Journal of Naval Architecture and Marine Engineering
June, 2007
http://jname.8m.net

ULTIMATE STRENGTH OF SQUARE PLATE WITH RECTANGULAR OPENING UNDER AXIAL COMPRESSION

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Abstract

Unstiffened plates are integral part of ship structures, offshore oil platforms, lock gates and floating docks. Openings are provided in these plates for access and maintenance. Provision of opening influences the ultimate strength of plate elements. In this paper the effect of increase in the size of rectangular opening along the loading direction on the ultimate strength is determined using nonlinear finite element analysis. A general purpose finite element software ANSYS is used for carrying out the study. The software is validated for the ultimate strength of unstiffened plate under axial compression. A parametric study is done for different plate slenderness ratios and by varying the area ratio of opening to plate to determine the effect of ultimate strength on the size of rectangular opening. It is found that increase in area ratio along the loading direction decreases the ultimate strength. The variation in ultimate strength varies linearly for plate slenderness ratio less than 2.23 and varies nonlinearly for plate slenderness ratio beyond 2.23 for area ratio ranging between 0.02 – 0.18. Based on nonlinear regression analysis, a design equation is proposed for square plate with rectangular opening under axial compression.

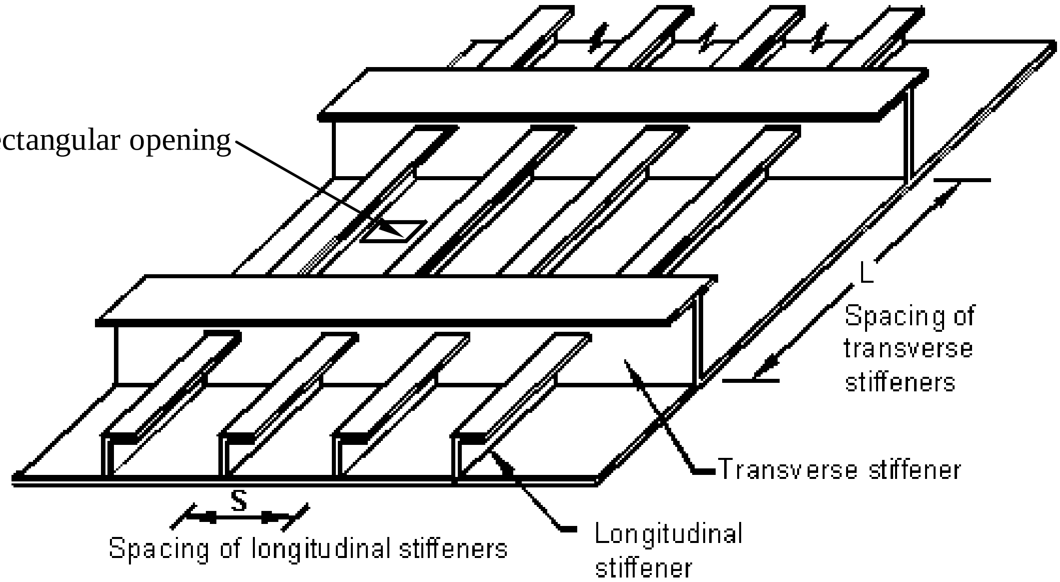
Keywords: Unstiffened Plate, Ultimate Strength, Rectangular Opening, Axial Compression, Design Equation

NOMENCLATURE

a	- Depth of Opening	t	- Thickness of Plate
b	- Width of Opening	w	- Lateral Deflection at the Centre of Plate
A	- Length of Plate	β	- Plate Slenderness Ratio
A_C	- Area of Opening	υ	- Poisson's Ratio
A_P	- Area of Plate	σ_u	- Ultimate Stress of Plate
AR	- Area Ratio (Area of Opening to Area of Plate)	σ_y	- Yield Stress of Plate
B	- Width of Plate		
E	- Young's Modulus of Elasticity		

1. Introduction

Thin plates in between stiffeners of a stiffened plate are integral part of ship structures, offshore oil platforms, lock gates and floating docks. These stiffened plates are designed to withstand the axial compression due to sagging and hogging moments. A typical ship deck in between the bulkheads and deep longitudinals is shown in Figure 1. The analysis of typical stiffened plate structure in a ship can be performed at grillage level, stiffened panel level between two adjacent transverses, and bare plate element level between longitudinal and transverse stiffeners. Local buckling and collapse of plating between is a basic failure mode and is important to evaluate the exact strength for safe design. The bending rigidities of the boundary edges of plates in between transverse frames and between longitudinal stiffeners are quite high compared to that of the plate itself. The rotational restraints along the plate edges can be considered to be small for plates subjected to axial compression. Hence, the plate elements in the present study are considered as simply supported along all the edges. Rectangular openings are necessitated for piping, ducts and other accessories for maintenance purposes. It is evident that opening reduces the ultimate strength of plates. But, the effect of rectangular opening is to be included in the ultimate strength formulations for efficient and safe design purposes.



2. I

A brief review of literature on the ultimate strength of plate elements with different openings and subjected to various loads is presented here. Bradfield (1980) presented the details of tests conducted on plates under axial compression with control on initial out-of-flatness and residual stresses. A large deflection elastic plastic analysis is carried out for tested specimens that showed good comparison. Narayanan and Chow (1984) developed design charts based on ultimate capacity of uniaxially compressed perforated plates with square and circular openings. Roberts and Azizian (1984) generated interaction curves for ultimate strength of square plates with central square and circular holes subjected to uniaxial compression, biaxial compression and pure shear. Narayanan and Chan (1985) presented design charts based on ultimate strength of plates containing circular holes under linearly varying edge displacements. Yettram and Brown (1985) studied the stability behaviour of flat square plates with central square perforations. Guedes Soares (1988) derived two design equations for merchant ships and warships which are weighted by the probability density function. Guedes Soares (1992) presented review of simple design methods for plate under uniaxial loads by incorporating partial safety factor which are applied on Ro-Ro ships, container ships and bulk carriers. Jwalamalini et al., (1992) developed the design charts for the stability of simply supported square plate with opening under in-plane loading as uniform compression and trapezoidal loading. Madasamy and Kalyanaraman (1994) presented the analysis of plated structures with rectangular cutouts and internal supports using the spline finite strip method. Motok (1997) carried out stress concentration studies on the contour of a plate opening of an arbitrary corner radius of curvature. Shanmugam (1997) reviewed the effects of openings in plate elements subjected to uniaxial compression, biaxial compression and pure shear in stiffened plates, shear webs and cold formed steel sections. Shanmugam et al., (1999) presented the design formula for axially compressed perforated plates with circular openings under axial compression for simply supported and clamped boundary conditions. Paik et al., (2001) presented ultimate strength formulations for ship plating under combined biaxial compression/tension, edge shear, and lateral pressure loads. Toulios and Caridis (2002) carried out a numerical study on the effect of aspect ratio on the buckling and collapse behaviour of flat bar stiffened plates loaded in uniaxial compression. Khaled El-Sawy et al., (2004) employed finite element method to determine the elasto-plastic buckling stress of uniaxially loaded simply supported square and rectangular plates with circular openings. The study recommended avoiding cutouts near the plate edge since this decreases considerably the critical buckling stress, especially when the failure occurs in the elasto-plastic buckling mode. Based on the literature review, it is observed that there is lack of studies on the ultimate strength of square plate with central rectangular opening along the loading direction under axial compression. Also, there is need for a design equation for the centrally located rectangular opening and that motivated the present study.

3. Numerical Study

Ultimate strength of unstiffened plate without opening is found to be maximum for an aspect ratio of A/B = 1.0 (Toulios and Caridis, 2002). So, an unstiffened plate of size 500 mm x 500 mm (A x B) is considered for the study. The thickness of plate is varied as 5 mm, 6 mm, 8 mm, 10 mm, 12 mm and 15 mm to obtain plate slenderness ratio $\left(\beta = \frac{B}{t}\sqrt{\frac{\sigma_y}{E}}\right)$ in the practical range of 1.0 - 4.5 used in ship construction. Rectangular opening is provided in the centre of the plate as shown in the Figure 2. The depth of opening (a) is kept constant as 100 mm throughout the study. The width of opening (b) is varied as 50 mm, 100 mm, 150 mm, 200 mm, 250 mm, 300 mm, 350 mm, 400 mm and 450 mm. Area ratio (AR) of rectangular opening is defined as the ratio of area of opening (A_C) to area of plate (A_P). In this study, the area ratio (AR) is varied as 0.02, 0.04, 0.06, 0.08, 0.10, 0.12, 0.14, 0.16 and 0.18. The details of the parametric study are given in Table 1. The yield strength of plate (σ_y) is assumed as 250 N/mm^2 with Young's modulus of elasticity (E) as 2 x 10^5 N/mm^2 and Poisson's ratio (υ) of 0.3. All the edges of the plate are assumed to be simply supported. The unloaded edges are allowed to deform inplane but remains straight. This is achieved by coupling the deformation of nodes in that direction. This condition is to generate the actual situation of unstiffened plate between longitudinal and transverse stiffeners. The reaction edge is constrained to obtain an equal force caused due to loading edge.

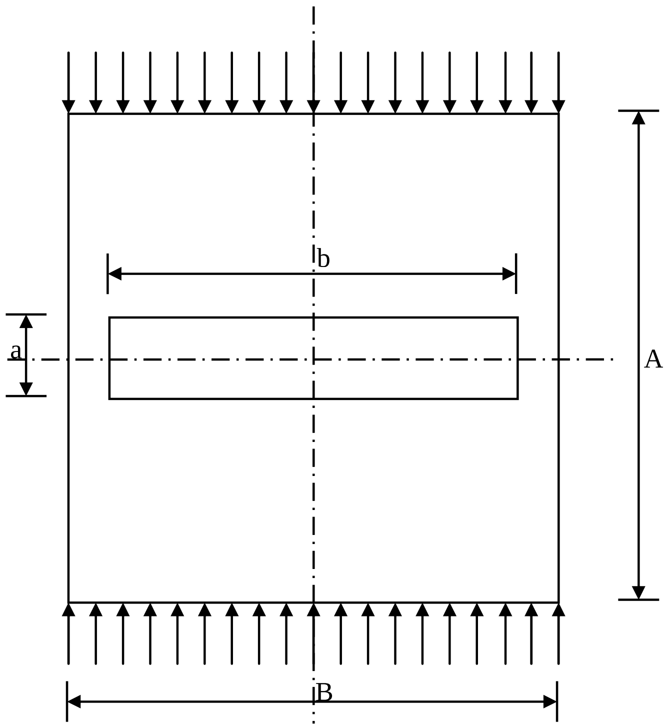


Figure 2: Unstiffened plate with centrally positioned rectangular opening

Table 1: Details of Parametric Study

Sl. No.	Specimen	Width of opening (b) mm	Thickness of plate, (t) mm	Plate Slenderness ratio, (β)	Area of Opening to Plate (AR = A_C/A_P)	Ultimate Load (P_u) kN	σ_u/σ_y
1	P1	50	5	3.54	0.02	356.64	0.57
			6	2.93		459.56	0.61
			8	2.27		719.32	0.72
			10	1.77		1117.48	0.89
			12	1.48		1373.70	0.92
			15	1.17		1717.21	0.92
2	P2	100	5	3.54	0.04	342.32	0.55
			6	2.93		441.60	0.59
			8	2.27		684.60	0.68
			10	1.77		1028.67	0.82
			12	1.48		1238.94	0.83
			15	1.17		1548.74	0.83
3	P3	150	5	3.54	0.06	325.61	0.52
			6	2.93		419.33	0.56
			8	2.27		652.74	0.65
			10	1.77		935.47	0.75
			12	1.48		1104.08	0.74
			15	1.17		1380.09	0.74
4	P4	200	5	3.54	0.08	303.45	0.49
			6	2.93		391.37	0.52
			8	2.27		622.29	0.62
			10	1.77		833.25	0.67
			12	1.48		964.21	0.64
			15	1.17		1205.31	0.64
5	P5	250	5	3.54	0.10	277.05	0.44
			6	2.93		360.09	0.48
			8	2.27		549.87	0.55
			10	1.77		674.34	0.54
			12	1.48		809.24	0.54
			15	1.17		1011.68	0.54
6	P6	300	5	3.54	0.12	247.25	0.40
			6	2.93		317.86	0.42
			8	2.27		422.28	0.42
			10	1.77		533.30	0.43
			12	1.48		639.99	0.43
			15	1.17		800.10	0.43
7	P7	350	5	3.54	0.14	206.86	0.31
			6	2.93		272.97	0.31
			8	2.27		310.21	0.31
			10	1.77		387.78	0.31
			12	1.48		465.36	0.31
			15	1.17		581.80	0.31
8	P8	400	5	3.54	0.16	153.17	0.20
			6	2.93		183.81	0.20
			8	2.27		198.96	0.20
			10	1.77		248.71	0.20
			12	1.48		298.46	0.20
			15	1.17		373.12	0.20
9	P9	450	5	3.54	0.18	75.78	0.10
			6	2.93		90.93	0.10
			8	2.27		98.58	0.10
			10	1.77		123.22	0.10
			12	1.48		147.87	0.10
			15	1.17		184.8499	0.10

4. Nonlinear Finite Element Analysis

A general purpose finite element software ANSYS is used for modeling, analysis and post processing of unstiffened plate with rectangular opening under axial compression. Modeling of unstiffened plate involves generation of a square of size 500 mm x 500 mm. To create the rectangular opening, a rectangle of size (a x b) is generated using key points and connecting it by means of area command available in preprocessor. Using the 'Subtract areas' option available in the 'Booleans' operation under the 'modeling' part, the rectangular area is deleted. Thus the geometry of an unstiffened plate with rectangular opening at the centre of the plate is developed. The lines are meshed set using the 'size controls' available with the 'mesh tool' in 'meshing' part. Four noded finite linear strain element (SHELL181) available in the ANSYS element library is used for discretisation of unstiffened plate. The element has six degrees of freedom per each node; three translations (U_X, U_Y and U_Z) and three rotations (R_X, R_Y and R_Z). This element is well suitable for analysing the linear, large rotation, and/large strain nonlinear applications. The finite element model of the square plate with rectangular opening is shown in Figure 3. Simply supported boundary conditions along all the edges of the plate are used in the analysis as shown in (Figure 3). All the nodes along the four edges of the plate are constrained for deflection and rotation along the thickness direction (U_Z, R_Z = 0). Apart from it, the reactive edge is constrained against axial deformation (U_Y = 0). All the nodes along the unloaded edges are coupled for inplane displacement (U_X) such that the displacements along the length of the plate are uniform. Both geometric and material nonlinearities are considered in the analysis. Large displacement static analysis with stress stiffening option is activated in geometric nonlinear analysis. Bilinear isotropic rate independent hardening with von Mises yield criteria is used in material nonlinear analysis.

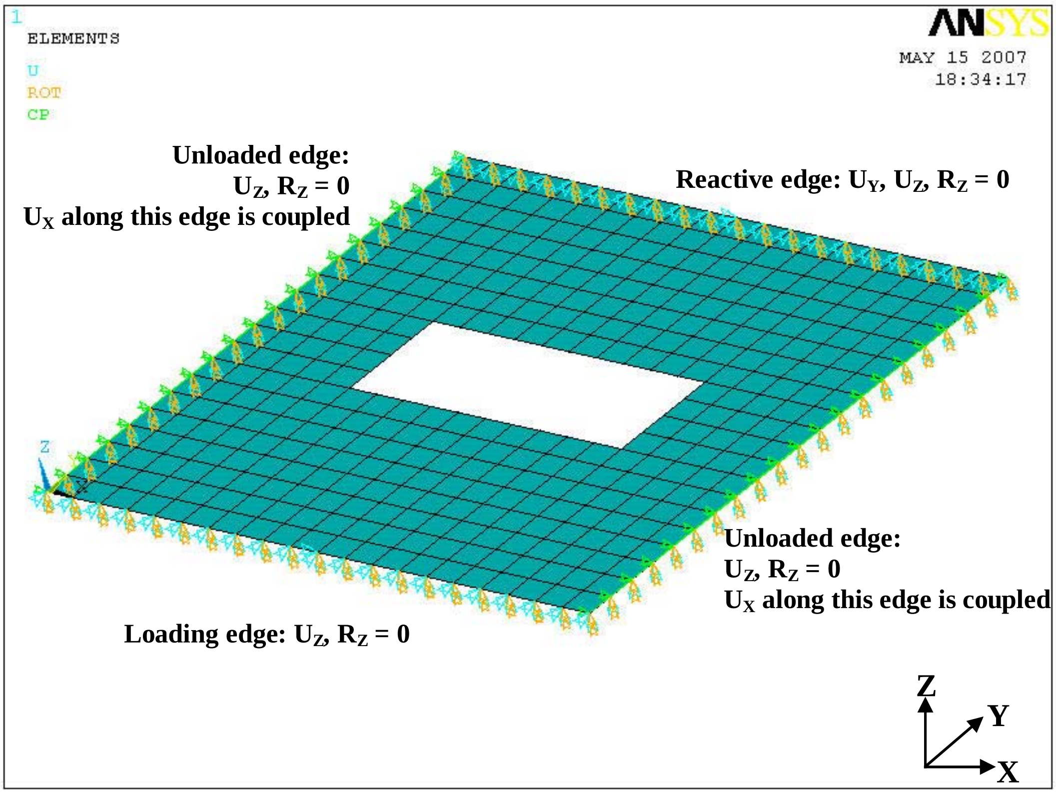


Figure 3: Finite element model of specimen P5

Equal increments of axial deformation (U_Y) of magnitude 0.1 mm are applied along the loading direction. Nonlinear equilibrium equations are solved using Newton Raphson iteration process.

Summation of axial force at all the nodes along the loading edge for every displacement increment gives the axial load acting on the specimen. Validation of the developed FE model is done with the published results of Paik et al., 2001. For this purpose, an unstiffened plate under axial compression is considered for the study. The size of the plate (A x B) is 500 mm x 500 mm. The thickness of the plate (t) is 3.2 mm. The yield strength of plate (σ_y) is considered as 264.6 N/mm^2 with Young's modulus of elasticity (E) as 2.058 x 10^5 N/mm^2 and Poisson's ratio (υ) of 0.3. Convergence study is performed to obtain an optimal element size to be used for the study. The four meshes used are coarse (5 x 5), medium (10 x 10), fine (20 x 20) and very fine (40 x 40) with the number of elements being 25, 100, 400 and 1600 respectively. The ultimate load and the CPU time for the varied meshes are given in Table 2. The results indicate a satisfactory convergence for fine mesh based on the ultimate load. Also much variation in ultimate is not observed for fine mesh with higher computing time. Thus an element of size of 25 mm x 25 mm is used for the mesh discretisation of the unstiffened plate with varied size of openings for the entire study.

Table 2: Convergence Study

Sl. No.	Mesh	No. of Elements	Ultimate Load (kN)	CPU Time (sec)
1	Coarse	25	400.11	5.86
2	Medium	100	393.66	8.19
3	Fine	400	392.93	68.22
4	Very Fine	1600	392.26	1478.98

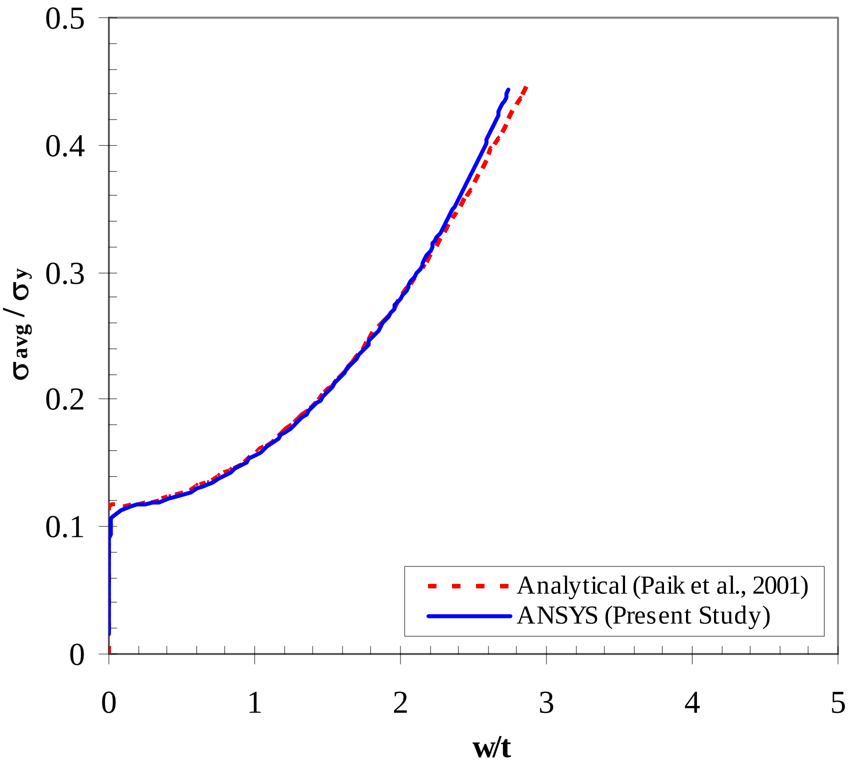


Figure 4: Load/deflection curves for unstiffened plate under axial compression

A comparison of load/deflection plot for the developed model with the published result is shown in Figure 4 and is found to be in good comparison. A typical plot for the specimen with thickness of 5 mm ($\beta = 3.54$) and varied area ratio is shown in Figure 5. Ultimate load of the specimens are determined from the peak of axial load/axial deformation plots (Figure 5). Similarly, ultimate load for the remaining specimens are obtained from axial load/axial deformation plots. The values of ultimate load for all the specimens with varied plate slenderness ratio, and area ratio are given in Table 2. The axial deformation contour, axial stress contour and von Mises equivalent stress contour for specimen P5 ($\beta = 3.54$) at ultimate load is shown in Figures 6, 7 and 8 respectively.

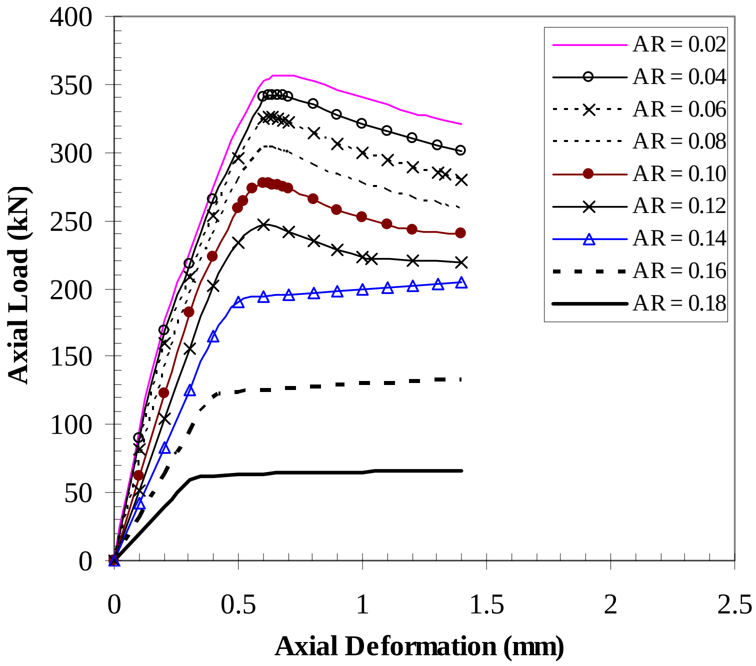


Figure 5: Axial load/axial deformation plot for specimen P5 ($\beta = 3.54$) with varied area ratio

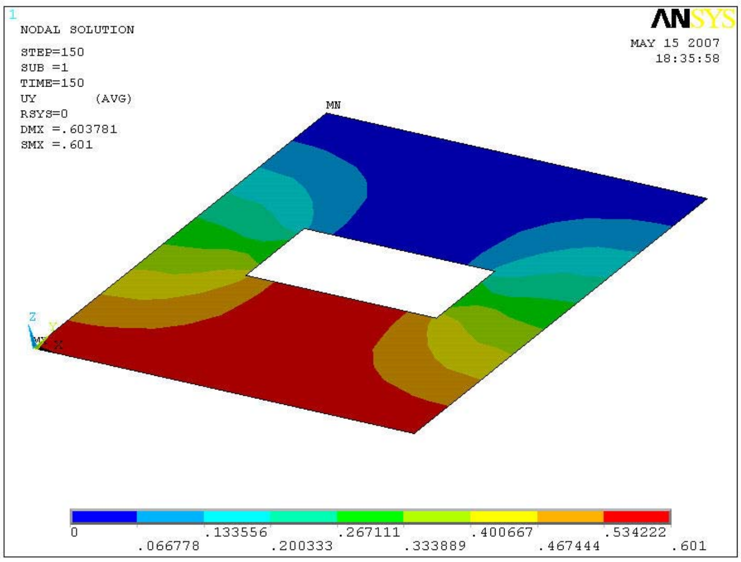


Figure 6: Axial deformation contour for specimen P5 ($\beta = 3.54$) at ultimate load

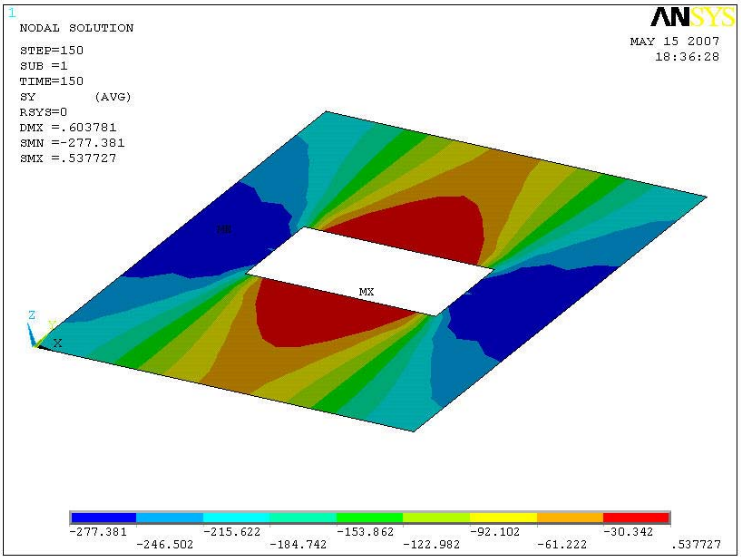


Figure 7: Axial stress contour for specimen P5 ($\beta = 3.54$) at ultimate load

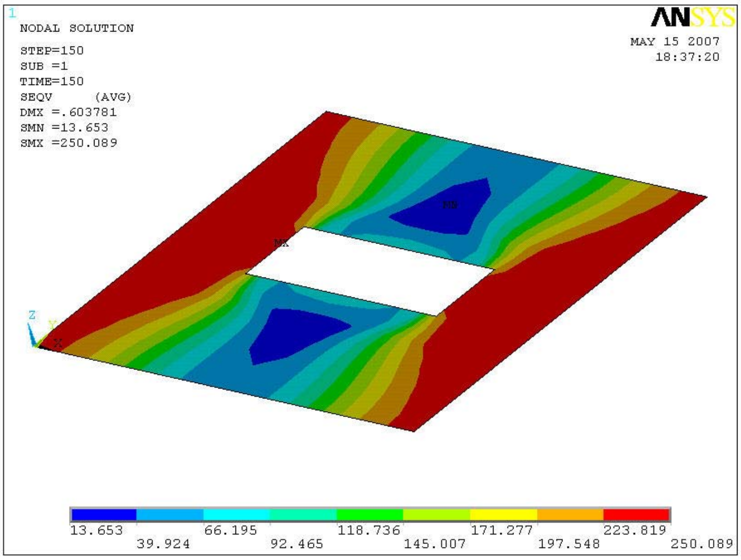


Figure 8: von Mises stress contour for specimen P5 ($\beta = 3.54$) at ultimate load

5. Results and Discussion

Design charts are prepared from the values of ultimate load obtained for all the specimens subjected to axial compression. The various charts developed are (i) effect of plate slenderness ratio on ultimate stress for varied a/b, (Figure 9) (ii) effect of area of opening to plate on ultimate stress for varied plate slenderness ratio, (Figure 10) (iii) effect of plate slenderness ratio on ultimate stress for varied b/B and (Figure 11) (iv) effect of ratio of opening on ultimate stress for varied plate slenderness ratio (Figure 12). It is observed that the specimens with rectangular opening of ratio a/b less than 0.33 for all plate slenderness ratios fail by yielding (Figure 9). Also, it is observed that for all plate slenderness ratios less than 1.77 fail by yielding irrespective of ratio of rectangular opening. The plot for σ_u/σ_y vs A_C/A_P (Figure 10) indicates that for plate slenderness ratio less than 2.23 the variation of ultimate strength varies linearly for area ratio ranging between 0.02 - 0.18, while the variation varies nonlinearly for

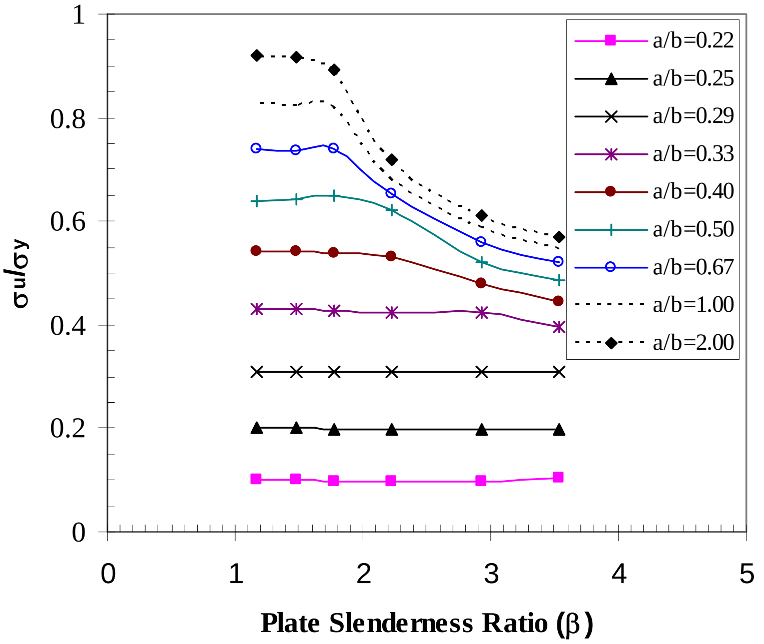


Figure 9: Effect of plate slenderness ratio on ultimate stress for varied a/b

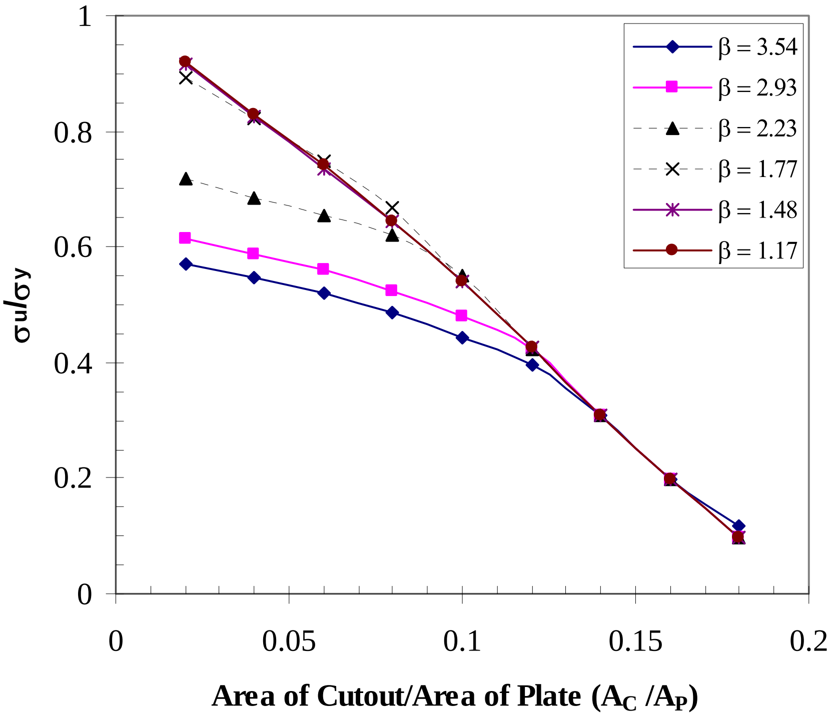


Figure 10: Effect of area of opening to plate on ultimate stress for varied plate slenderness ratio

plate slenderness ratio more than 2.23. It is also observed that for plate slenderness ratio ranging 1.17 - 3.54. From Figure 11, it is observed that the specimens with rectangular opening of ratio b/B more than 0.60 for all plate slenderness ratios fail by yielding. Also, it is found that for all plate slenderness ratios less than 1.77 fail by yielding irrespective of ratio of rectangular opening. The plot for σ_u/σ_y vs a/b (Figure 12) indicates that for plate slenderness ratio the variation of ultimate strength varies nonlinearly. It is also observed that for ratio of opening (a/b) less than 0.33 the variation of ultimate stress is linear for plate slenderness ratio ranging 1.17 - 3.54 beyond which it is nonlinear up to a/b = 1. Significant increase in ultimate strength for all plate slenderness ratios is not observed beyond a/b = 1.0.

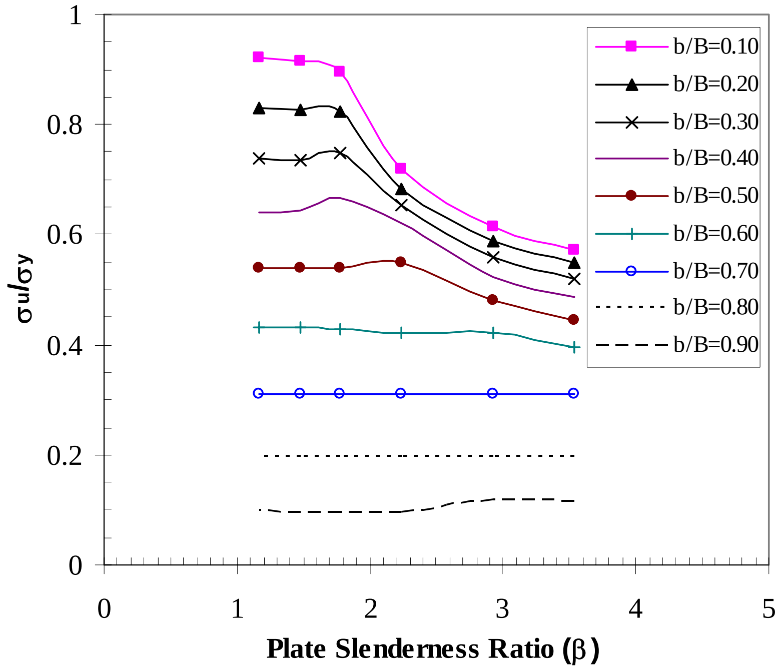


Figure 11: Effect of plate slenderness ratio on ultimate stress for varied b/B

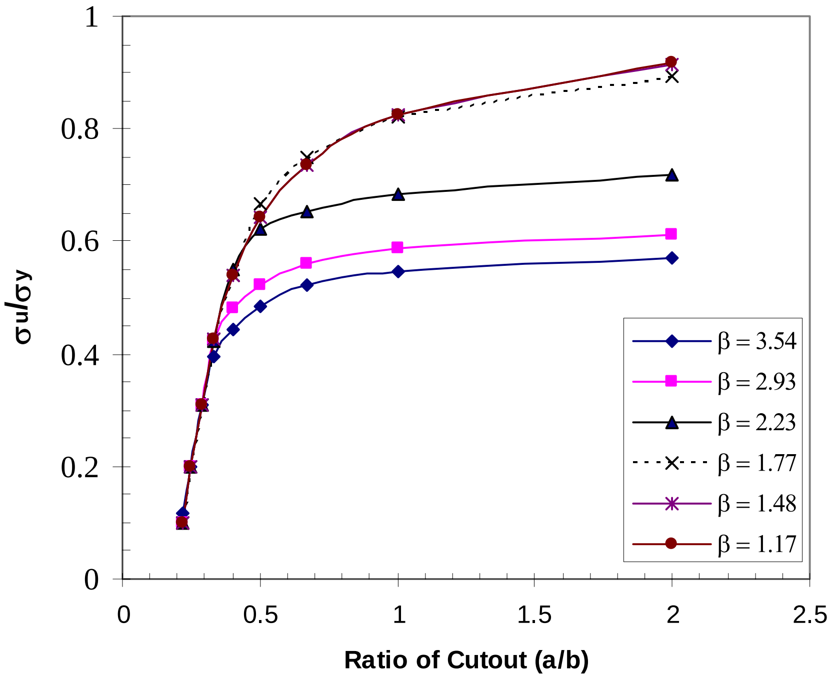


Figure 12: Effect of ratio of opening on ultimate stress for varied plate slenderness ratio

Shanmugam et al., (1999) used best fit regression analysis in developing the design formula to predict the ultimate load of square plates with centrally placed square or circular shapes and subjected to uniaxial and biaxial compression. Similar approach is employed to develop the ultimate strength design formula for unstiffened plates with rectangular openings subjected to axial load. From the parametric study, it is ascertained that interaction between the parameters like plate slenderness ratio (β), ratio of opening (a/b) and area ratio (AR) influence the strength of unstiffened plates. The effect of yield stress (σ_y) and Young's modulus of elasticity (E) of the plate are taken care in the calculation of plate slenderness ratio. The interaction of these parameters is important and there is a need for simple formulae for the design. Interaction equations are derived for the ultimate strength design of unstiffened plates subjected to axial load from the parametric studies using statistical analysis software SPSS. From the present study it is observed that relationship between plate slenderness ratio (β), ratio of opening (a/b) and area ratio (AR) on the ultimate strength of unstiffened plate under axial load varies nonlinearly. Hence nonlinear regression analysis is adopted to predict the relationship these variables. A nonlinear regression can estimate models with arbitrary relationship between the dependent variable (σ_u/σ_y) and a set of independent variables (β, a/b, AR). This is accomplished using iterative estimation algorithms. For each iteration, parameter estimates and residual sum of squares is obtained. For the assumed model, sum of squares for regression, residual, uncorrected total and corrected total, parameter estimates, asymptotic standard errors, and asymptotic correlation matrix of parameter estimates are evaluated. The best fit for any assumed relationship between dependent and independent parameters can be ensured only if R-squared [1-(Residual sum of squares/Corrected sum of squares)] value is more than 0.95. The following design equations are developed using nonlinear regression analysis based on the present study:

$$\frac{\sigma_u}{\sigma_y} = -0.137\beta - 0.029\left(\frac{a}{b}\right) - 0.147\beta^{-0.459}\left(\frac{a}{b}\right)^{-1.297} + 1.168 \qquad (1)$$

$$\frac{\sigma_u}{\sigma_y} = -0.321\beta - 7.231(AR) + 0.874\beta^{0.849}(AR)^{0.384} + 1.234 \qquad (2)$$

$$\frac{\sigma_u}{\sigma_y} = -0.321\beta - 1.446\left(\frac{b}{B}\right) + 0.471\beta^{0.849}\left(\frac{b}{B}\right)^{0.384} + 1.234 \qquad (3)$$

It is found that for the above mentioned proposed design equations (1), (2) and (3), the R-squared value is found to be 0.98006, 0.98983 and 0.98983 respectively and hence best fits the data obtained using nonlinear finite element analysis. The developed formula is simple and reliable, and can be used for the purpose of design by practical engineers.

6. Summary and Conclusions

The effect of rectangular central opening of a square plate on the ultimate strength under axial compression is found. The effect of plate slenderness ratio, area ratio of opening to plate and ratio of opening on ultimate strength is determined using nonlinear finite element analysis. The variation of ultimate strength is found to be linear for plate slenderness ratio less than 2.23 and nonlinear for plate slenderness ratio beyond 2.23 for area ratio ranging between 0.02 – 0.18. A design equation is proposed based on nonlinear regression analysis for rectangular opening under axial compression. The study has to be further extended to determine the effect of initial imperfections and residual stresses.

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