COMBINED CONVECTIVE AND VISCOUS DISSIPATION EFFECTS ON PERISTALTIC FLOW OF ELLIS FLUID IN A NON-UNIFORM TUBE

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Abstract:
This paper investigates the peristaltic transport of Ellis fluid in a non-uniform tube with combined effects of convective and viscous dissipation analytically. Mathematical modeling is carried out by utilizing long wavelength and low Reynolds number assumptions. Analysis of wall properties is also included. The expressions for velocity, temperature profile, and stream function are obtained subjected to convective boundary conditions. The peristaltic transport significant characteristics are delineated with plots for various Ellis fluid parameter values. The higher rigidity and stiffness values increased the number of streamlines due to liquid circulation in the shear-thinning example, whereas the viscous damping force parameter had the reverse trend.

Keywords: Peristaltic transport, Ellis fluid, convective boundary condition, viscous dissipation, non-uniform tube.

1. Introduction

Peristalsis is a protuberant mechanism that pushes material forward in a tube or channel bypassing fluid through an effecting contractile circle over the tube. Peristaltic motion problems are commonly used in channel/tube processes in both physiology and engineering. Swallowing food through the esophagus, the evolution of chyme in the gastrointestinal tract, ovum movement in the female fallopian tube, vasomotion of small blood vessels, urine transit from the kidney to the bladder, spermatozoa transit in the efferent ducts of the male reproductive tract. The industrial applications of peristalsis include corrosive fluid transportation and sanitary fluid carriage.

The association between shear stress and shear rate in non-Newtonian fluids is nonlinear. In recent years, non-Newtonian fluids have become nearly indispensable in medical science, chemical products, and technology. As a result, a non-Newtonian fluid in a system has become increasingly popular. The Williamson fluid model, the Casson fluid model, the power-law fluid model, the Prandtl model, the Jeffrey fluid model, the Powell-Eyring fluid model, the Rabinowitsch fluid model, and the Ellis fluid model were all presented considering the importance of the flow regime aspects of non-Newtonian fluids. The classical work of Raju and Devanathan (1972) initiated a framework to study the non-Newtonian fluids by considering the power-law model. Motivated by this, numerous researchers have worked in this direction. The important studies of recent years comprise (Nagarani and Lewis, 2012, Rathod and Mahadev, 2012, Vajarvelu et al. 2013, Sankad and Asha Patil. 2016, Ebaid et al. 2017, Devaki et al., 2018, Hayat et al. 2019, Hina Sadaf and Nadeem, 2017, Mahadev et al. 2022).

Many complex processes are involved in heat transmissions, such as analyzing skin injuries, destroying undesired cancer tissues, using a dilution approach to examine blood flow, hemodialysis processes, oxygenation, drug delivery systems, manufacturing paper, vasodilation, food processing, metabolic heat generation, and radiation between a surface and its surroundings. Blood flow may be increased when a man engages in arduous physical activity or when the body is exposed to and highly hot environment. To accommodate the increased blood flow, the artery’s diameters must be suitably enlarged. When the temperature of the atmosphere exceeds 20°C, heat transfer comes from the surface of the skin by dissipation through perspiration, and when the temperature is less than 20°C, heat in the human body declines via both conduction and radiation.
In the last few decades, the literature on peristalsis with heat transfer is considerable, and several studies on this approach may be consulted. (Nadeem et al. 2014, 2014, Akram et al. 2020, 2020, Akbar et al. 2021). Channakote et al., (2021) addressed the heat transfer influence on the peristaltic flow of Rabinowitsch fluid in a conduit with porous walls. Vajravelu et al. (2011) reported the interaction of heat transfer on the peristaltic transport of a Jeffrey fluid in a vertical porous stratum. Rathod et al. (2014) discussed the peristaltic pumping of fractional second-grade fluid with heat transfer in a cylindrical tube. Wahed et al. (2019) studied the MHD peristaltic flow of Jeffery fluid with the Joule heat effect using the multi-step differential transform method. Asha et al. (2019) addressed the peristalsis with a combined effect of hall and heat transfer on the MHD flow of Jeffrey fluid in an asymmetric vertical porous channel. Rafiq and Abbas (2021) reported the effects of viscous dissipation and heat radiation on the Rabinowitsch fluid model with wall characteristics that obey the peristaltic process.

The Ellis fluid model is one of the fluid models in which the shear stress and shear rate have a nonlinear relationship. The generalization of power-law and Bingham (viscoelastic) models is the Ellis model. Furthermore, the study of Ellis fluid model is a well-established model for describing the non-Newtonian nature of the fluid, and also it is used in the study to calculate the flow characteristics of peristaltic transport in the channel/tube. Ellis fluid has the advantage of being able to predict Newtonian behavior at low shear stresses and power-law behavior at high shear stresses. Ellis fluid and power-law connections are prominent constitutive equations for representing non-Newtonian fluids’ shear-thinning behavior. This aspect of the Ellis fluid model allows it to accurately depict the rheological properties of much polymeric fluid divergent. The application of these fluid models is very useful in predicting the rheology of several bio-fluids such as blood, cervical mucus, respiratory mucus, chime, etc.


Because of the above studies, one can perceive that no examination of the combined convective and viscous dissipation effects on peristaltic flow of Ellis fluid in a cylindrical tube with wall properties has been present in the available literature. With the aforementioned motive in mind, we intended to investigate the effects of viscous dissipation on the peristaltic transport of the Ellis fluid model in a non-uniform vertical tube. The energy equation takes into consideration the convective slip effect. To simplify the normalized constitutive equations, a long-wavelength and low Reynolds number approximation are adopted. The effects of relevant attention parameters on velocity, temperature, and streamlines for Ellis fluid prototypes are commonly encountered and deliberated using graphs. The current study qualitatively depicts the biofluid rheology of a complicated flow through a tube with a muscle wall, and it has a wide variety of applications for describing blood flow dynamics in vessels while assuming the necessary wall properties. The research also predicts physiological flow characteristics (namely blood flow) in a tube/channel with the (muscular) wall.

2. Mathematical Formulation

Consider the peristalsis of Ellis fluid in a non-uniform tube in the presence of a compliant wall for the two-dimensional flow of an incompressible fluid. The geometry of the problem is shown in fig. 1.

The mass, momentum, and energy conservation equations can be described as:

\[
\frac{1}{r} \frac{\partial (ru^*)}{\partial r} + \frac{\partial w^*}{\partial z} = 0, \tag{1}
\]

\[
\rho \left( \frac{\partial u^*}{\partial t} + u^* \frac{\partial u^*}{\partial r^*} + w^* \frac{\partial u^*}{\partial z^*} \right) = -\frac{\partial p^*}{\partial r^*} + \frac{1}{r} \frac{\partial (\tau_{11}^*)}{\partial r^*} + \frac{\partial (\tau_{13}^*)}{\partial z^*} - \frac{\tau_{22}^*}{r^*}, \tag{2}
\]

Combined convective and viscous dissipation effects on peristaltic flow of Ellis fluid in a non-uniform tube
\[
\rho \left( \frac{\partial w^*}{\partial t} + u^* \frac{\partial w^*}{\partial r} + w^* \frac{\partial w^*}{\partial z} \right) = -\frac{\partial p^*}{\partial z} + \frac{1}{r^*} \frac{\partial (r^* \tau_{13}^*)}{\partial r} + \frac{\partial \tau_{33}^*}{\partial z}, \tag{3}
\]

\[
\rho c_p \left( \frac{\partial T^*}{\partial t} + u^* \frac{\partial T^*}{\partial r} + w^* \frac{\partial T^*}{\partial z} \right) = k_1 \left( \frac{\partial^2 T^*}{\partial r^*^2} + \frac{1}{r^*} \frac{\partial T^*}{\partial r} + \frac{\partial^2 T^*}{\partial z^*^2} \right) + \tau_{11}^* \frac{\partial u^*}{\partial r} + \tau_{33}^* \frac{\partial u^*}{\partial z} + \tau_{13}^* \left( \frac{\partial w^*}{\partial r} + \frac{\partial u^*}{\partial z} \right), \tag{4}
\]

In the transverse coordinates, \( u^* \) and \( w^* \) are the velocity components, respectively, \( \rho \) is density, \( p \) is pressure, \( T^* \) is temperature, \( k_1 \) is thermal conductivity, \( c_p \) is specific heat, \( t^* \) is the time, \( \tau_{11}^*, \tau_{13}^*, \tau_{33}^*, \tau_{12}^* \) are stress tensor components.

**Fig. 1:** The geometry of the problem

The useful boundary conditions for the study concern problem are:

\[
w^* = 0, \quad \text{at} \quad H^* = a(z^*) + b \sin \frac{2\pi}{\lambda} \left( z^* - ct^* \right), \tag{5}
\]

\[
\frac{\partial w^*}{\partial r} = 0, \quad \text{at} \quad r^* = 0,
\]

where \( H^* \) is the non-uniform wave in which \( a(z^*) \) is the non-uniform radius, and \( b \) is the wave amplitude.

\[
-k \frac{\partial T^*}{\partial r} = \eta_d (T^* - T_0) \quad \text{at} \quad r^* = H^*, \quad \frac{\partial T^*}{\partial r} = 0, \quad \text{at} \quad r^* = 0. \tag{6}
\]

We introduce the following non-dimensional parameters:
where \( \delta \) is the wavenumber, \( \phi \) is the amplitude ratio, \( Ec \) is the Eckert number, and \( pr \) is the Prandtl number respectively. With the help of Eq. (7) and under the conditions of long wavelength and low Reynolds number, equations. (1)- (6) taking the following form:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( ru \right) + \frac{\partial w}{\partial z} = 0 .
\] (8)

\[
\frac{\partial p}{\partial r} = 0 ,
\] (9)

\[
\frac{\partial p}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \tau_{13} \right) .
\] (10)

\[
\left[ \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^{2} \theta}{\partial r^{2}} \right] = -Br \tau_{13} \left( \frac{\partial w}{\partial r} \right) .
\] (11)

The governing equation of motion of flexible walls may be quantified (Kumar et al. 2013, Srinivas et al. 2009).

\[
L(H^{+}) = p^{+} - p_{0} .
\] (12)

The operator characterizes the motion of the stretched membrane, such that

\[
L = -\sigma \frac{\partial^{2}}{\partial z^{2}} + m \frac{\partial^{2}}{\partial t^{2}} + C' \frac{\partial}{\partial t}^{2}\tau .
\] (13)

Furthermore, we infer that \( p_{0} = 0 \) since \( p_{0} \) reflects the pressure exerted on the outside of the wall's surface due to muscle tension. After dimensionless it becomes

\[
\frac{\partial p}{\partial z} = \frac{\partial L(h)}{\partial z} = E_{3} \frac{\partial^{3} h}{\partial z^{3}} + E_{2} \frac{\partial^{3} h}{\partial z^{2} \partial t} + E_{3} \frac{\partial^{2} h}{\partial z \partial t} = A(z,t) .
\] (14)

The component of \( \tau_{13} \) is attained by making use of Eq. (10) in the stress tensor specified in Refs. Ali et al. 2019 and Sarvana et al. 2020).

\[
\frac{\partial w}{\partial r} = \tau_{13} + \eta \left( \tau_{13} \right)^{n-1} \tau_{13} .
\] (15)

The non-dimensional form of the boundary conditions is

\[
\frac{\partial w}{\partial r} = 0 \quad \text{at} \quad r = 0 , \quad w = 0 \quad \text{at} \quad r = h .
\] (16)
\[
\frac{\partial \theta}{\partial t} + \kappa \theta = 0 \quad \text{at} \quad r = h, \quad \frac{\partial \theta}{\partial r} = 0 \quad \text{at} \quad r = 0. \tag{17}
\]
Longitudinal momentum Eq. (10) subjected to the boundary condition \(\tau_{13} = 0\) at the symmetry line \(r = 0\) yield
\[
\tau_{13} = \frac{r}{2} \frac{\partial p}{\partial z} = A(z,t). \tag{18}
\]

3. Solution Procedure

The exact solution of Eqs. (11) and (15) by imposing the boundary conditions Eq. (16) and Eq. (17) are obtained as:
\[
w = \left(r^2 - h^2\right) \frac{A}{4} + \left(r^{n+1} - h^{n+1}\right) \frac{A^n \eta}{2^n (n+1)} \tag{19}
\]
\[
\theta = \frac{BrA^2}{64} + \left(h^4 - r^4\right) + Br\left(h^{n+3} - r^{n+3}\right) \frac{A^{n+1} \eta}{2^n (n+3)^2} + \frac{Br}{\kappa} \left(\frac{h^2 A^2}{16} + \frac{h^{n+2} A^{n+1} \eta}{2^{n+1} (n+3)}\right) \tag{20}
\]
The corresponding stream function can be obtained by using the formula
\[
u = -\frac{1}{r} \frac{\partial \psi}{\partial r}, \quad w = \frac{1}{r} \frac{\partial \psi}{\partial r} \quad \text{at} \quad r = h. \tag{21}
\]
Where,
\[
A(z,t) = \phi(-2\pi)^3 \cos[2\pi(z - ct)] \left(E_1 + E_2\right) + E_3(2\pi)^2 \sin[2\pi(z - ct)]. \tag{22}
\]

4. Results and Discussion

The major goal of this study is to look into the peristaltic flow of Ellis fluid in a cylindrical tube with wall properties. The convective slip effect is taken into account in the energy equation. As a result, this section will explore the impact of various parameters on the axial velocity \(w\) and temperature \(\theta\). The results of pseudo plastic, dilatant, and Newtonian fluids, as well as other parameters, are briefly discussed. Furthermore, streamlined graphs are used to highlight the trapping phenomenon.

Figs. (2) and (3) show the deviation of the velocity profile \(w\) as the rigidity parameter \(E_1\) and stiffness parameter \(E_2\) are increased, respectively. For shear-thinning and viscous situations, values of \(E_1\) and \(E_2\) are noticeable. Because of the wall properties, less resistance is accessible to the flow, and thus velocity increases; however, the reverse behavior is represented in the case of shear thickening. Furthermore, in the case of shear thinning, the velocity profile has a bigger curve, but the curve for viscous fluid has a smaller variation. Fig. 4 illustrates the velocity profiles for the viscous damping force parameter \(E_3\). For increasing \(E_3\) values, the velocity curve gets smaller deviations and decreases for increasing values of dilatant but has the opposite behavior for pseudo-plastic and viscous fluids.

Fig. 5 and 6 show the influence of different parameters on temperature. Fig. 5 shows that as the Brinkman number \(Br\) rises, the temperature profile is an increasing function for a pseudo plastic, dilatant, and Newtonian fluid. Higher \(Br\) values, in general, suggest greater viscous dissipation, or more heat generation due to resistance in the flow induced by shear, which increases the fluid temperature.
Fig. 2: Velocity profile for various values of $E_1, \eta$ at $E_2 = 1, E_3 = 0.5, \phi = 0.01, \lambda = 0.02, \kappa = 0.24, t = 0.025, l = 0.34, n = 3$

Fig. 3: Velocity profile for various values of $E_2, \eta$ at $E_1 = 1, E_3 = 0.5, \phi = 0.01, \lambda = 0.02, \kappa = 0.24, t = 0.025, l = 0.34, n = 3$

Fig. 4: Velocity profile for various values of $E_3, \eta$

$E_1 = 1, E_2 = 0.5, \phi = 0.01, \lambda = 0.02, \kappa = 0.24, t = 0.025, l = 0.34, n = 3$. 

Fig. 6 shows the temperature profile for varying Biot number values. As the dimensionless Biot number $\kappa$ is increased, the temperature profile for all three cases decreases (Pseudo plastic, dilatant, and viscoelastic fluid). The reason for this is that as the $\kappa$ increases, the thermal conductivity declines, leading to a reduction in the temperature profile.

The temperature profile for the elasticity parameter is graphically depicted in Figs. 7-9. For all rising values of rigidity parameter $E_1$, stiffness parameter $E_2$, and viscous fluid $E_3$, the temperature profile increases. In the dilatant (pseudo plastic) case, variations for the damping force parameter are also similar. Variations in the damping force are also closer in the case of pseudo plastic.
Fig. 5: Temperature profile for various values of \( Br, \eta \) at 
\( E_1 = 1, E_2 = 1, E_3 = 0.5, t = 0.025, l = 0.24, n = 2 \) 
\( k = 2.4, z = 0.02, Br = 2, \phi = 0.01, \lambda = 0.07 \)

Fig. 6: Temperature profile for various values of \( \kappa, \eta \) at 
\( E_1 = 1.5, E_3 = 0.5, \phi = 0.01, \) 
\( \lambda = 0.07, k = 2.4, \kappa = 2, l = 0.24, Br = 2 \)

Fig. 7: Temperature profile for various values of \( E_1, \eta \) at 
\( Br = 2, E_2 = 1, E_3 = 0.5, t = 0.25, l = 0.24, n = 2 \) 
\( k = 2.4, z = 0.02, Br = 2, \phi = 0.01, \lambda = 0.07, \kappa = 2 \)

Fig. 8: Temperature profile for various values of \( E_2, \eta \) at 
\( E_1 = 1.5, E_3 = 0.5, \phi = 0.01, \) 
\( \lambda = 0.07, k = 2.4, \kappa = 2, l = 0.24, Br = 2 \)

Fig. 9: Temperature profile for various values of \( E_3, \eta \) at 
\( E_1 = 0.5, E_1 = 0.5, \phi = 0.01, \lambda = 0.02, \kappa = 0.24, t = 0.025, l = 0.34, n = 3, Br = 2 \).
Combined convective and viscous dissipation effects on peristaltic flow of Ellis fluid in a non-uniform tube

(a) $E_1 = 0.5, \eta = -0.5$
(b) $E_1 = 1, \eta = -0.5$

Fig. 10: Streamlines for $\phi = 0.03, \lambda = 0.07, l = 0.22, n = 3, t = 0.25, E_1 = 0.5, E_3 = 1.5, k = 2.4$.

(a) $E_2 = 0.5, \eta = -0.5$
(b) $E_2 = 1, \eta = -0.5$

Fig. 11: Streamlines for $\phi = 0.03, \lambda = 0.07, l = 0.22, n = 3, t = 0.25, E_1 = 0.5, E_3 = 1.5, k = 2.4$.

(a) $E_3 = 0.5, \eta = -0.5$
(b) $E_3 = 1, \eta = -0.5$

Fig. 12: Streamlines for $\phi = 0.03, \lambda = 0.07, l = 0.22, n = 3, t = 0.25, E_1 = 0.5, E_2 = 1.5, k = 2.4$.
Combined convective and viscous dissipation effects on peristaltic flow of Ellis fluid in a non-uniform tube

(a) $E_1 = 0.5, \eta = 0$

(b) $E_1 = 1, \eta = 0$

Fig.13: Streamlines for $\phi = 0.03, \lambda = 0.07, l = 0.22, n = 3, t = 0.25, E_2 = 0.5, E_3 = 1.5, k = 2.4$.

(a) $E_2 = 0.5, \eta = 0$

(b) $E_2 = 1, \eta = 0$

Fig.14: Streamlines for $\phi = 0.03, \lambda = 0.07, l = 0.22, n = 3, t = 0.25, E_1 = 0.5, E_3 = 1.5, k = 2.4$.

(a) $E_3 = 1, \eta = 0$

(b) $E_3 = 1.5, \eta = 0$

Fig.15: Streamlines for $\phi = 0.03, \lambda = 0.07, l = 0.22, n = 3, t = 0.25, E_1 = 0.5, E_2 = 1.5, k = 2.4$. 
Combined convective and viscous dissipation effects on peristaltic flow of Ellis fluid in a non-uniform tube.

(a) $E_1 = 0.5, \eta = 0.5$

(b) $E_1 = 1, \eta = 0.5$

Fig.16: Streamlines for $\phi = 0.03, \lambda = 0.07, l = 0.22, n = 3, t = 0.25, E_2 = 0.5, E_3 = 1.5, k = 2.4$.

(a) $E_2 = 0.5, \eta = 0.5$

(b) $E_2 = 1, \eta = 0.5$

Fig.17: Streamlines for $\phi = 0.03, \lambda = 0.07, l = 0.22, n = 3, t = 0.25, E_1 = 0.5, E_3 = 1.5, k = 2.4$.

(a) $E_3 = 1, \eta = 0.5$

(b) $E_3 = 1.5, \eta = 0.5$

Fig.18: Streamlines for $\phi = 0.03, \lambda = 0.07, l = 0.22, n = 3, t = 0.25, E_1 = 0.5, E_2 = 0.5, k = 2.4$.
5. Conclusion

The impact of viscous dissipation and convective analysis on the peristaltic transport of the Ellis fluid model in a non-uniform tube is investigated in this research. Analytical solutions are obtained for the velocity and temperature profile. The flow behavior is investigated using graphs. The most significant findings are summarized here.

- For pseudo-plastic and viscous liquids, the elastic nature of the walls delivers less resistance to flow, and increasing rigidity parameter $E_1$, stiffness parameter $E_2$, and viscous damping force parameter $E_3$ rises the velocity profile, but for dilatant liquids, the opposite conduct is depicted.
- The velocity profile is found to be an increasing function of the rigidity parameter, stiffness parameter, and viscous damping force parameter for shear-thinning and viscous fluids due to the lower resistance given by the walls, whereas the reverse trend is observed for shear thickening fluids.
- For shear thinning, shear thickening, and viscous fluid, the temperature profile is increasing as the Brinkman number increases. A physically larger value of $Br$ fluid temperature rises because shear in the flow generates more heat due to friction, raising the fluid temperature.
- The temperature profile declines as the Biot number increases.
- The trapping bolus width increases as the rigidity and stiffness parameters for the shear thickness scenario of fluid increase. For a viscous fluid, the trapping bolus size rises somewhat, but the quantity of trapped bolus remains constant.
- The Brinkman number is known to connect with the effects of viscous dissipation, and so it aids in rising the liquid temperature in all scenarios.

References


