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Bayesian Estimation under Different Loss Functions Using Gamma Prior for the Case of Exponential Distribution

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Abstract

The Bayesian estimation approach is a non-classical estimation technique in statistical inference and is very useful in real world situation. The aim of this paper is to study the Bayes estimators of the parameter of exponential distribution under different loss functions and compared among them as well as with the classical estimator named maximum likelihood estimator (MLE). Since exponential distribution is the lifetime distribution, we have studied exponential distribution using gamma prior. Here the gamma prior is used as the prior distribution of exponential distribution for finding the Bayes estimator. In our study we also used different symmetric and asymmetric loss functions such as squared error loss function, quadratic loss function, modified linear exponential (MLINEX) loss function and non-linear exponential (NLINEX) loss function. We have used simulated data using R-coding to find out the mean squared error (MSE) of different loss functions and hence found that non-classical estimator is better than classical estimator. Finally, mean square error (MSE) of the estimators of different loss functions are presented graphically.

Keywords: Bayes estimator; Maximum likelihood estimator (MLE); Squared error (SE) loss function; Modified linear exponential (MLINEX) loss function; Non-Linear exponential (NLINEX) loss function.

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1. Introduction

The exponential distribution plays an important role in lifetime data analysis and waiting time or queuing problems [1]. Many authors have developed inference procedures for exponential model. For example, Kulldorff devoted a large part of book to the estimation of the parameters of the exponential distribution [2] based on completely or partially grouped data. Sarhan found the empirical Bayes estimators of exponential model [3]. Janeen explained the empirical Bayes estimators of the parameter of exponential distribution based on record values [4]. To know more details

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the work of Balakrishnan *et al*. and Al-Hemyari of exponential distribution [4,5] can be seen.

Rahman *et al*. studied the bayes estimators under conjugate prior for power function distribution [7]. We have studied this under gamma prior for exponential distribution to see the comparative situation. We can use the exponential distribution to estimate the stress-strength parameters and reliability of representing survival of head and neck cancer patients [8].

A continuous random variable X is said to have one parameter exponential distribution with parameter θ (θ>0) if its probability density function (pdf) is given by Roy [9].

$$f(x;\theta) = \begin{cases} \theta e^{-\theta x} & ; 0 \le x \le \infty \ , \theta > 0 \\ 0 & ; otherwise \end{cases} \tag{1}$$

Here we are interested to find the Bayes estimator of parameter θ under different loss functions.

1.1 *Prior and Posterior density function of parameter* θ

For Bayesian estimation we need to specify a prior distribution for the parameter. Consider a gamma prior for the parameter θ having density function [10]

$$\pi(\theta) = \frac{\beta^\alpha}{\Gamma\alpha} e^{-\beta\theta} \theta^{\alpha-1} ; \theta, \alpha, \beta > 0 \tag{2}$$

Then the posterior density function of θ for the given random sample X is given by Mood *et al.* [11]

$$f(\theta / x) = \frac{\left[\prod_{i=1}^{n} f(x_i/\theta)\right]\pi(\theta)}{\int\left[\prod_{i=1}^{n} f(x_i/\theta)\right]\pi(\theta)d\theta} = \frac{\prod_{i=1}^{n} \theta e^{-\theta x_i} \frac{\beta^\alpha}{\Gamma\alpha} e^{-\beta\theta}\theta^{\alpha-1}}{\int_0^\infty \prod_{i=1}^{n} \theta e^{-\theta x_i} \frac{\beta^\alpha}{\Gamma\alpha} e^{-\beta\theta}\theta^{\alpha-1} d\theta} = \frac{\theta^n e^{-\theta\sum x_i} \frac{\beta^\alpha}{\Gamma\alpha} e^{-\beta\theta}\theta^{\alpha-1}}{\int_0^\infty \theta^n e^{-\theta\sum x_i} \frac{\beta^\alpha}{\Gamma\alpha} e^{-\beta\theta}\theta^{\alpha-1} d\theta}$$

$$= \frac{e^{-\left(\beta+\sum x_i\right)\theta} \theta^{\alpha+n-1}}{\int_0^\infty e^{-\left(\beta+\sum x_i\right)\theta} \theta^{\alpha+n-1} d\theta} = \frac{e^{-\left(\beta+\sum x_i\right)\theta} \theta^{\alpha+n-1}}{\frac{\Gamma(\alpha+n)}{(\beta+\sum x_i)^{\alpha+n}}}$$

$$\therefore f(\theta / x) = \frac{(\beta+\sum x_i)^{\alpha+n}}{\Gamma(\alpha+n)} e^{-(\beta+\sum x_i)\theta} \theta^{(\alpha+n)-1} \tag{3}$$

This implies that $f(\theta / x) \sim Gamma[\alpha+n, \beta+\sum x_i]$

2. Different Estimators of Parameter θ

Here, Bayes estimator of θ for different loss functions along with maximum likelihood estimator has been determined.

2.1. ***Maximum likelihood estimator (MLE) of parameter θ***

Suppose, $X = (X_1, X_2, ..., X_n)$ be a random sample of size *n* drawn from exponential distribution. Let $(x_1, x_2, \dots, x_n)$ is the observe value of $(X_1, X_2, \dots, X_n)$. Then the likelihood function of θ based on $(x_1, x_2 \dots x_n)$ is given by Mood *et al.* [11]

$$L(\theta / x) = \prod_{i=s}^{n} f(x_i;\theta) = \prod_{i=1}^{n} \theta e^{-\theta x_i} = \theta^n e^{-\theta \sum x_i} \qquad (4)$$

The natural logarithm of likelihood function is given by

$$\ln L(\theta / x) = n \ln \theta - \theta \sum x_i$$

Now the MLE of θ is obtained by solving the following equation

$$\frac{\partial \ln L(\theta / x)}{\partial \theta} = 0$$

$$\Rightarrow \frac{n}{\theta} - \sum x_i = 0$$

$$\Rightarrow \frac{n}{\theta} = \sum x_i$$

Hence, $\hat{\theta}_{MLE} = \frac{n}{\sum x_i}$ is the MLE of θ.

2.2. ***Bayes estimator of parameter θ for squared error (SE) loss function***

Here we have determined Bayes estimator of θ for squared error loss function [10] defined by

$$L(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^2 \qquad (5)$$

For squared error loss function Bayes estimator is the mean of posterior density function. From (3) posterior density function is a gamma distribution with parameter $(\alpha + n)$ and $(\beta + \sum x_i)$. Here, the mean of posterior density function is $\frac{\alpha + n}{(\beta + \sum x_i)}$.

Hence, $\hat{\theta}_{BSE} = \frac{\alpha + n}{(\beta + \sum x_i)}$ is the Bayes estimator of θ under squared error loss function.

2.3. ***Bayes Estimator of θ for Quadratic Loss (QL) function***

Let, the quadratic loss function is defined as [12]

$$L(\hat{\theta}; \theta) = \left(\frac{\hat{\theta} - \theta}{\theta} \right)^2 \qquad (6)$$

Under this loss function the Bayes estimator of θ is obtained by solving the equation

$$\frac{\partial}{\partial \hat{\theta}} \int L(\hat{\theta}; \theta) f(\theta / x) d\theta = 0$$

$$\Rightarrow \frac{\partial}{\partial \hat{\theta}} \int \left(\frac{\hat{\theta} - \theta}{\theta} \right)^2 \frac{(\beta + \sum x_i)^{\alpha+n}}{\Gamma(\alpha + n)} e^{-(\beta + \sum x_i)\theta} \theta^{(\alpha+n)-1} d\theta = 0$$

$$\Rightarrow \int \frac{2(\hat{\theta}-\theta)}{\theta^2} \frac{(\beta+\sum x_i)^{\alpha+n}}{\Gamma(\alpha+n)} e^{-(\beta+\sum x_i)\theta} \theta^{(\alpha+n)-1} d\theta = 0$$

$$\Rightarrow \frac{(\beta+\sum x_i)^{\alpha+n}}{\Gamma(\alpha+n)} \hat{\theta} \int e^{-(\beta+\sum x_i)\theta} \theta^{(\alpha+n-2)-1} d\theta = \frac{(\beta+\sum x_i)^{\alpha+n}}{\Gamma(\alpha+n)} \int e^{-(\beta+\sum x_i)\theta} \theta^{(\alpha+n-1)-1} d\theta$$

$$\Rightarrow \frac{(\beta+\sum x_i)^{\alpha+n}}{\Gamma(\alpha+n)} \hat{\theta} \frac{\Gamma(\alpha+n-2)}{(\beta+\sum x_i)^{\alpha+n-2}} = \frac{(\beta+\sum x_i)^{\alpha+n}}{\Gamma(\alpha+n)} \frac{\Gamma(\alpha+n-1)}{(\beta+\sum x_i)^{\alpha+n-1}}$$

$$\Rightarrow \hat{\theta} = \frac{\Gamma(\alpha+n-1)}{(\beta+\sum x_i)^{\alpha+n-1}} \frac{(\beta+\sum x_i)^{\alpha+n-2}}{\Gamma(\alpha+n-2)} = \frac{(\alpha+n-2)}{(\beta+\sum x_i)}$$

Hence, $\hat{\theta}_{BQL} = \dfrac{(\alpha+n-2)}{(\beta+\sum x_i)}$ is the Bayes estimator of θ under quadratic loss function.

2.4. *Bayes estimator of θ for MLINEX loss function*

Let, the MLINEX loss function [12] is defined as

$$L(\hat{\theta}-\theta) = \omega\left[\left(\frac{\hat{\theta}}{\theta}\right)^c - c\ln\left(\frac{\hat{\theta}}{\theta}\right) - 1\right], \omega > 0, c \neq 0 \qquad (7)$$

For MLINEX loss function the Bayes estimator of θ is obtained by

$$\hat{\theta}_{BML} = \left[E(\theta^{-c}/x)\right]^{-\frac{1}{c}} \qquad (8)$$

$$E(\theta^{-c}/x) = \int_0^\infty \theta^{-c} f(\theta/x) d\theta = \frac{(\beta+\sum x_i)^{\alpha+n}}{\Gamma(\alpha+n)} \int_0^\infty e^{-(\beta+\sum x_i)\theta} \theta^{(\alpha+n-c)-1} d\theta$$

$$= \frac{(\beta+\sum x_i)^{\alpha+n}}{\Gamma(\alpha+n)} \frac{\Gamma(\alpha+n-c)}{(\beta+\sum x_i)^{\alpha+n-c}}$$

$$\therefore E(\theta^{-c}/x) = \frac{\Gamma(\alpha+n-c)}{\Gamma(\alpha+n)} (\beta+\sum x_i)^c$$

Hence, $\hat{\theta}_{BML} = \left[\dfrac{\Gamma(\alpha+n-c)}{\Gamma(\alpha+n)}\right]^{-\frac{1}{c}} (\beta+\sum x_i)^{-1}$ is the Bayes estimator of θ under MLINEX loss function.

2.5. *Bayes Estimator of parameter θ for NLINEX Loss function*

Let, the NLINEX loss function [13] of the form

$$L(D) = k\left(\exp(cD) + cD^2 - cD - 1\right), k > 0, c > 0 \qquad (9)$$

Here, D represents the estimation error i.e., $D = \hat{\theta} - \theta$. For NLINEX loss function Bayes estimator [11] of θ is given by

$$\hat{\theta}_{BNL} = -[\ln E_\theta\{\exp(-c\theta)\} - 2E_\theta(\theta)]/(c+2) \qquad (10)$$

Where, $E_\theta(.)$ stands for posterior expectation

Now, $E_\theta\{\exp(-c\theta)\} = \int_0^\infty e^{-c\theta} f(\theta/x)d\theta = \frac{(\beta+\sum x_i)^{\alpha+n}}{\Gamma(\alpha+n)}\int_0^\infty e^{-(\beta+c+\sum x_i)\theta}\theta^{(\alpha+n)-1}d\theta$

$$= \frac{(\beta+\sum x_i)^{\alpha+n}}{\Gamma(\alpha+n)}\frac{\Gamma(\alpha+n)}{(\beta+c+\sum x_i)^{\alpha+n}} == \left(\frac{\beta+c+\sum x_i}{\beta+\sum x_i}\right)^{-(\alpha+n)}$$

$$\therefore E_\theta\{\exp(-c\theta)\} = \left(1+\frac{c}{\beta+\sum x_i}\right)^{-(\alpha+n)}$$

$$\ln E_\theta\{\exp(-c\theta)\} = -(\alpha+n)\ln\left(1+\frac{c}{\beta+\sum x_i}\right) \tag{11}$$

$$Again, E_\theta(\theta) = \int_0^\infty \theta f(\theta/x)d\theta = \frac{(\beta+\sum x_i)^{\alpha+n}}{\Gamma(\alpha+n)}\int_0^\infty e^{-(\beta+\sum x_i)\theta}\theta^{(\alpha+n+1)-1}d\theta$$

$$= \frac{(\beta+\sum x_i)^{\alpha+n}}{\Gamma(\alpha+n)}\frac{\Gamma(\alpha+n+1)}{(\beta+\sum x_i)^{\alpha+n+1}}$$

$$\therefore E_\theta(\theta) = \frac{\alpha+n}{\beta+\sum x_i} \tag{12}$$

Using (11) and (12) in (10) we obtain,

$$\hat{\theta}_{BNL} = -\left[-(\alpha+n)\ln\left(1+\frac{c}{\beta+\sum x_i}\right) - 2\frac{(\alpha+n)}{(\beta+\sum x_i)}\right]/(c+2)$$

Hence, $\hat{\theta}_{BNL} = (\alpha+n)\left[\ln\left(1+\frac{c}{\beta+\sum x_i}\right) + \frac{2}{(\beta+\sum x_i)}\right]/(c+2)$ is the Bayes estimator of θ under NLINEX loss function.

3. Empirical Analysis

In order to compare estimators $\hat{\theta}_{MLE}, \hat{\theta}_{BSE}, \hat{\theta}_{BQL}, \hat{\theta}_{BML}$ and $\hat{\theta}_{BNL}$ we have considered MSE of the estimators. The MSE of estimator $\hat{\theta}$ is defined as:

$$MSE(\hat{\theta}) = E[\hat{\theta}-\theta]^2 = Var(\hat{\theta}) + [Bias(\hat{\theta})]^2$$

In our study 6000 Samples have generated for each case. To obtain the variance of $\hat{\theta}$, we have used the true (assume) value of the parameter θ under consideration. Again we have obtained the estimated value, MSE and Bias of the estimators by using Monte Carlo simulation method using R-Code from the exponential distribution. The results and their graphs (Using MS- Excel) are presented in Table 1.

Table 1. Estimated value and MSE of different estimators of θ of Exponential distribution where α = 0.5, β = 1.0, θ = 0.5 and c = 1.0.

Sample Size n	Criteria	$\hat{\theta}_{MLE}$	$\hat{\theta}_{BSE}$	$\hat{\theta}_{BQL}$	$\hat{\theta}_{BML}$	$\hat{\theta}_{BNL}$
5	Estimated Value	0.690	0.821	0.180	0.557	0.429
	MSE	0.147	0.081	0.045	0.047	0.073
10	Estimated Value	0.354	0.408	0.343	0.611	0.418
	MSE	0.042	0.035	0.024	0.026	0.032
15	Estimated Value	0.545	0.905	0.499	0.490	0.385
	MSE	0.024	0.021	0.016	0.017	0.019
20	Estimated Value	0.748	0.546	0.369	0.458	0.386
	MSE	0.016	0.015	0.013	0.013	0.014
25	Estimated Value	0.765	0.455	0.458	0.494	0.455
	MSE	0.012	0.011	0.009	0.010	0.011
30	Estimated Value	0.730	0.537	0.436	0.420	0.428
	MSE	0.009	0.009	0.008	0.009	0.009

From Table 1 we have observed that, the MSE of $\hat{\theta}_{MLE}$ is very high for small sample size but decline sharply and become closer to other estimators with increase in sample size. Among Bayes estimators $\hat{\theta}_{BQL}$ gives smaller value of MSE when sample size is small but for large sample they are almost identical (Fig. 1).

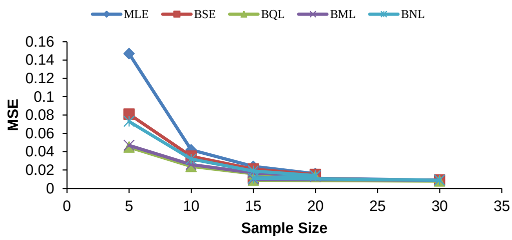

Fig. 1. Graph of MSEs of different estimators of θ of Exponential distribution where α = 0.5, β = 1.0, θ = 0.5 and c = 1.0.

Table 2. Estimated value and MSE of different estimators of θ for Exponential distribution where α = 1.0, β= 1.5, θ = 1.0 and c =1.0.

Sample Size n	Criteria	$\hat{\theta}_{MLE}$	$\hat{\theta}_{BSE}$	$\hat{\theta}_{BQL}$	$\hat{\theta}_{BML}$	$\hat{\theta}_{BNL}$
5	Estimated Value	1.029	0.899	0.745	1.257	0.744
	MSE	0.576	0.136	0.156	0.112	0.115
10	Estimated Value	0.699	0.915	0.511	1.094	1.152
	MSE	0.171	0.089	0.086	0.074	0.082
15	Estimated Value	0.748	0.738	0.717	0.757	0.888
	MSE	0.104	0.064	0.059	0.056	0.061
20	Estimated Value	1.008	0.843	0.606	0.714	0.725
	MSE	0.065	0.049	0.047	0.045	0.046
25	Estimated Value	0.711	0.894	0.971	1.029	0.815
	MSE	0.053	0.039	0.038	0.037	0.039
30	Estimated Value	0.649	0.851	0.758	0.986	1.051
	MSE	0.039	0.033	0.032	0.031	0.032

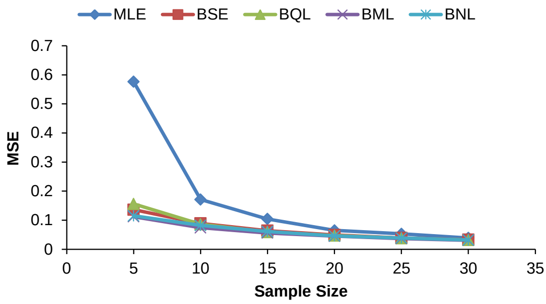

Fig. 2. Graph of MSEs of different estimators of θ for Exponential distribution where α = 1.0, β= 1.5, θ = 1.0 and c =1.0.

Table 3 represents the variation in the performance of the estimators for different sample size. More or less similar patterns are observed here as a previous table that is MSE of $\hat{\theta}_{MLE}$ is higher than all other estimators. Here MSE of $\hat{\theta}_{BNL}$ is least in the class of all other Bayes estimators. Also MSE of $\hat{\theta}_{BQL}$ and $\hat{\theta}_{BML}$ are very close to each other for large sample (Fig. 3).

Table 3. Estimated value and MSE of different estimators of θ for Exponential distribution where α = 1.5, β= 2.0, θ = 1.0 and c =2.0.

Sample Size *n*	Criteria	Estimators				
		$\hat{\theta}_{MLE}$	$\hat{\theta}_{BSE}$	$\hat{\theta}_{BQL}$	$\hat{\theta}_{BML}$	$\hat{\theta}_{BNL}$
5	Estimated Value	0.663	1.155	0.629	0.590	1.684
	MSE	0.611	0.109	0.136	0.109	0.083
10	Estimated Value	1.233	1.321	1.117	1.035	0.683
	MSE	0.166	0.079	0.076	0.069	0.067
15	Estimated Value	1.542	1.350	0.634	1.009	1.020
	MSE	0.092	0.058	0.056	0.054	0.053
20	Estimated Value	0.923	0.898	1.187	0.915	1.682
	MSE	0.063	0.046	0.042	0.042	0.042
25	Estimated Value	0.943	0.915	1.037	0.855	0.959
	MSE	0.050	0.039	0.035	0.035	0.034
30	Estimated Value	1.171	1.397	0.745	1.216	1.038
	MSE	0.039	0.033	0.029	0.029	0.029

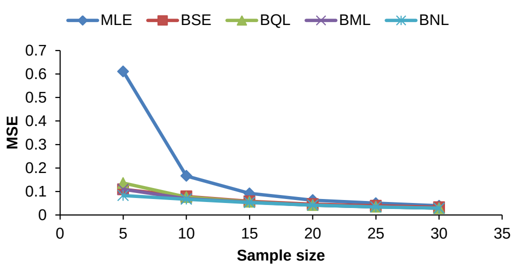

Fig. 3. Graph of MSEs of different estimators of parameter θ of Exponential distribution where α = 1.5, β= 2.0, θ = 1.0 and c =2.0.

For different sample sizes Table 4 represents the minimum values of MSE for $\hat{\theta}_{BNL}$ and some cases it is very near to that of $\hat{\theta}_{BSE}$. On the other hand $\hat{\theta}_{MLE}$ keeps its traditional fashion as previous cases.

Table 4. Estimated value and MSE of different estimators of θ for Exponential distribution where α = 2.0, β= 2.0, θ = 1.5 and c =2.0.

Sample Size n	Criteria	Estimators				
		$\hat{\theta}_{MLE}$	$\hat{\theta}_{BSE}$	$\hat{\theta}_{BQL}$	$\hat{\theta}_{BML}$	$\hat{\theta}_{BNL}$
5	Estimated Value	1.746	1.182	0.791	0.965	1.471
	MSE	1.282	0.156	0.317	0.249	0.151
10	Estimated Value	2.062	1.405	1.735	1.055	1.578
	MSE	0.369	0.133	0.164	0.148	0.123
15	Estimated Value	1.641	1.230	0.987	1.535	1.672
	MSE	0.224	0.109	0.120	0.109	0.096
20	Estimated Value	1.243	1.911	1.384	1.587	1.405
	MSE	0.143	0.088	0.094	0.088	0.079
25	Estimated Value	1.448	1.481	1.411	2.215	1.628
	MSE	0.106	0.073	0.076	0.075	0.068
30	Estimated Value	1.603	1.807	0.975	1.696	1.802
	MSE	0.088	0.063	0.066	0.064	0.060

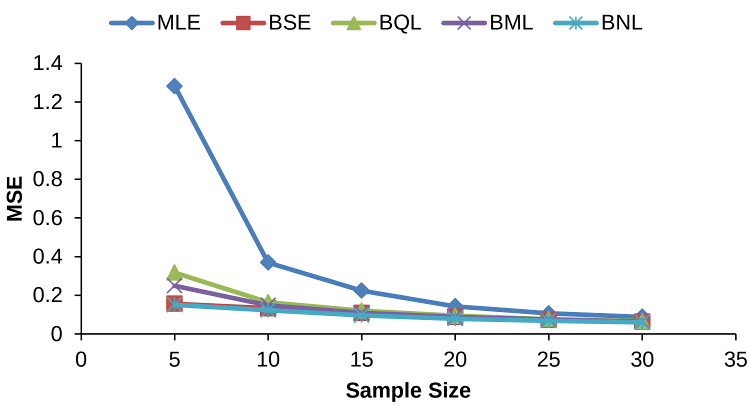

Fig. 4. Graph of MSEs of different estimators of θ for Exponential distribution where α = 2.0, β= 2.0, θ = 1.5 and c =2.0.

Table 5 shows the smallest values of MSE for $\hat{\theta}_{BSE}$ than all other estimators in the study. From above table the relationship among different estimators according to MSE is: $MSE(\hat{\theta}_{BSE}) \leq MSE(\hat{\theta}_{BNL}) \leq MSE(\hat{\theta}_{BQL}) \leq MSE(\hat{\theta}_{BML}) \leq MSE(\hat{\theta}_{MLE})$

Table 5. Estimated value and MSE of different estimators of θ for Exponential distribution where α = 2.0, β= 3.0, θ = 2.0 and c =3.0.

Sample Size n	Criteria	Estimators $\hat{\theta}_{MLE}$	$\hat{\theta}_{BSE}$	$\hat{\theta}_{BQL}$	$\hat{\theta}_{BML}$	$\hat{\theta}_{BNL}$
5	Estimated Value	3.229	1.273	0.866	0.927	1.017
	MSE	2.176	0.521	1.151	1.169	0.755
10	Estimated Value	3.541	1.499	1.207	1.407	1.325
	MSE	0.659	0.291	0.558	0.563	0.415
15	Estimated Value	1.766	1.607	1.323	1.329	1.359
	MSE	0.395	0.199	0.350	0.356	0.277
20	Estimated Value	2.095	1.569	1.769	1.397	1.874
	MSE	0.251	0.154	0.247	0.253	0.207
25	Estimated Value	2.545	1.909	1.304	1.324	2.016
	MSE	0.194	0.132	0.187	0.194	0.163
30	Estimated Value	1.789	1.455	1.566	1.791	1.775
	MSE	0.158	0.110	0.157	0.156	0.137

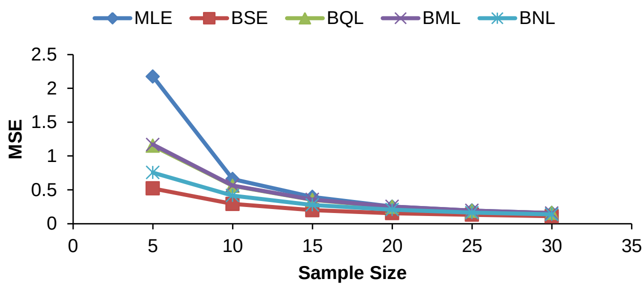

Fig. 5. Graph of MSEs of different estimators of θ for Exponential distribution where α = 2.0, β= 3.0, θ = 2.0 and c =3.0.

It is clear from Table 6 that MSE of $\hat{\theta}_{BSE}$ is smaller than other estimators (Fig. 6) and some cases it is very near to that of MSE of $\hat{\theta}_{BNL}$ for different values of parameter θ.

Table 6. Estimated value and MSE of different estimators of θ for Exponential distribution when n = 20, α = 1.5, β= 2.0, and c =3.0.

θ	Criteria	Estimators $\hat{\theta}_{MLE}$	$\hat{\theta}_{BSE}$	$\hat{\theta}_{BQL}$	$\hat{\theta}_{BML}$	$\hat{\theta}_{BNL}$
1.0	Estimated Value	1.138	0.842	0.715	0.570	0.871
	MSE	0.064	0.046	0.044	0.045	0.039
1.5	Estimated Value	1.607	1.278	1.813	1.248	1.198
	MSE	0.145	0.083	0.099	0.104	0.083
2.0	Estimated Value	2.587	1.702	2.224	1.345	1.466
	MSE	0.253	0.143	0.206	0.204	0.168
2.5	Estimated Value	2.644	2.012	2.276	1.980	1.634
	MSE	0.398	0.239	0.369	0.370	0.321
3.0	Estimated Value	2.728	2.231	2.035	2.259	2.678
	MSE	0.584	0.392	0.628	0.620	0.587

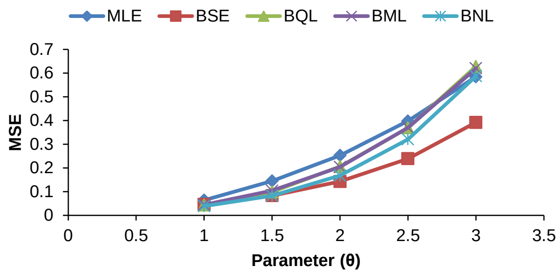

Fig. 6. Graph of MSEs of different estimators of θ for Exponential distribution when n = 20, α = 1.5, β= 2.0, and c =3.0.

4. Conclusion

From above analysis and graphical representation we have concluded that except for few cases Bayes estimator under Squared Error (SE) loss function and NLINEX loss function are better than other estimators in the study. We also concluded that, non-classical estimator (the class of Bayes estimator under different loss functions) is better than classical estimator (MLE). We can apply Bayes estimator to calculate posterior probability on the basis of prior information. In real life Byes estimator can be used for forecasting insurance loss payments.

Appendix A. R- coding for MLE of θ

```
n<-5
theta<-1
s<-6000
sim<-matrix(rep(0,1*s),nrow=s)
for(i in 1:s)
  {
u<-runif(n,min=0,max=1)
x<-(-((1/theta)*log(1-u)))
MLE<-(n/sum(x))
sim[i,]<-MLE
    }
mean<-mean(sim[,1])
bias<-(mean-theta)
MSE<-mean((sim[,1]-theta)^2)
```

Appendix B. R-coding for Bayes Estimator of θ for MLINEX Loss function

```
n<-5
alpha<-0.5
beta<-1
```

```
theta<-1
s<-6000
sim<-matrix(rep(0,1*s),nrow=s)
for(i in 1:s)
  {
u<-runif(n,min=0,max=1)
x<-(-((1/theta)*log(1-u)))
BQL<-((alpha+n-2)/(beta+sum(x)))
sim[i,]<-BQL
    }
mean<-mean(sim[,1])
bias<-(mean-theta)
MSE<-mean((sim[,1]-theta)^2)
```

Appendix C. R- coding for Bayes Estimator of θ for MLINEX Loss function

```
n<-5
alpha<-0.5
beta<-1
theta<-1
c<-1
s<-6000
sim<-matrix(rep(0,1*s),nrow=s)
for(i in 1:s)
  {
u<-runif(n,min=0,max=1)
x<-(-((1/theta)*log(1-u)))
BML<-((((gamma(alpha+n-c))/(gamma(alpha+n)))^-(1/c))*((beta+sum(x))^-1))
sim[i,]<-BML
    }
mean<-mean(sim[,1])
bias<-(mean-theta)
MSE<-mean((sim[,1]-theta)^2)
```

Appendix D. R-coding of Bayes Estimator of θ for NLINEX Loss function

```
n<-5
alpha<-0.5
beta<-1
theta<-1
c<-1
s<-6000
sim<-matrix(rep(0,1*s),nrow=s)
for(i in 1:s)
  {
u<-runif(n,min=0,max=1)
x<-(-((1/theta)*log(1-u)))
```

```
BNL<-((alpha+n)*(log(1+c/(beta+sum(x)))+2/(beta+sum(x))))/(c+2)
sim[i,]<-BNL
   }
mean<-mean(sim[,1])
bias<-(mean-theta)
MSE<-mean((sim[,1]-theta)^2)
```

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