JSR Publications

Available Online

J. Sci. Res. **2** (1), 77-86 (2010)

JOURNAL OF SCIENTIFIC RESEARCH

www.banglajol.info/index.php/JSR

Sometimes Pooled Testimation in the Inverse Gaussian Model for Measure of Dispersion

G. Prakash[1*] and D. C. Singh[2]

[1]Evaluation Division, State Planning Institution, Lucknow, U. P., India

[2]Department of Statistics, Harish Chandra P. G. College, Varanasi, U. P., India

Abstract

This paper suggests sometimes pool estimators for the measure of dispersion in the inverse Gaussian distribution and their properties are studied in terms of the relative bias and relative efficiency under two different loss functions.

Keywords: Sometimes pool estimator; Level of significance; Relative bias; Relative efficiency; Effective interval.

DOI: 10.3329/jsr.v2i1.2809 J. Sci. Res. **2** (1), 77-86 (2010)

1. Introduction

The inverse Gaussian distribution has useful applications in a wide variety of fields such as Biology, Economics and Medicine [1-3]. It plays an important role in reliability theory and life testing problems. Let x be the inverse Gaussian variate with the parameters μ (measure of location) and σ (measure of dispersion), having probability density function

$$f(x) = \sqrt{\left(2\pi\,\sigma\,x^{3}\right)^{-1}}\ \exp\left(-\frac{(x-\mu)^{2}}{2\sigma\,\mu^{2}x}\right);\ x>0,\ \sigma,\mu>0\,. \tag{1}$$

Here, σ^{-1} stands as the shape parameter. Let $x_{i1}, x_{i2}, x_{i3}, \ldots, x_{in_i};\ i = 1, 2$ be two independent random samples of size n_1 and n_2 drawn from two inverse Gaussian distributions. The maximum likelihood estimates of μ_i and $\sigma_i^{-1};\ i = 1,2$ are given as

$$\hat{\mu}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} x_{ij} = \bar{x}_i$$

and

[*] *Corresponding author*: ggyanji@yahoo.com

$$\hat{\sigma}_i^{-1} = \frac{n_i}{v_i} \; ; \; v_i = \sum_{j=1}^{n_i} \left(\frac{1}{x_{ij}} - \frac{1}{\bar{x}_i} \right) .$$

Also, $\bar{x}_i$ and $\hat{\sigma}_i = v_i (n_i - 1)^{-1}$ are the unbiased estimates of μ_i and σ_i; i = 1, 2, respectively. The pool unbiased estimator for σ is given as

$$\bar{\sigma} = \frac{v_1 + v_2}{n_1 + n_2 - 2} . \tag{2}$$

In many real life situations the overestimation and underestimation are not of equal consequences. For such situations a symmetric loss function like as a square error loss (SELF) function is not appropriate [4–6]. A useful asymmetric loss function was introduced by [4], named as the LINEX loss function. The invariant version of the LINEX loss function (ILLF) [7] for the parameter θ is given as

$$L(\Delta) = e^{a\Delta} - a\Delta - 1 \; ; \; a \neq 0 \text{ and } \Delta = \left(\hat{\theta} - \theta\right)\theta^{-1} . \tag{3}$$

The LINEX loss function rises approximately exponentially on one side of zero and approximately linearly on the other side. Here, $\hat{\theta}$ is an estimate of the parameter θ. The sign and magnitude of 'a' represent the direction and degree of asymmetry respectively. The positive (negative) value of 'a' is used when overestimation is more (less) serious than underestimation. For 'a' close to zero, the LINEX loss is approximately squared error and therefore almost symmetric. Recently, some shrinkage testimators for the inverse dispersion of the inverse Gaussian distribution under the ILLF has been studied elsewhere [7].

Han and Bancroft [8] have studied the sometimes pool estimator for the mean of a Normal distribution. They have considered the situation when two independent random samples are available from two Normal distributions with means μ_1 and μ_2 and the common variance. The problem of pooling in different situations has also been considered by other workers [9-12]. Rai [13] has estimated the mean life of Exponential distribution. Sometimes pool estimator for shape parameter of the Pareto distribution under the SELF has been proposed by [14].

In the present article, we have studied the performances of the sometimes pool estimators for σ under the SELF and ILLF.

2. The Proposed Class of Estimators

We consider a class of estimators for σ of the model (1) as

$$Y = C\hat{\sigma}_1 \; ; \; C \in R^{+} . \tag{4}$$

The risk of Y under the SELF and ILLF are obtained respectively as

$$R_{(S)}(Y) = \sigma_1^2 \left\{ C^2 \frac{\varphi_1 + 1}{\varphi_1} - 2C + 1 \right\}, \tag{5}$$

and

$$R_{(L)}(Y) = e^{-a}\left(1-\frac{aC}{\varphi_1}\right)^{-\varphi_1} + a(1-C) - 1\ ;\ \varphi_1 = \frac{n_1-1}{2}\ . \tag{6}$$

The suffix S and L stand respectively for the risk under the SELF and ILLF criteria. The values of C which minimize the risks $R_{(S)}(Y)$ and $R_{(L)}(Y)$ respectively are given as

$$C_1 = \frac{\varphi_1}{\varphi_1+1} \quad \text{and} \quad C_2 = \frac{\varphi_1}{a}\left\{1-\exp\left(-\frac{a}{\varphi_1+1}\right)\right\}.$$

The minimum risk estimators of σ in the class Y with their respective risks are given as

Estimator	Risks
$Y_1 = C_1\hat{\sigma}_1$	$R_{(S)}(Y_1) = \dfrac{\sigma_1^2}{\varphi_1+1}$ $R_{(L)}(Y_1) = e^{-a}\left(\dfrac{\varphi_1+1}{\varphi_1+1-a}\right)^{\varphi_1} - \left(\dfrac{\varphi_1+1-a}{\varphi_1+1}\right)$
$Y_2 = C_2\hat{\sigma}_1$	$R_{(S)}(Y_2) = \sigma_1^2\left\{C_2^2\dfrac{\varphi_1+1}{\varphi_1} - 2C_2 + 1\right\}$ $R_{(L)}(Y_2) = (\varphi_1+1)\left(\exp\left(-\dfrac{a}{\varphi_1+1}\right) - 1\right) + a$

3. The Proposed Pool Class of Estimators

We consider a class of the pooled estimator for σ of the distribution (1) as

$$\hat{S} = l\,\bar{\sigma}\ ;\ l \in R^{+}\ . \tag{7}$$

The expressions of the risks for $\hat{S}$ under the SELF and ILLF, respectively are given as

$$R_{(S)}(\hat{S}) = \sigma_1^2\left\{l^2\frac{\varphi_1+\varphi_2+1}{\varphi_1+\varphi_2} - 2l + 1\right\};\ \varphi_2 = \frac{n_2-1}{2} \tag{8}$$

and

$$R_{(L)}\left(\hat{S}\right) = e^{-a}\left(1-\frac{a\,l}{\varphi_1+\varphi_2}\right)^{-\varphi_1-\varphi_2} + a\,(1-l)\,-1\,. \tag{9}$$

The values of l for which the risks $R_{(S)}\left(\hat{S}\right)$ and $R_{(L)}\left(\hat{S}\right)$ are minimum, are given respectively as

$$l_1 = \frac{\varphi_1+\varphi_2}{\varphi_1+\varphi_2+1} \quad \text{and} \quad l_2 = \frac{\varphi_1+\varphi_2}{a}\left\{1-\exp\left(-\frac{a}{\varphi_1+\varphi_2+1}\right)\right\}.$$

The improved classes of pooled estimator for σ with their respective risks are given as

Estimator	Risks
$\hat{S}_1 = l_1\,\bar{\sigma}$	$R_{(S)}\left(\hat{S}_1\right) = \dfrac{\sigma_1^2}{\varphi_1+\varphi_2+1}$
$\hat{S}_2 = l_2\,\bar{\sigma}$	$R_{(L)}\left(\hat{S}_2\right)=\left(\varphi_1+\varphi_2+1\right)\left(\exp\left(-\dfrac{a}{\varphi_1+\varphi_2+1}\right)-1\right)+a$

4. The Proposed Sometimes Pool Estimators and their Properties

Our interest is to estimate the parameter σ_1 when it is suspect but not known for certain that $\sigma_1 = \sigma_2$. Before pooling the two sample estimates for the estimation of the parameter σ, the test of hypothesis $H_0 : \sigma_1 = \sigma_2$ may be performed at some pre-assigned level of significance α. The test statistic for H_0 is given as

$$F = \frac{(n_2-1)\,v_1}{(n_1-1)\,v_1} \sim F_{(n_1-1),\,(n_2-1)}. \tag{10}$$

The proposed sometimes pooled estimator is given as

$$\hat{\sigma}_{ST_i} = \begin{cases} l_i\,\bar{\sigma} & \textit{if } f_1 \le F \le f_2 \\ C_i\,\hat{\sigma}_1 & \textit{else} \end{cases}, \tag{11}$$

where i = 1,2 and f_1, f_2 are the lower and upper $100\alpha/2\%$ points of the F distribution with (n_1-1) and (n_2-1) degrees of freedom. The hypothesis H_0 is rejected when $F \le f_1$ or $F \ge f_2$ and $P_{H_0}\left[F \le f_1 \text{ or } F \ge f_2\right] = \alpha$. Thus if $\alpha = 0$, $\hat{\sigma}_{ST_i} = l_i\,\bar{\sigma}$ and it is always pool estimator. If $\alpha = 1$, $\hat{\sigma}_{ST_i} = C_i\,\hat{\sigma}_1$ and it is never pool estimator. Otherwise it is sometimes pool estimator.

The relative biases for the estimators $\hat{\sigma}_{ST_i};\ i = 1,2$ are obtained as

$$RB\left(\hat{\sigma}_{ST_i}\right) = \frac{E\left(\hat{\sigma}_{ST_i}\right)}{\sigma_1} - 1$$
$$= G\left\{F_1,\ F_2,\ \left(\Delta_{i1} - \Delta_{i0}\right),\ 0,\ 0\right\} + C_i - 1\,, \tag{12}$$

where $\Delta_{i1} = \dfrac{l_i\left(\delta + Z_2\right)}{\delta\left(\varphi_1 + \varphi_2\right)}$, $\Delta_{i0} = \dfrac{C_i}{\varphi_1}$, $F_1 = \left(\dfrac{n_2 - 1}{n_1 - 1}\right)\dfrac{\delta}{f_2}$, $\delta = \dfrac{\sigma_1}{\sigma_2}$, $F_2 = \left(\dfrac{n_2 - 1}{n_1 - 1}\right)\dfrac{\delta}{f_1}$,

$$G\left\{F_1,\ F_2,\ w_1, w_2,\ r\right\} = \frac{\Gamma\left(\varphi_1 + \varphi_2 + 1 + r\right)}{\Gamma\varphi_1\,\Gamma\varphi_2}\int_{F_1}^{F_2}\frac{w_1\,Z_2^{\varphi_2 - 1}}{\left(1 + Z_2 - w_2\right)^{\varphi_1 + \varphi_2 + 1 + r}}\,dZ_2$$

and w_1 and w_2 are functions of Z_2.

The expressions of the risks under the SELF and ILLF for $\hat{\sigma}_{ST_i};\ i = 1,2$ are obtained as

$$R_{(S)}\left(\hat{\sigma}_{ST_i}\right) = \sigma_1^2\left\{G\left\{F_1,\ F_2,\ \left(\Delta_{i1}^2 - \Delta_{i0}^2\right),\ 0,\ 1\right\}\right.$$
$$\left. - 2\,G\left\{F_1,\ F_2,\ \left(\Delta_{i1} - \Delta_{i0}\right),\ 0,\ 0\right\} + 1 - C_i\right\} \tag{13}$$

and

$$R_{(L)}\left(\hat{\sigma}_{ST_i}\right) = e^{-a}\left\{G\left\{F_1,\ F_2,\ 1,\ a\left(\Delta_{i1} - \Delta_{i0}\right),\ -1\right\}\right.$$
$$\left. + G\left\{0,\ \infty,\ 1,\ a\,\Delta_{i0}, -1\right\}\right\} + a\left(1 - C_i\right) - 1$$
$$- a\,G\left\{F_1,\ F_2,\ \left(\Delta_{i1} - \Delta_{i0}\right),\ 0,\ 0\right\}\ ;\ i = 1,2. \tag{14}$$

The relative efficiencies for the pooled estimator $\hat{\sigma}_{ST_1}$ with respect to Y_1 and $\hat{\sigma}_{ST_2}$ with respect to Y_2 under the SELF and ILLF criteria are defined as

$$RE_{(S)}\left(\hat{\sigma}_{ST_i},\ Y_i\right) = R_{(S)}\left(Y_i\right) / R_{(S)}\left(\hat{\sigma}_{ST_i}\right)$$

and

$$RE_{(L)}\left(\hat{\sigma}_{ST_i},\ Y_i\right) = R_{(L)}\left(Y_i\right) / R_{(L)}\left(\hat{\sigma}_{ST_i}\right)\ .$$

The expressions of the relative biases and relative efficiencies are the functions of n_1, n_2, δ, a and α. For the selected set of values of $(n_1,\ n_2) = 05, 08, 10$; $\delta = 0.60\,(0.20)\,1.80$; $a = \pm 0.25, \pm 0.50$ and $\alpha = 0.01, 0.05, 0.10, 0.15$ the relative biases and relative efficiencies have been calculated. The 16-point Gauss-Legendre quardature formula is used to solve the integrations involved in relative biases and relative efficiencies. The relative biases are not presented here and the relative efficiencies have been presented in the Tables 1 to 4 for α = 0.01 and 0.05.

Table 1. Relative efficiency between $\hat{\sigma}_{ST_1}$ and Y_1 under SELF.

$\alpha = 0.01$		δ						
n_1	n_2	0.60	0.80	1.00	1.20	1.40	1.60	1.80
	05	1.5406	2.1106	2.7387	2.4039	2.2887	2.1675	2.0579
05	08	1.4563	2.4582	3.4010	2.9121	2.6405	2.3816	2.1672
	10	1.4395	2.6305	3.7701	3.1776	2.7969	2.4526	2.1826
	05	1.4683	1.9306	2.4863	2.1903	2.0973	1.9972	1.9059
08	08	1.3558	2.1473	2.9177	2.5092	2.2952	2.0899	1.9199
	10	1.2717	2.3718	3.4216	2.8622	2.4941	2.1754	1.9335
	05	1.4421	1.8927	2.4432	2.1527	2.0603	1.9615	1.8072
10	08	1.3378	1.9756	2.6143	2.2598	2.1014	1.9869	1.8154
	10	1.2376	2.2284	3.1354	2.6236	2.3130	2.0419	1.8319
$\alpha = 0.05$								
	05	1.4924	1.9694	2.5446	2.2366	2.1335	2.0255	1.9296
05	08	1.4482	2.2873	3.0479	2.5793	2.3458	2.1401	1.9790
	10	1.4212	2.4401	3.3005	2.7331	2.4267	2.1756	1.9893
	05	1.4266	1.7779	2.2942	2.0447	1.9778	1.8977	1.8114
08	08	1.3226	1.9333	2.5948	2.2581	2.1007	1.9439	1.8121
	10	1.2472	2.0786	2.8991	2.4678	2.2177	1.9921	1.8166
	05	1.4124	1.7403	2.2506	2.0132	1.9513	1.8231	1.7171
10	08	1.3163	1.7975	2.3847	2.0981	1.9770	1.8486	1.7273
	10	1.2257	1.9396	2.7256	2.3484	2.1186	1.9038	1.7363

Table 2. Relative efficiency between $\hat{\sigma}_{ST_2}$ and Y_2 under SELF for $\alpha = 0.01$.

$a = -0.50$		δ						
n_1	n_2	0.60	0.80	1.00	1.20	1.40	1.60	1.80
	05	1.7843	2.2528	2.8015	2.4094	2.2705	2.1382	2.0233
05	08	1.7431	2.7441	3.6105	3.0089	2.6881	2.4029	2.1740
	10	1.6963	3.0198	4.1381	3.3815	2.9162	2.5223	2.2233
	05	1.5696	1.8743	2.3225	2.0231	1.9370	1.8521	1.7771
10	08	1.4889	2.1963	2.8413	2.3841	2.1610	1.9630	1.8036
	10	1.4164	2.3859	3.1923	2.6053	2.2725	1.9969	1.8177

Table 2 (*continued*)

$a = -0.25$								
	05	1.6618	2.1804	2.7664	2.4026	2.2754	2.1489	2.0370
05	08	1.5903	2.6249	3.5512	2.9945	2.6877	2.4074	2.1800
	10	1.5383	2.8687	4.0485	3.3491	2.9018	2.5149	2.2191
	05	1.5089	1.8400	2.3087	2.0257	1.9480	1.8680	1.7958
10	08	1.4030	2.1359	2.8219	2.3931	2.1801	1.9853	1.8264
	10	1.3243	2.3117	3.1732	2.6198	2.2951	2.0199	1.8292
$a = 0.25$								
	05	1.4203	2.0355	2.7053	2.4031	2.3022	2.1878	2.0815
05	08	1.3084	2.3971	3.4532	2.9896	2.7099	2.4360	2.2085
	10	1.2554	2.5894	3.8974	3.3100	2.8944	2.5168	2.2237
	05	1.3879	1.7726	2.2855	2.0372	1.9779	1.9082	1.8420
10	08	1.2385	2.0187	2.7920	2.4235	2.2319	2.0429	1.8540
	10	1.1504	2.1694	3.1498	2.6673	2.3576	2.0805	1.8644
$a = 0.50$								
	05	1.3302	1.9642	2.6814	2.4129	2.3265	2.2184	2.1146
05	08	1.1799	2.2908	3.4176	3.0022	2.7348	2.4618	2.2319
	10	1.1300	2.4635	3.8394	3.3056	2.9024	2.5264	2.2325
	05	1.3077	1.7398	2.2767	2.0468	1.9974	1.9333	1.8702
10	08	1.1603	1.9625	2.7822	2.4459	2.2657	2.0793	1.9197
	10	1.0689	2.1021	3.1472	2.7023	2.3994	2.1196	1.9910

Table 3. Relative efficiency between $\hat{\sigma}_{ST_1}$ and Y_1 under ILLF for $\alpha = 0.01$.

$a = -0.50$					δ			
n_1	n_2	0.60	0.80	1.00	1.20	1.40	1.60	1.80
	05	1.3799	1.9940	2.7103	2.3035	2.2241	2.1205	2.0190
05	08	1.2934	2.3075	3.3753	2.8072	2.5730	2.3237	2.1086
	10	1.2572	2.4599	3.7182	3.0405	2.6992	2.3645	2.0947
	05	1.3317	1.8079	2.4141	2.0501	1.9887	1.9083	1.8294
10	08	1.2364	1.8978	2.6097	2.1720	2.0387	1.9644	1.7661
	10	1.1312	2.1173	3.1418	2.5496	2.2669	2.0002	1.7875
$a = -0.25$								
	05	1.4500	2.0422	2.7135	2.2743	2.1796	2.0705	1.9683
05	08	1.3588	2.3689	3.3763	2.7616	2.5169	2.2713	2.0639
	10	1.3151	2.5291	3.7263	3.0026	2.6536	2.3257	2.0650
	05	1.3790	1.8422	2.4205	2.0323	1.9578	1.8708	1.7891
10	08	1.2800	1.9281	2.6121	2.1420	2.0007	1.8861	1.7306
	10	1.1769	2.1623	3.1626	2.4996	2.2124	1.9524	1.7482

Table 3 (*continued*)

$a = 0.25$								
	05	1.6195	2.1545	2.7380	2.2382	2.1187	2.0013	1.8985
05	08	1.5262	2.5234	3.7011	2.7075	2.4441	2.2038	2.0083
	10	1.4683	2.7087	3.7829	2.9645	2.5998	2.2813	2.0345
	05	1.4915	1.9201	2.4405	2.0055	1.9081	1.8107	1.7250
10	08	1.3823	1.9997	2.6217	2.0991	1.9442	1.8792	1.6781
	10	1.2881	2.2697	3.1716	2.4263	2.1317	1.8829	1.6925
$a = 0.50$								
	05	1.7166	2.2149	2.7559	2.2286	2.0992	1.9788	1.8760
05	08	1.6274	2.6120	3.8272	2.6942	2.4228	2.1844	1.9935
	10	1.5633	2.8147	3.8252	2.9593	2.5871	2.2718	2.0304
	05	1.5558	1.9622	2.4530	1.9961	1.8890	1.7875	1.7005
10	08	1.4405	2.0399	2.6374	2.0850	1.9242	1.8179	1.6597
	10	1.3534	2.3306	3.2197	2.4008	2.1033	1.8590	1.6741

Table 4. Relative efficiency between $\hat{\sigma}_{ST_2}$ and Y_2 under ILLF for $\alpha = 0.01$.

$a = -0.50$					δ			
n_1	n_2	0.60	0.80	1.00	1.20	1.40	1.60	1.80
	05	1.6066	2.1385	2.7699	2.4378	2.3240	2.2003	2.0865
05	08	1.5492	2.5666	3.5634	3.0552	2.7567	2.4670	2.2262
	10	1.5103	2.7989	4.0536	3.4129	2.9685	2.5651	2.2513
	05	1.4730	1.8150	2.3065	2.0414	1.9729	1.8967	1.8255
10	08	1.3671	2.1024	2.8365	2.4358	2.2284	2.0290	1.8627
	10	1.2947	2.2694	3.1904	2.6717	2.3479	2.0621	1.8797
$a = -0.25$								
	05	1.5747	2.1235	2.7522	2.4189	2.3047	2.1824	2.0710
05	08	1.4984	2.5385	3.5312	3.0214	2.7253	2.4422	2.2085
	10	1.4518	2.7625	4.0103	3.3681	2.9306	2.5384	2.2347
	05	1.4610	1.8106	2.3014	2.0358	1.9672	1.8916	1.8214
10	08	1.3438	2.0897	2.8211	2.4210	2.2160	2.0204	1.8578
	10	1.2657	2.2549	3.1751	2.6562	2.3356	2.0548	1.8671

Table 4 (*continued*)

$a = 0.25$								
	05	1.5035	2.0915	2.7158	2.3814	2.2669	2.1481	2.0416
05	08	1.3901	2.4780	3.4648	2.9541	2.6645	2.3948	2.1748
	10	1.3294	2.6863	3.9258	3.2831	2.8596	2.4889	2.2047
	05	1.4348	1.8015	2.2912	2.0248	1.9560	1.8817	1.8134
10	08	1.2942	2.0632	2.7892	2.3910	2.1912	2.0034	1.8484
	10	1.2045	2.2230	3.1413	2.6236	2.3106	2.0403	1.8522
$a = 0.50$								
	05	1.4636	2.0746	2.6975	2.3628	2.2485	2.1315	2.0275
05	08	1.3328	2.4461	3.4308	2.9207	2.6349	2.3720	2.1589
	10	1.2658	2.6469	3.8845	3.2426	2.8263	2.4660	2.1912
	05	1.4204	1.7967	2.2860	2.0193	1.9505	1.8769	1.8096
10	08	1.2678	2.0493	2.7729	2.3760	2.1790	1.9951	1.8440
	10	1.1723	2.2057	3.1228	2.6066	2.2978	2.0330	1.8498

5. Recommendation

The absolute relative bias (ARB) of $\hat{\sigma}_{ST_1}$ first decreases and then increases steadily as n_1 increases for all considered values of δ for fixed α and n_2. The bias is almost negligible near δ = 100. The values of ARB of $\hat{\sigma}_{ST_1}$ decrease (increase) as $\alpha\ (n_2)$ increases for all considered values of δ except δ near 1.00.

The bias of $\hat{\sigma}_{ST_2}$ is almost negligible for all considered values of δ. The ARB of $\hat{\sigma}_{ST_2}$ decreases as n_1 increases for all δ when other parametric values are fixed but it increases as n_2 increases for all δ (except δ near 1.00). The opposite trend has been seen when $\alpha\ (\le 0.10)$ increases. The decreasing trend also has been seen as $'a'$ increases for $\delta \ge 0.80$ when other values are fixed.

5.1. *Under SELF risk criterion*

The estimator $\hat{\sigma}_{ST_i};\ i=1,2$ is more efficient than the estimator $Y_i;\ i=1,2$ respectively in the effective interval $0.60 \le \delta \le 1.80$ for all considered values of the parametric space and attains maximum efficiency at the point $\delta = 1.00$.

The relative efficiency $RE_{(S)}\left(\hat{\sigma}_{ST_1},\ Y_1\right)$ increases (decreases) as $n_2(n_1)$ increases for $\delta \ge 0.80$ when other values are fixed. In addition, the gain in efficiency decreases with increase of α when $\delta \ge 0.80$ for all considered values of the parametric space.

The relative efficiency $RE_{(S)}\left(\hat{\sigma}_{ST_2},\ Y_2\right)$ increases (decreases) as $n_2(\alpha)$ increases for $0.80 \le \delta \le 1.6$ (when other parametric values are fixed). The gain in efficiency decreases as n_1 increases. The similar trend has been seen with the increase of $'a'$ when δ is small and other parametric values are fixed.

5.2. Under ILLF risk criterion

The estimator $\hat{\sigma}_{ST_i}$; $i=1,2$ is more efficient than the estimator Y_i; $i=1,2$ respectively in the interval $\delta \geq 0.60$ for all considered values of the parametric space and attains maximum efficiency at the point $\delta = 1.00$.

The relative efficiency $RE_{(L)}\left(\hat{\sigma}_{ST_1}, Y_1\right)$ decreases for $\delta \geq 0.60$ when n_1 or α. In addition, the gain in efficiency increases when n_2 increases for $\delta \geq 0.80$. It also increases with increase of $'a'$ for $\delta \leq 1.00$ when other values are fixed.

The relative efficiency $RE_{(L)}\left(\hat{\sigma}_{ST_2}, Y_2\right)$ decreases as $'a'$ increases for all considered values of δ when $\alpha > 0.05$ and for $\delta \leq 1.00$ when $\alpha \leq 0.05$. Other properties of $\hat{\sigma}_{ST_2}$ are similar to $\hat{\sigma}_{ST_1}$.

On the basis of relative efficiencies we conclude that the estimator $\hat{\sigma}_{ST_2}$ dominates $\hat{\sigma}_{ST_1}$ for all considered values of the parametric space except small sample size and δ near 1.00 when $'a'$ is negative i.e., when overestimation is less serious than underestimation. On other hand when overestimation is more serious than underestimation, $\hat{\sigma}_{ST_2}$ is preferable over $\hat{\sigma}_{ST_1}$ for large $\delta \geq 1.20$.

References

1. J. L. Folks and R. S. Chhikara, Journal of the Royal Statistical Society, Series–B, **40**, 263 (1978).
2. J. L. Folks and R. S. Chhikara, The Inverse Gaussian distribution (Marcel Dekker, New York, 1989).
3. V. Seshadri, The Inverse Gaussian distribution. Statistical Theory and Applications, (Springer–Verlag, New York, 1998).
4. H. R Varian, A Bayesian approach to real estate assessment. In studies in Bayesian econometrics and statistics in honor of L.J. Savage, Eds S.E. Feinberg and A. Zellner (Amsterdam North Holland, 1975) p. 195.
5. A. Zellner, Journal of the American Statistical Association, **81**, 446 (1986). doi:10.2307/2289234
6. J. Rojo, Communication in Statistics – Theory and Methods, **16**, 3745 (1987).
7. G. Prakash and D. C. Singh, Austrian Journal of Statistics, **35 (4)**, 463 (2006).
8. C. P. Han and T. A. Bancroft, Journal of American Statistical Association, **63**, 1333 (1968). doi:10.2307/2285888
9. F. Mosteller, Journal of the American Statistical Association, **43**, 231 (1948). doi:10.2307/2280369
10. T. Kitagawa, University California Publication of Statistics, **3**, 147 (1963).
11. B. K. Kale and T. A. Bancroft, Biometrices, **23**, 335 (1967). doi:10.2307/2528166
12. J. Singh, Journal of American Statistical Association, **66**, 82 (1971). doi:10.2307/2284852
13. O. Rai, Communication in Statistics – Theory and Methods, **25**, 2057 (1996).
14. B. N. Pandey, B. P. Singh and A. K. Srivastava, Aligarh Journal of Statistics, **25**, 35 (2005).