JSR Publications

Available Online

J. Sci. Res. **6** (3), 457-466 (2014)

JOURNAL OF SCIENTIFIC RESEARCH

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Hall Effects on Steady MHD Heat and Mass Transfer Free Convection Flow along an Inclined Stretching Sheet with Suction and Heat Generation

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Received 8 March 2014, accepted in final revised form 30 July 2014

Abstract

The heat and mass transfer of a steady flow of an incompressible electrically conducting fluid over an inclined stretching plate under the influence of an applied uniform magnetic field with heat generation and suction and the effects of Hall current are investigated. Using suitable similarity transformations the governing boundary layer equations for momentum, thermal energy and concentration are reduced to a set of coupled ordinary differential equations which are then solved numerically by the shooting method along with Runge-Kutta fourth-fifth order integration scheme. The numerical results concerned with the velocity, secondary velocity, temperature and concentration profiles effects of various parameters on the flow fields are investigated and presented graphically. The results have possible technological applications in liquid-based systems involving stretchable materials.

Keywords: MHD; Inclined stretching sheet; Suction; Heat generation; Hall current.

doi: http://dx.doi.org/10.3329/jsr.v6i3.16903 J. Sci. Res. **6** (3), 457-466 (2014)

1. Introduction

The steady MHD boundary layer flow over an inclined stretching sheet with suction and heat generation has much interested in last few decades. The effects of magnetic field on free convective flows are of importance in liquid metals, electrolytes, and ionized gases. Due to presence of a strong magnetic field the conduction mechanism in ionized gases is different from that in a metallic substance. An electric current in ionized gases is generally carried out by electrons which undergo successive collisions with other charged or neutral particles. However, in the presence of a strong electric field, the electrical conductivity is affected by a magnetic field. Also MHD laminar boundary layer flow over a stretching sheet has noticeable applications in glass blowing, continuous casting, paper production, hot rolling, wire drawing, drawing of plastic films, metal and polymer extrusion, metal spinning and spinning of fibers. During its manufacturing process a stretched sheet

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interacts with the ambient fluid thermally and mechanically. Both the kinematics of stretching and the simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products. In the extrusion of a polymer sheet from a die, the sheet is sometime stretched. By drawing such a sheet in a viscous fluid, the rate of cooling can be controlled and the final product of the desired characteristics can be achieved. Elbashbeshy and Sedki [1] studied the effect of chemical reaction on mass transfer over a stretching surface embedded in a porous medium, Jhankal and Kumar [2] have analyzed the MHD boundary layer flow past a stretching plate with heat transfer, Bhattacharyya [3] discussed the mass transfer on a continuous flat plate moving in a parallel or reversely to a free stream in the presence of a chemical reaction, Ferdows and Qasem [4] studied the effects of order of chemical reaction on a boundary layer flow with heat and mass transfer over a linearly stretching sheet, Fadzilah *et al.* [5] analyzed the MHD boundary layer flow and heat transfer of a viscous and electrically conducting fluid over a stretching sheet with an induced magnetic field, Ibrahim and Shanker [6] studied the unsteady MHD boundary layer flow and heat transfer due to stretching sheet in the presence of heat source or sink by Quasi-linearization technique, Ahmmed and Sarker [7] considered the MHD natural convection flow of fluid from a vertical flat plate considering temperature dependent viscosity, Samad and Mohebujjaman [8] investigated the case along a vertical stretching sheet in presence of magnetic field and heat generation, Abel and Mahesh [9] presented an analytical and numerical solution for heat transfer in a steady laminar flow of an incompressible viscoelastic fluid over a stretching sheet with power-law surface temperature, including the effects of variable thermal conductivity and non-uniform heat source and radiation, Tan *et al.* [10] studied various aspects of this problem, such as the heat, mass and momentum transfer in viscous flows with or without suction or blowing, Raptis *et al.* [11] have studied the viscous flow over a non-linearly stretching sheet in the presence of a chemical reaction and magnetic field, Cortell [12] studied the magneto hydrodynamics flow of a power-law fluid over a stretching sheet, The effect of chemical reaction on free-convective flow and mass transfer of a viscous, incompressible and electrically conducting fluid over a stretching sheet was investigated by Afify [13] in the presence of transverse magnetic field, Sonth *et al.* [14] studied the heat and mass transfer in a visco-elastic fluid flow over an accelerating surface with heat source/sink and viscous dissipation, Sakiadis [15] who developed a numerical solution for the boundary layer flow field over a continuous solid surface moving with constant speed. In the present paper, we have investigated the effects of Hall current of an electrically conducting viscous incompressible fluid flow along the linearly stretching inclined sheet in the presence of heat and mass transfer as well as a uniform magnetic field which is normal to the sheet with heat generation and suction.

2. Mathematical Formulation of the Problem

Consider a two dimensional steady laminar MHD viscous incompressible electrically conducting fluid along an inclined stretching sheet with an acute angle γ. X direction is taken along the leading edge of the inclined stretching sheet and Y is normal to it and extends parallel to X-axis. A magnetic field of strength B_0 is introduced to the normal to the direction to the flow. The uniform plate temperature T_w ($>T_\infty$), where T_∞ is the temperature of the fluid far away from the plate. Let u, v and w be the velocity components along the X and Y axis and secondary velocity component along the Z axis respectively in the boundary layer region. The sketch of the physical configuration and coordinate system are shown in Fig.1.

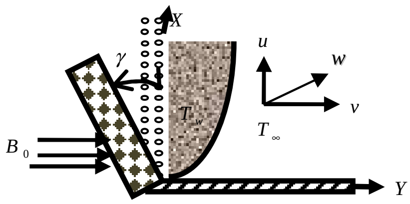

Fig. 1. Physical configuration and coordinate system.

Under the above assumptions and usual boundary layer approximation, the dimensional governing equations of continuity, momentum, energy and concentration under the influence of externally imposed magnetic field are:

Equation of continuity: [10]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)\cos\gamma+g\beta^*(C-C_\infty)\cos\gamma-\frac{\sigma B_0^2}{\rho(1+m^2)}(u+mw) \tag{2}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\nu\frac{\partial^2 w}{\partial y^2}+\frac{\sigma B_0^2}{\rho(1+m^2)}(mu-w) \tag{3}$$

Energy Equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{Q^*}{\rho c_p}(T-T_\infty),\alpha=\frac{k}{\rho c_p} \tag{4}$$

Concentration Equation:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2} \quad (5)$$

Boundary conditions are:

$$u=Ax,\ v=v_0,\ w=Ax, T=T_w, C=C_w \quad at \quad y=0$$
$$u=0, w=0, T=T_\infty, C=C_\infty \ \ as \quad y\to\infty$$

To convert the governing equations into a set of similarity equations, we introduce the following similarity transformation:

$$w=Axg_0(\eta),\ \eta=y\sqrt{\frac{A}{\nu}},\ \psi=x\sqrt{\nu A}\,f(\eta), \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \varphi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}$$

where, A (>0) is a constant, and v_0 is a velocity component at the wall having positive value to indicate suction.

By using the above transformations, the Eqs.(2) –(5) are reduced to non-dimensional, nonlinear and coupled ordinary differential equations :

$$f''' + ff'' - f'^2 + Gr\theta\cos\gamma + Gm\varphi\cos\gamma - \frac{M}{1+m^2}f' - \frac{Mm}{1+m^2}g_0 = 0 \quad (6)$$

$$g_0'' + fg_0' - f'g_0 - \frac{M}{1+m^2}g_0 + \frac{Mm}{1+m^2}f' = 0 \quad (7)$$

$$\theta'' + \Pr f\theta' + \Pr Q\theta = 0 \quad (8)$$

$$\varphi'' + S_c f\varphi' = 0 \quad (9)$$

The transform boundary conditions:

$$f=-F_{w,}\ g_0=1, f'=1, \theta=\varphi=1\ at\ \eta=0,\ f=f'=g_0=\theta=\varphi=0\ \ as\ \eta\to\infty$$

3. Results and Discussion

The ordinary differential Eqs. (6) - (9) subject to the boundary conditions are solved numerically by Runge -Kutta fourth - fifth order method. These higher order non-linear differential Eqs. (6) - (9) are converted into simultaneous linear differential equations of first order and further transformed into initial value problem by applying the shooting technique. The numerical calculation for the primary velocity, secondary velocity, temperature and concentration profiles across the boundary layer for different values of the mentioned parameters are shown graphically in Figs. 2- 21 to illustrate the influence of physical parameters viz., the suction parameter F_w, magnetic parameter M, Hall parameter *m*, heat generation parameter *Q*, Buoyancy parameter *Gr* and *Gm*, Schmidt number *Sc,* Prandlt Number *Pr*, inclination parameter γ.

The effects of *M* and *m* on the primary and secondary velocity profiles are shown in Figs. 2, 3, 9 and 10. It can be clearly seen that an increase in the magnetic parameter decreases the primary velocity and increases the secondary velocity. This result agrees with the expectations, since the magnetic field exerts a retarding effect on the free convective flow. This field may control the flow characteristics, an increase in *M* results in thinning of the boundary layer. The effect of the Hall parameter has an increasing effect on the primary velocity, whereas the negligible increasing effect on the secondary velocity profile. Again Figs. 4, 5, 8, and 11 display the effects of *Q*, γ, *Pr* and F_w. From these figures it is observed that the primary velocity is decreased for the increasing values of *Q*, γ, *Pr* and F_w but increased for increasing values of *Gm* and *Gr* as shown in Figs. 6 and 7. Again, Figs.12, 13, and 19 depict the temperature and concentration profiles for *M* and *m*. From Fig.12 it is observed that, the increasing and decreasing effect on the thermal boundary layer up to certain values of η for increasing values of *M* but reverse trend arises for *m* which is shown in Fig. 13. Again the concentration is decreased for increasing values of *M* as shown in Fig.19. Also, Figs.14 to 18 display the thermal boundary layer for various values of entering parameters. From these figures it is observed that, the temperature profile is increased up to certain interval of η and then decreased which are shown in Figs. 14, 15 and 18 for the increasing values of *Q*, γ, and *Pr* whereas reverse trend arises for the effect of *Gr* and *Gm* are shown in Figs. 16 and 17.

Fig.18 clearly demonstrates that the thermal boundary layer thickness increases as the *Pr* increases up to certain values of η implying smaller heat transfer. It is due to fact that higher values of *Pr* means decreasing thermal conductivity and therefore it is able to diffuse away from the plate more slowly than smaller values of *Pr*, hence the rate of heat transfer is reduced as a result the heat of the fluid in the boundary layer increases. The concentration profile for the effects of S_c and F_w are shown in Figs. 20 and 21. It is seen that the concentration is decreased for the increasing values of Schmidt number and suction parameter.

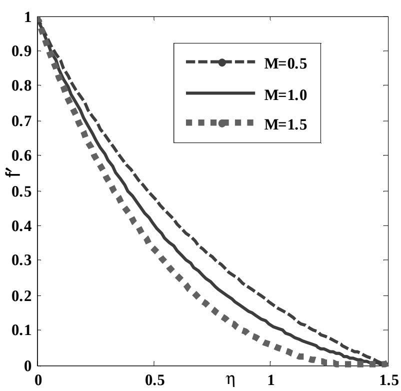

Fig. 2. Primary velocity profile for *M*.

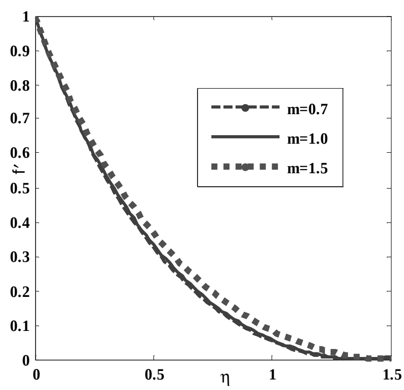

Fig. 3. Primary velocity profile for m.

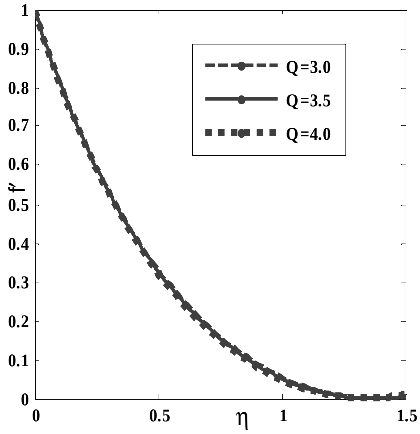

Fig. 4. Primary velocity profile for m.

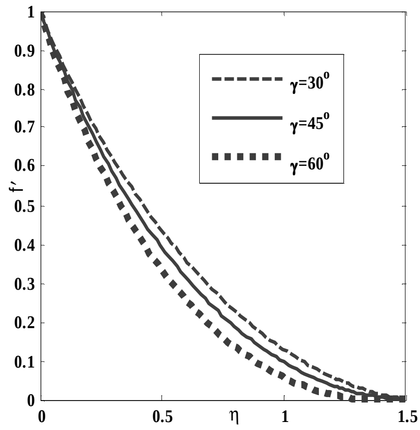

Fig. 5. Primary velocity profile for γ.

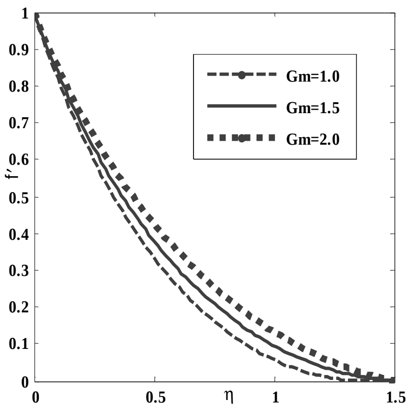

Fig. 6. Primary velocity profile for m.

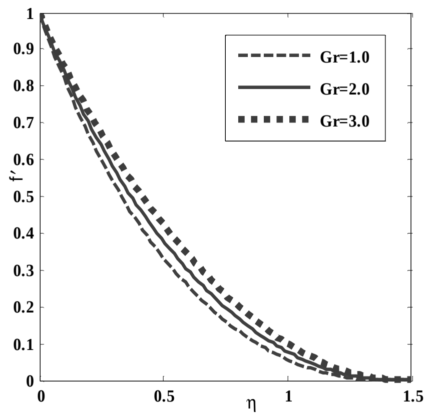

Fig. 7. Primary velocity profile for m.

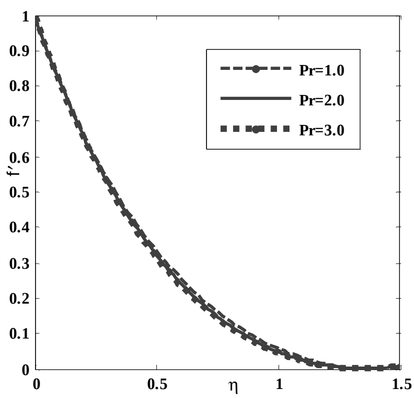

Fig. 8. Primary velocity profile for m.

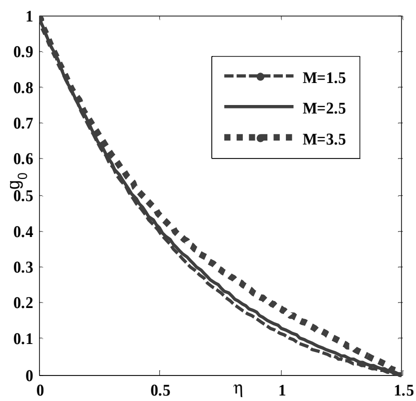

Fig. 9. Secondary velocity profile for M.

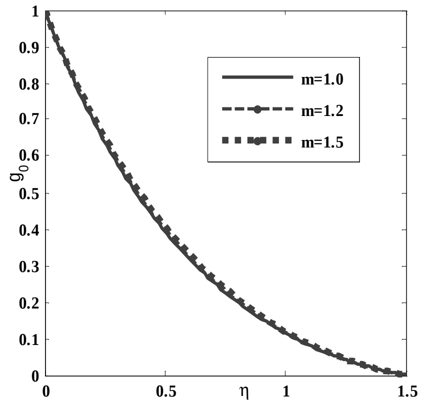

Fig. 10. Secondary velocity profile for m.

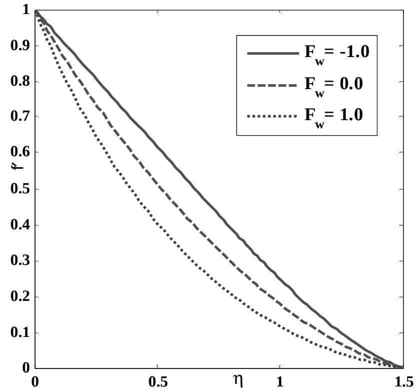

Fig. 11. Primary velocity profile for m.

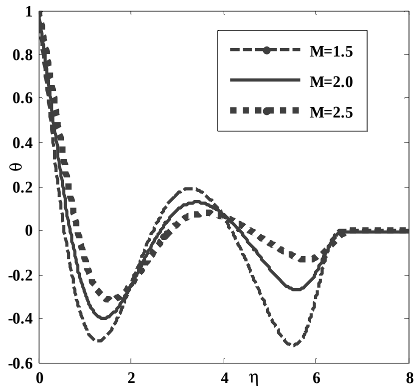

Fig. 12. Temperature profile for M.

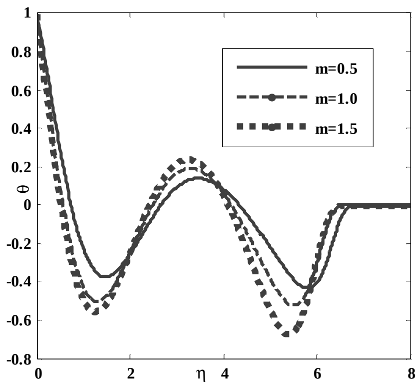

Fig. 13. Temperature profile for m.

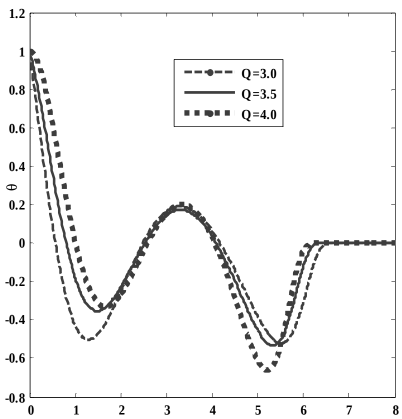

Fig. 14. Temperature profile for Q.

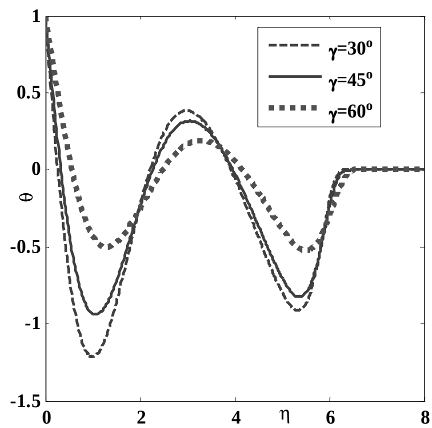

Fig. 15. Temperature profile for γ.

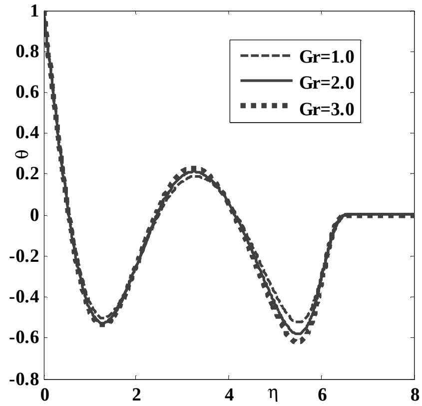

Fig. 16. Temperature profile for Gr.

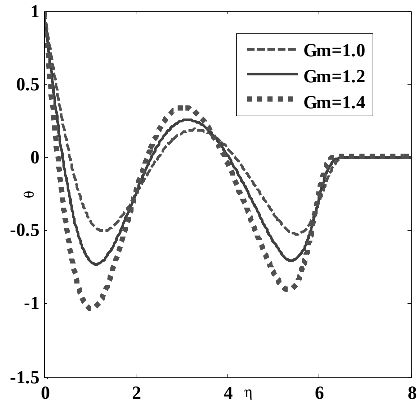

Fig. 17. Temperature profile for Gm.

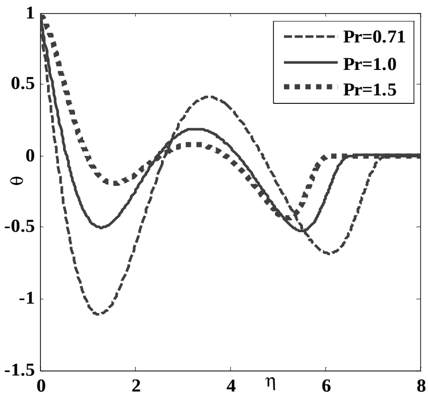

Fig. 18. Temperature profile for Pr.

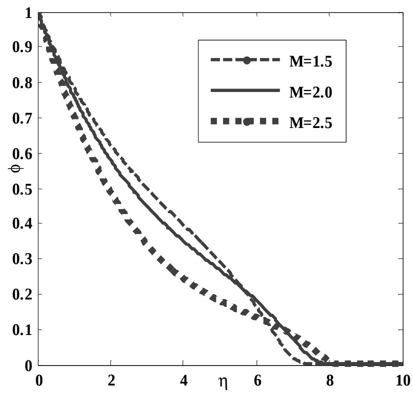

Fig. 19. Concentration profile for M.

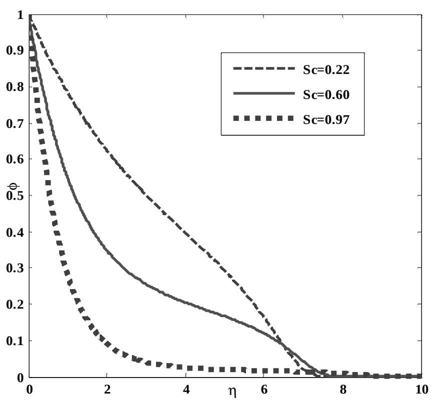

Fig. 20. Concentration profile for Sc.

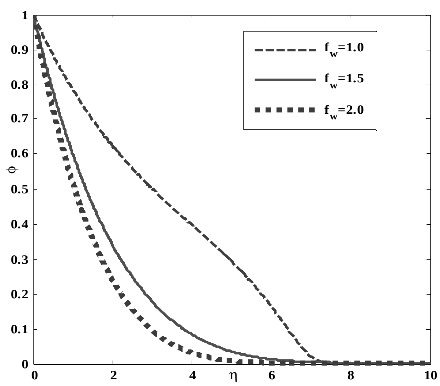

Fig. 21. Concentration profile for f_w.

5. Conclusion

Following are the conclusions made from the above analysis:

- The primary velocity decreases and secondary velocity increases with increasing magnetic parameter causing of Lorentz force The primary velocity is increased for Hall parameter and bouncy parameter but decreased for heat generation parameter, Prandtl number, suction parameter and angle of inclination.
- Thermal boundary layer is increased up to certain values of η and then decreased for the values of magnetic parameter, heat generation parameter, Prandtl number, and angle of inclination whereas reverse trend arises for Hall parameter and bouncy parameter.
- The concentration is decreased for the increasing values of Schmidt number, magnetic parameter and suction parameter.

Nomenclature

MHD	Magnetohydrodynamics
c_p	Specific heat of with constant pressure
g	Gravitational acceleration
g_0	Secondary velocity
f'	Velocity profile
M	Magnetic parameter, $M = \frac{\sigma B_0^2}{\rho A}$
m	Hall parameter
ν	Kinematic viscosity
γ	Inclination of the plate
η	Similarity variable
α	Thermal diffusivity
β	Thermal expansion coefficient
β^*	Coefficient of expansion with concentration
ρ	Density
σ	Electric conductivity
θ	Dimensionless temperature
Q	Dimensionless heat generation parameter, $Q = \frac{Q^*}{A\rho c_p}$
Q^*	Heat source parameter
S	Dummy parameter
U	Velocity component in x-direction
v	Velocity component in y-direction
v	Velocity component in y-direction
w	Velocity component in z-direction
T	Temperature
K	Thermal conductivity
D_m	Thermal molecular diffusivity
C	Concentration
C_∞	Concentration of the fluid outside the boundary layer
Pr	Prandtl number, $\Pr = \frac{\nu}{\alpha}$
S_c	Schmidt number, $S_c = \frac{\nu}{D_m}$
Gr	Buoyancy parameter, $Gr = \frac{Sg\beta(T_w - T_\infty)}{A^2 x}$
Gm	Modified buoyancy parameter, $Gm = \frac{Sg\beta^*(C_w - C_\infty)}{A^2 x}$
T_w	Temperature at the Plate
B_0	Constant magnetic field intensity
T_∞	Temperature of the fluid outside the boundary layer

Subscripts

w	Quantities at wall
∞	Quantities at the free stream

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