mixture of Rayleigh distributions. Aslam *et al.* [2] studied Bayesian analysis of a three component mixture of Rayleigh distributions with unknown mixing proportions. Boudjerd *et al*. [4] applied Bayesian estimation of the Rayleigh distribution under different loss function. Sindhu *et al*. [10] employed Bayesian

* *Corresponding author:* johnstat@cu.ac.bd

inference of mixture of two Rayleigh distributions. After analyzing its significance, this paper has been intended to get the best estimate of the parameter of this distribution considering varied loss functions using simulated and real data.

A continuous random variable *X* is said to have a Rayleigh [11] distribution if its probability density function is given by

$$f(x,\lambda) = \frac{2x}{\lambda^2} e^{-\frac{x^2}{\lambda^2}} \quad ; \; x \geq 0, \lambda > 0 \qquad (1)$$

$$= 0 \quad ; \text{otherwise.}$$

Where, λ^2 is the only one scale parameter of this distribution.

Replacing $\frac{1}{\lambda^2} = \theta$, we get

$$f(x,\lambda) = 2x\theta e^{-\theta x^2} ; \; x \geq 0, \theta > 0 \qquad (2)$$

$$= 0 \quad ; \text{otherwise}$$

Now Monte Carlo techniques for generating sample from Rayleigh distribution is applied with the help of inverse transform method.

2. Sample Generation from Rayleigh Distribution

Let X be a Rayleigh variate having the p.d.f

$$f(x;\lambda) = \frac{2x}{\lambda^2} e^{-\frac{x^2}{\lambda^2}} \quad ; \text{x} \geq 0, \lambda > 0$$

$$= 0 \quad ; \text{otherwise}$$

The cumulative distribution function of this distribution is

$$F_X(x) = \int_0^x f(x)dx \qquad (3)$$

$$\Rightarrow F_X(x) = \int_0^x \frac{2x}{\lambda^2} e^{-\frac{x^2}{\lambda^2}} dx$$

Let $\frac{x^2}{\lambda^2} = p$

Now from equation (3) we have

$$\text{F}_X(x) = 1 - e^{-\frac{x^2}{\lambda^2}}.$$

Now by inverse transformation method we have

$$U = 1 - e^{-\frac{x^2}{\lambda^2}}$$

$$\Rightarrow x = \left\{-\lambda^2 \ln(1-U)\right\}^{\frac{1}{2}}$$

Where, *U* is uniformly distributed over the range [0,1]. Using this relationship a sample of size *n* can easily be generated from Rayleigh distribution.

3. Different Estimators of Parameter and Their Bayes Risk

Bayesian approach under different loss function has been applied for estimating the parameters of the Rayleigh distribution.

3.1. *Prior and posterior density function*

We get from (1) the p.d.f of Rayleigh distribution is

$$f(x,\lambda) = \frac{2x}{\lambda^2}\, e^{-\frac{x^2}{\lambda^2}} \qquad ;\ x \ge 0, \lambda > 0$$

$$= 0 \qquad ;\ \text{otherwise}$$

Let the prior density function of θ is a gamma distribution defined as

$$g(\theta) = \frac{\beta^{\alpha}}{\Gamma\alpha}\, e^{-\beta\theta}\, \theta^{\alpha-1}\ ; \quad \alpha, \beta, \theta > 0 \tag{4}$$

Where, α and β is the shape and scale parameter respectively.
Now the posterior density function of θ for the given sample is

$$f(\theta/x) = \frac{\prod_{i=1}^{n} f(x_i,\theta) g(\theta)}{\int \prod_{i=1}^{n} f(x_i,\theta) g(\theta) d\theta}$$

$$\Rightarrow f(\theta/x) = \frac{\left(\sum_{i=1}^{n} x_i^2 + \beta\right)^{n+\alpha}}{\Gamma(n+\alpha)}\, e^{-\theta\left(\sum_{i=1}^{n} x_i^2 + \beta\right)}\, \theta^{n+\alpha-1}.$$

Therefore,

$$f(\theta/x) \sim G\left(n+\alpha, \sum_{i=1}^{n} x_i^2 + \beta\right) \tag{5}$$

3.2. *Bayes estimator under squared error (SE) loss function*

Now let the loss function be squared [5] error defined as

$$L\left(\hat{\theta};\theta\right) = \left(\hat{\theta} - \theta\right)^2 \tag{6}$$

Hence under SE loss function Bayes estimator of θ is

$$\hat{\theta}_{BSE} = \frac{n+\alpha}{\sum_{i=1}^{n} x_i^2 + \beta} \tag{7}$$

Therefore , $\hat{\lambda}^2_{BSE} = \frac{1}{\hat{\theta}_{BSE}} = \frac{\sum_{i=1}^{n} x_i^2 + \beta}{n+\alpha}$, is the Bayes estimator of λ^2 under SE loss. Squared error loss function is applicable when the loss is symmetric in nature.

3.3. *Bayes estimator under LINEX loss function*

Let us consider the following LINEX [3] loss function which is applied in real estate assessment

$$L(D) = k\left[e^{c(\hat{\theta}-\theta)} - c\left(\hat{\theta}-\theta\right) - 1\right] \; ; k > 0, c \neq 0 \,. \tag{8}$$

Where, $D = \left(\hat{\theta}-\theta\right)$ represents the estimation error and c is the shape parameter of the loss function.

For LINEX loss function, the Bayes estimator of θ is given by

$$\hat{\theta}_{BL} = -\frac{1}{c}\ln E_\theta\left(e^{-c\theta}\right)$$

Now,

$$E_\theta\left(e^{-c\theta}\right) = \int e^{-c\theta} f(\theta/x)\, d\theta$$

$$\Rightarrow E_\theta\left(e^{-c\theta}\right) = \left(1 + \frac{c}{\sum_{i=1}^{n} x_i^2 + \beta}\right)^{-(n+\alpha)}$$

Therefore,

$$\hat{\theta}_{BL} = -\frac{1}{c}\ln E_\theta\left(e^{-c\theta}\right) = \frac{n+\alpha}{c}\left(\frac{c}{\sum_{i=1}^{n} x_i^2 + \beta} - \frac{c^2}{2\left(\sum_{i=1}^{n} x_i^2 + \beta\right)^2} + \frac{c^3}{3\left(\sum_{i=1}^{n} x_i^2 + \beta\right)^3} - \ldots\ldots\right).$$

So, we neglecting the 3nd and higher order term, then we get

$$\hat{\theta}_{BL} = \frac{n+\alpha}{\sum_{i=1}^{n} x_i^2 + \beta} - \frac{c(n+\alpha)}{2\left(\sum_{i=1}^{n} x_i^2 + \beta\right)^2}$$

Hence, $$\hat{\lambda}_{BL}^2 = \frac{1}{\hat{\theta}_{BL}} = \frac{\sum_{i=1}^{n} x_i^2 + \beta}{n+\alpha} - \frac{2\left(\sum_{i=1}^{n} x_i^2 + \beta\right)^2}{c(n+\alpha)}, \tag{9}$$

is the Bayes estimator under LINEX loss function.

3.4. *Bayes estimator under MLINEX loss function*

When the loss is asymmetric, then the MLINEX loss function is applied. MLINEX loss function is the modification of LINEX loss function. Now, let us consider the MLINEX loss [6] function defined as

$$L\left(\hat{\theta};\theta\right)=\omega\left[\left(\frac{\hat{\theta}}{\theta}\right)^{c}-c\log\left(\frac{\hat{\theta}}{\theta}\right)-1\right] \quad ;\omega>0, c\neq 0 \tag{10}$$

For MLINEX loss function Bayes estimator of θ is obtained from

$$\hat{\theta}_{BML}=\left[E\left(\theta^{-c}/x\right)\right]^{-\frac{1}{c}}.$$

Here

$$E\left(\theta^{-c}\right)=\int\theta^{-c}f\left(\theta/x\right)d\theta.$$

Hence, $\hat{\lambda}^2_{BML}=\left[\frac{\Gamma(n+\alpha-c)}{\Gamma(n+\alpha)}\right]^{\frac{1}{c}}\left(\sum_{i=1}^{n}x_i^{\,2}+\beta\right)$, since $\frac{1}{\lambda^2}=\theta$ (11)

which is the Bayes estimator under MLINEX loss function.

3.5. *Bayes risk of the estimator*

Let $X=\left(X_1, X_2, \ldots\ldots\ldots\ \ldots, X_n\right)$ be a random sample of size n drawn from a density function $f(x/\theta)$. Let $L\left(\hat{\theta},\theta\right)$ be the loss function in estimating θ by $\hat{\theta}$ with density function $g(\theta)$. The risk of the estimator $\hat{\theta}$ is defined by $R\left(\hat{\theta},\theta\right)=E_\theta\left[L\left(\hat{\theta},\theta\right)\right]$ and the Bayes risk [7] of estimator $\hat{\theta}$ with respect to the loss function $L\left(\hat{\theta},\theta\right)$ and prior density $g(\theta)$, denoted by $R_g\left(\hat{\theta},\theta\right)$ is defined as

$$R_g\left(\hat{\theta},\theta\right)=E_g\left[R\left(\hat{\theta},\theta\right)\right]=\begin{cases}\int R\left(\hat{\theta},\theta\right)g(\theta)d\theta & ;\text{if } \theta \text{ is continuous}\\ \sum_\theta R\left(\hat{\theta},\theta\right)g(\theta) & ;\text{if } \theta \text{ is discrete}\end{cases}$$

4. Empirical Study

The estimated values of the parameter and Bayes risk of the estimators are computed by Monte-Carlo simulation method from Rayleigh distribution. A short algorithm of the above simulation is given below:

i) Generate a random sample of size n using the following formula $x = \left\{-\lambda^2 \ln(1-U)\right\}^{\frac{1}{2}}$, where U is uniformly distributed over the range [0,1] and specific value of λ.

ii) Obtain Bayes estimator under different loss function.

iii) Repeat the above steps 1000 times and denote the Bayes estimates of λ^2 as $\hat{\lambda}_j^2$, $j = 1,2,...,1000$.

iv) Simulated Bayes estimate of $\lambda^2 = \frac{1}{1000}\sum_{j=1}^{1000} \hat{\lambda}_j^2$.

v) Calculate the simulated Bayes risk (SBR) of $\hat{\lambda}^2$ if $R\left(\hat{\lambda}^2\right)$.

The numerical results and their graphs are as follows.

Table 1. Estimated value and Bayes risk of different estimators of the parameter λ^2 of Rayleigh distribution when $\alpha = 2, \beta = 1, \lambda^2 = 1 \, and \; c = 1$.

n	Criteria	$\hat{\lambda}^2_{BSE}$	$\hat{\lambda}^2_{BL}$	$\hat{\lambda}^2_{BML}$
5	Estimate	0.612	0.693	0.714
	Bayes risk	0.250	0.194	0.182
10	Estimate	0.907	0.950	0.989
	Bayes risk	0.109	0.102	0.100
15	Estimate	1.134	1.165	1.205
	Bayes risk	0.118	0.127	0.142
20	Estimate	0.709	0.732	0.742
	Bayes risk	0.185	0.172	0.166
25	Estimate	0.890	0.909	0.924
	Bayes risk	0.112	0.108	0.106
30	Estimate	0.770	0.786	0.794
	Bayes risk	0.153	0.146	0.142

Fig. 1 depicts that, for $\alpha = 2, \beta = 1, \lambda^2 = 1, c = 1$ and different values of n, the Bayes estimator under MLINEX loss function provides better estimate than the other loss functions except n=15.

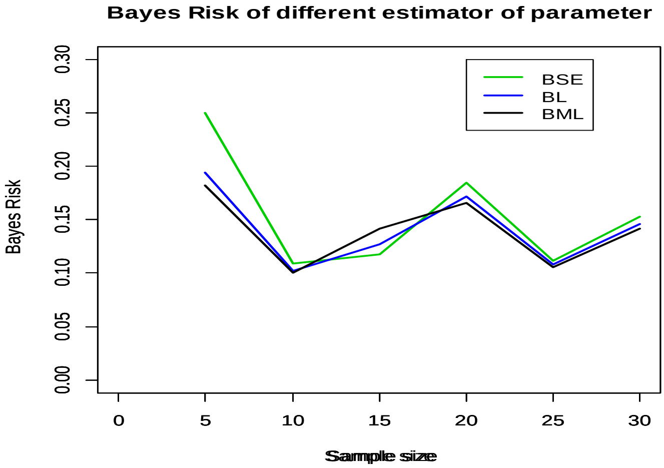

Fig. 1. Graph of Bayes risk of different estimators of the parameter λ^2 of Rayleigh distribution when $\alpha = 2, \beta = 1, \lambda^2 = 1 \, and \; c = 1$.

Table 2. Estimated value and Bayes risk of different estimators of the parameter λ^2 of the Rayleigh distribution when $\alpha = 3, \beta = 2, \lambda^2 = 1 and c = 1$.

n	Criteria	$\hat{\lambda}^2_{BSE}$	$\hat{\lambda}^2_{BL}$	$\hat{\lambda}^2_{BML}$
5	Estimate	0.661	0.730	0.755
	Bayes risk	0.265	0.223	0.210
10	Estimate	0.914	0.954	0.990
	Bayes risk	0.157	0.152	0.150
15	Estimate	1.127	1.156	1.193
	Bayes risk	0.166	0.174	0.187
20	Estimate	0.721	0.744	0.754
	Bayes risk	0.228	0.216	0.210
25	Estimate	0.894	0.912	0.927
	Bayes risk	0.161	0.158	0.155
30	Estimate	0.770	0.786	0.794
	Bayes risk	0.203	0.196	0.192

For $\alpha = 3, \beta = 2, \lambda^2 = 1, c = 1$, Bayes estimator under MLINEX loss function provide better estimate than other loss functions except n=15.

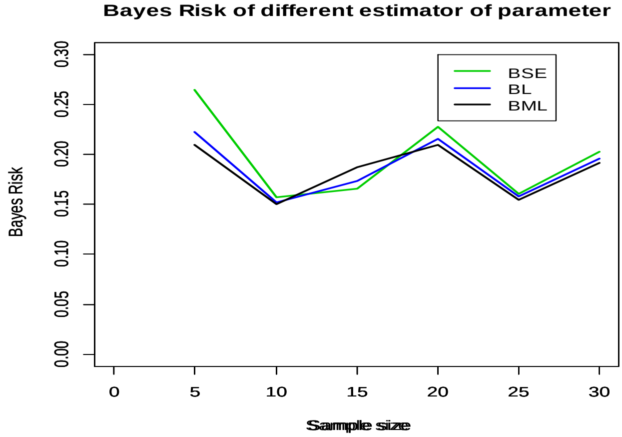

Fig. 2. Graph of Bayes risk of different estimators of the parameter λ^2 of the Rayleigh distribution when $\alpha = 3, \beta = 2, \lambda^2 = 1 and c = 1$.

Table 3. Estimated value and Bayes risk of different estimators of the parameter λ^2 of the Rayleigh distribution when $\alpha = 2, \beta = 1, \lambda^2 = 1 and c = 2$.

n	Criteria	$\hat{\lambda}^2_{BSE}$	$\hat{\lambda}^2_{BL}$	$\hat{\lambda}^2_{BML}$
5	Estimate	0.612	0.799	0.783
	Bayes risk	0.300	0.191	0.197
10	Estimate	0.907	0.999	1.038
	Bayes risk	0.159	0.150	0.151
15	Estimate	1.134	1.197	1.245
	Bayes risk	0.168	0.189	0.210
20	Estimate	0.709	0.757	0.761
	Bayes risk	0.235	0.209	0.207
25	Estimate	0.89	0.928	0.942
	Bayes risk	0.162	0.155	0.153
30	Estimate	0.77	0.802	0.808
	Bayes risk	0.203	0.189	0.187

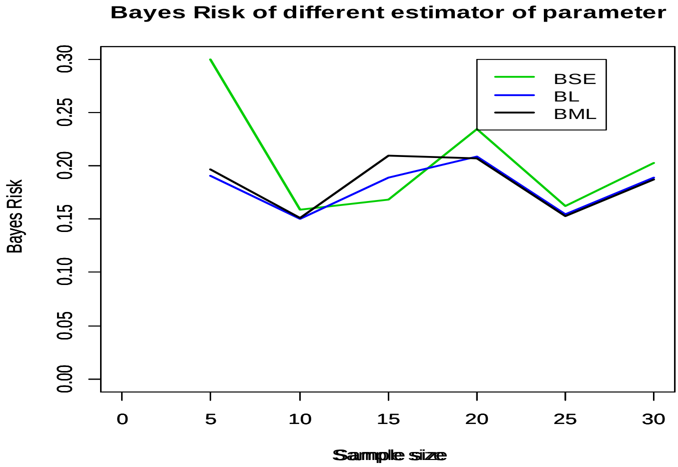

Fig. 3. Graph of Bayes risk of different estimators of the parameter λ^2 of the Rayleigh distribution when $\alpha = 2, \beta = 1, \lambda^2 = 1 and c = 2$.

Fig. 3 shows that the Bayes risk of the Bayesian approach under MLINEX loss function is lower than others loss function except for sample size n = 15.

Table 4. Estimated value and Bayes risk of different estimators of the parameter λ^2 of the Rayleigh distribution when $\alpha = 2, \beta = 4, \lambda^2 = 1.5 and c = 2$.

n	Criteria	$\hat{\lambda}^2{}_{BSE}$	$\hat{\lambda}^2{}_{BL}$	$\hat{\lambda}^2{}_{BML}$
5	Estimate	0.884	1.055	1.130
	Bayes risk	0.529	0.348	0.287
10	Estimate	0.882	0.974	1.009
	Bayes risk	0.532	0.426	0.391
15	Estimate	0.952	1.015	1.045
	Bayes risk	0.450	0.385	0.357
20	Estimate	0.624	0.673	0.67
	Bayes risk	0.918	0.834	0.839
25	Estimate	0.717	0.756	0.759
	Bayes risk	0.764	0.704	0.699
30	Estimate	0.617	0.65	0.648
	Bayes risk	0.929	0.872	0.876

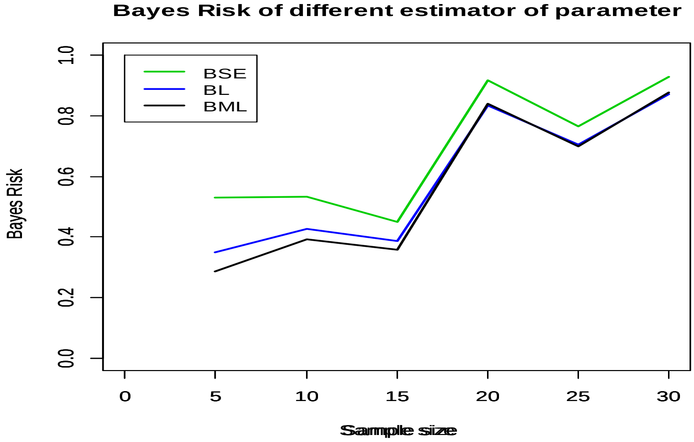

Fig. 4. Graph of Bayes risk of different estimators of the parameter λ^2 of the Rayleigh distribution when $\alpha = 2, \beta = 4, \lambda^2 = 1.5 and c = 2$.

Fig. 4 represent that, Bayes estimator under MLINEX loss function have smaller risk than other loss functions for all cases of sample size.

Table 5. Estimated value and Bayes risk of different estimates of the parameter λ^2 of the Rayleigh distribution when $\alpha = 1, \beta = 3, \lambda^2 = 1.5 and c = 1$.

n	Criteria	$\hat{\lambda}^2_{BSE}$	$\hat{\lambda}^2_{BL}$	$\hat{\lambda}^2_{BML}$
5	Estimate	0.865	0.957	1.038
	Bayes risk	0.553	0.444	0.363
10	Estimate	0.872	0.920	0.959
	Bayes risk	0.545	0.487	0.443
15	Estimate	0.949	0.982	1.013
	Bayes risk	0.453	0.419	0.387
20	Estimate	0.606	0.631	0.636
	Bayes risk	0.949	0.906	0.896
25	Estimate	0.706	0.725	0.734
	Bayes risk	0.781	0.75	0.737
30	Estimate	0.605	0.621	0.625
	Bayes risk	0.951	0.922	0.916

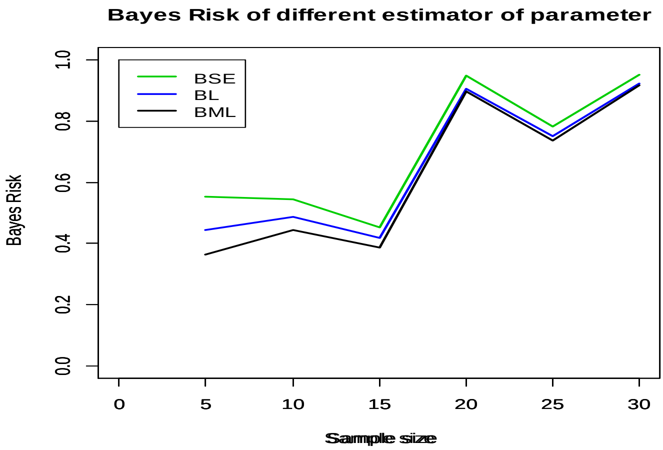

Fig. 5. Graph of Bayes risk of different estimators of the parameter λ^2 of the Rayleigh distribution when $\alpha = 1, \beta = 3, \lambda^2 = 1.5 \, and \, c = 1$.

It is seen from the Fig. 5, for $\alpha = 1, \beta = 3, \lambda^2 = 1.5, c = 1$ and different values of n, the Bayes estimator under MLINEX loss function provides better estimate than the other loss functions.

Table 6. Estimated value and Bayes risk of different estimators of the parameter λ^2 of the Rayleigh distribution when $n = 20, \beta = 2, \lambda^2 = 1 \, and \, c = 1$.

α	Criteria	$\hat{\lambda}^2_{BSE}$	$\hat{\lambda}^2_{BL}$	$\hat{\lambda}^2_{BML}$
1	Estimate	0.924	0.948	0.970
	Bayes risk	0.156	0.153	0.151
2	Estimate	1.030	1.053	1.079
	Bayes risk	0.151	0.153	0.156
3	Estimate	0.832	0.855	0.870
	Bayes risk	0.178	0.171	0.167
4	Estimate	0.737	0.759	0.769
	Bayes risk	0.219	0.208	0.203
5	Estimate	0.747	0.768	0.778
	Bayes risk	0.214	0.204	0.199
6	Estimate	0.852	0.872	0.886
	Bayes risk	0.172	0.166	0.163

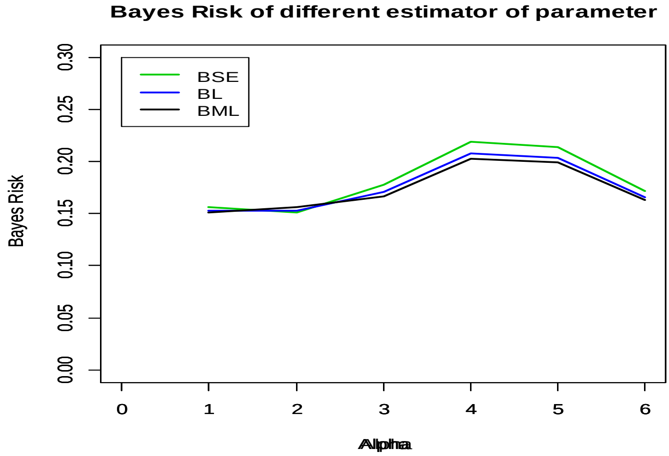

Fig. 6. Graph of Bayes risk of different estimators of the parameter λ^2 of the Rayleigh distribution when $n = 20, \beta = 2, \lambda^2 = 1 and c = 1$.

Although for varying α and the sample size moderately large, we get the same result i.e. the Bayes estimator under MLINEX loss function give better result than the other loss functions except α =2.

Table 7. Estimated value and Bayes risk of different estimators of the parameter λ^2 of the Rayleigh distribution when $n = 20, \alpha = 3, \lambda^2 = 1 and c = 1$.

β	Criteria	$\hat{\lambda}^2_{BSE}$	$\hat{\lambda}^2_{BL}$	$\hat{\lambda}^2_{BML}$
1	Estimate	0.800	0.822	0.836
	Bayes risk	0.585	0.182	0.177
2	Estimate	0.985	1.007	1.030
	Bayes risk	0.150	0.150	0.151
3	Estimate	0.876	0.898	0.916
	Bayes risk	0.165	0.160	0.157
4	Estimate	0.856	0.879	0.895
	Bayes risk	0.171	0.165	0.161
5	Estimate	0.943	0.965	0.986
	Bayes risk	0.153	0.151	0.150
6	Estimate	1.137	1.159	1.189
	Bayes risk	0.169	0.175	0.186

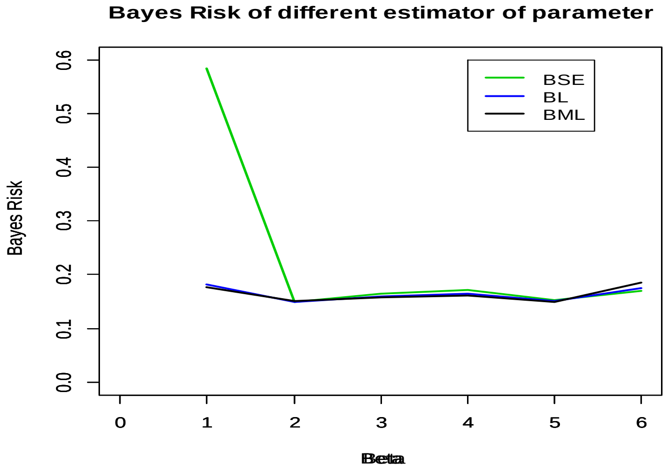

Fig. 7. Graph of Bayes risk of different estimators of the parameter λ^2 of the Rayleigh distribution when $n = 20, \alpha = 3, \lambda^2 = 1 and c = 1$.

Fig. 7 displays that, Bayes estimator under MLINEX loss function give smaller Bayes risk than Bayes estimator under SE and LINEX loss function except $\beta = 2 \& 6$.

Table 8. Estimated value and Bayes risk of different estimators of the parameter λ^2 of the Rayleigh distribution when $n = 20, \alpha = 2, \beta = 1 \ and c = 1$.

λ^2	Criteria	$\hat{\lambda}^2_{BSE}$	$\hat{\lambda}^2_{BL}$	$\hat{\lambda}^2_{BML}$
0.5	Estimate	1.604	1.627	1.681
	Bayes risk	1.369	1.421	1.544
1.0	Estimate	0.885	0.908	0.927
	Bayes risk	0.163	0.158	0.155
1.5	Estimate	0.556	0.580	0.582
	Bayes risk	1.041	0.997	0.992
2.0	Estimate	0.759	0.783	0.795
	Bayes risk	1.690	1.632	1.632
2.5	Estimate	0.507	0.531	0.531
	Bayes risk	4.122	4.028	4.026
3.0	Estimate	0.479	0.503	0.502
	Bayes risk	6.504	6.384	6.389

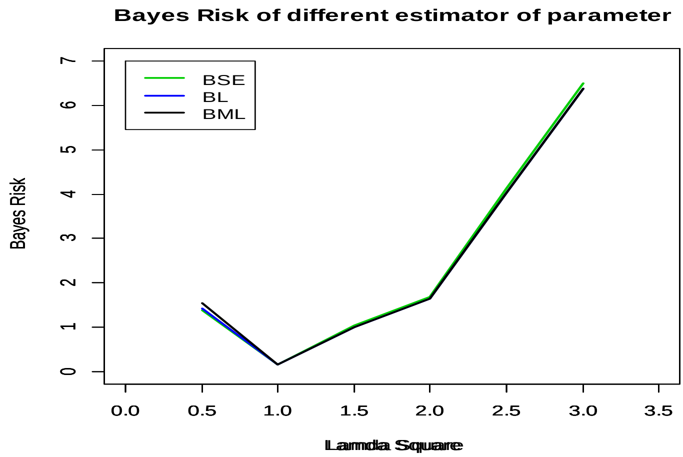

Fig. 8. Graph of Bayes risk of different estimators of the parameter λ^2 of the Rayleigh distribution when $n = 20, \alpha = 2, \beta = 1$ $and c = 1$.

Fig. 8 shown for varying λ^2 and n moderately large; the Bayes estimator under MLINEX loss functions gives the better result than the other loss functions.

The variation in the performance of the estimators for various sample sizes and varied parameters are observed from Tables 1-8. From Figs. 1-8, the Bayes risk of the estimators of different loss function, the MLINEX loss function is minimum. Therefore, it can be concluded that, Bayes estimator under MLINEX loss function is better than all other estimators in the study.

5. Real Study

For fitting the distribution, weather data have been used in this paper. Wind speed (kph) data have been chosen in the period of 2014-2017 of Dhaka Airport.

Table 9. Estimated value and Bayes risk of different estimators of the parameter λ^2 for different value of β.

β	Criteria	$\hat{\lambda}^2_{BSE}$	$\hat{\lambda}^2_{BL}$	$\hat{\lambda}^2_{BML}$
1	Estimate	5.152	5.053	5.257
	Bayes risk	1.921	1.972	0.040
2	Estimate	5.172	5.236	5.277
	Bayes risk	1.906	1.911	0.039
3	Estimate	5.192	5.241	5.297
	Bayes risk	1.891	1.741	0.039
4	Estimate	5.212	5.251	5.318
	Bayes risk	1.877	1.217	0.039

β	Criteria	$\hat{\lambda}^2_{BSE}$	$\hat{\lambda}^2_{BL}$	$\hat{\lambda}^2_{BML}$
5	Estimate	5.232	5.121	5.338
	Bayes risk	1.863	1.310	0.038
6	Estimate	5.252	5.431	5.359
	Bayes risk	1.848	1.390	0.038

Table 9 shows that for all cases Bayes risk of Bayesian approach under MLINEX loss function is the smallest than other approaches.

Table 10. Estimated value and Bayes risk of different estimators of the parameter λ^2 for different value of α .

α	Criteria	$\hat{\lambda}^2_{BSE}$	$\hat{\lambda}^2_{BL}$	$\hat{\lambda}^2_{BML}$
1	Estimate	5.277	5.412	5.387
	Bayes risk	1.795	1.798	0.037
2	Estimate	5.172	5.492	5.277
	Bayes risk	1.906	1.980	0.039
3	Estimate	5.070	5.074	5.172
	Bayes risk	2.022	1.982	0.041
4	Estimate	4.973	4.998	5.070
	Bayes risk	2.143	1.990	0.043
5	Estimate	4.879	5.020	4.973
	Bayes risk	2.268	2.309	0.045
6	Estimate	4.788	4.882	4.879
	Bayes risk	2.398	2.212	0.047

Table 10 represents that for different values of α, Bayes estimate under MLINEX loss function have the smallest Bayes risk than other Bayesian approaches.

If we want to predict about the wind speed on specific region then Rayleigh distribution has been used and for fitting the distribution if we use Bayesian approach under MLINEX loss function then it will give better result. Because MLINEX loss function shows the smallest Bayes risk.

6. Conclusion

In this study, we have considered the Bayesian estimation approach to estimate the parameter of Rayleigh distribution. In Bayesian approach, squared error (SE), linear exponential (LINEX) and modified linear exponential (MLINEX) loss functions have been used. We conducted a comprehensive simulation and real data to judge the relative performance of the Bayes estimator under different loss functions at different sample sizes and varied parameters of prior distribution. From simulated results and real study, smallest risk has been observed by Bayesian approach under MLINEX loss function than other loss functions about all cases. Also Figs. 1-8, downward shape has been displayed by

MLINEX loss function than SE and LINEX loss functions. That means, Bayesian approach under MLINEX loss function gives better results than other loss functions. Therefore, Bayesian approach under MLINEX loss function can be suggested to estimate the parameter of Reyleigh distribution.

References

1. S. A. Ahmed, and H. O. Mahammed, Jordan J. Mech. Ind. Eng. **6**, 135 (2012).
2. M. Aslam, M.Tahir, Z. Hussain, and B. Al-Zahrani, PLOS, **10**, 1 (2015).
3. S. C. Biswas, J. Mahanta, and M. K. Roy, Int. J. Stat. Med. Res. **7**, 57 (2018).
4. K. Boudjerda, A.Chadli, A. Merradji, and H. Fellag, Electronic J. Appl. Stat. Anal. **10**, 50 (2017). https://doi.org/10.6000/1929-6029.2018.07.02.4
5. M. R. Hasan and A. R. Baizid, J. Sci. Res. **9**, 67 (2016). https://doi.org/10.3329/jsr.v1i1.29308
6. J. Mahanta and M. B. A. Talukder, Open Sci. J. Stat. Applicat. **5**, 40 (2018).
7. A. M. Mood, F. A. Graybill, and D. C. Boes, Introduction to the Theory of Statistics, 3rd Edition (McGraw-Hill Book Company, USA, 1974).
8. W. Rattanapitikon, Ocean Eng. **37**, 1554 (2010). https://doi.org/10.1016/j.oceaneng.2010.09.013
9. M. Saleem, and M. Aslam, J. Appl. Stat. Sci. **16**, 493 (2008).
10. T. N. Sindhu, N. Feroze, M. Aslam, and A. Shafiq, Punjab Unv. J. Math. **48**, 49 (2016).
11. C. Walck, Hand-Book on Statistical Distributions for Experimentalists (University of Stockholm, 2007).

Appendix A. R code for estimating different estimators and their Bayes risk by simulated data

```
Ray<-function(n,c,a,b,m)
{
u<-runif(n)
x<-(-m*log(1-u))^(1/2)
bse<-(sum(x^2)+b)/(n+a)
rbse<-((sum(x^2)+b)^2)/((n+a)*(n+a+1))-bse^2
bl<-bse-2*(sum(x^2)+b)*bse/c
k<-(gamma(n+a+c)/gamma(n+a))*((sum(x^2)+b)^c)
rbl<-exp(bl)*k-c*bl+c*bse-1
bml<-((gamma(n+a-c)/gamma(n+a))^(1/c))*(sum(x^2)+b)
rbml<-bml*(gamma(n+a-c)/gamma(n+a))*((sum(x^2)+b)^c)-c*log(bml)-1
list(bse,rbse,bl,rbl,bml,rbml)
}
Ray(100,2,2,1,.1)
```

Appendix B. R code for estimating different estimators and their Bayes risk by real data

```
library(foreign)
```

```
data<-read.table("D:/d.txt")
Ray<-function(c,a,b)
{
x<-data[,1]
n<-length(x)
bse<-(sum(x^2)+b)/(n+a)
rbse<-1/(((sum(x^2)+b)^2)/((n+a)*(n+a+1))-bse^2)
bl<-bse-2*(sum(x^2)+b)*bse/c
k<-(gamma(n+a+c)/gamma(n+a))*((sum(x^2)+b)^c)
rbl<-1/(exp(bl)*k-c*bl+c*bse-1)
bml<-((gamma(n+a-c)/gamma(n+a))^(1/c))*(sum(x^2)+b)
rbml<-1/(bml*(gamma(n+a-c)/gamma(n+a))*((sum(x^2)+b)^c)-c*log(bml)-1)
list(bse,rbse,bl,rbl,bml,rbml)
}
Ray(1,1,2)
```