Journal of Naval Architecture and Marine Engineering
June, 2021
https://doi.org/10.3329/jname.v18i1.53380 http://www.banglajol.info

MASS TRANSFER EFFECT ON VISCOUS DISSIPATIVE MHD FLOW OF NANOFLUID OVER A STRETCHING SHEET EMBEDDED IN A POROUS MEDIUM

B. C. Parida[1*], B. K. Swain[2], N. Senapati[3]
[1]Department of Mathematics, Utkal University, Bhubaneswar, Odisha, India, 751004, Email: bcparida87@gmail.com
[2]Department of Mathematics, IGIT, Sarang, Dhenkanal, Odisha, India, 759146, Email: bharatkeshari1@gmail.com
[3]Department of Mathematics, Ravenshaw University, Cuttack, Odisha, India, 753003, Email:nityananda.senapati@gmail.com

Abstract:
Present analysis elucidates the steady free convective flow of nanofluid over a stretching sheet embedded in a porous medium. Here nanofluid consists of water as base fluid and copper as nanoparticle. Mass transfer analysis with chemical reaction is the main concern of this study. Viscous dissipation is also taken into account to explore the heat transfer analysis more. The governing equations are re-modelled as a system of ordinary differential equation adopting similarity transformation and treated numerically by 4th order Runge-Kutta method along with Shooting technique. The effects of pertinent parameters such as Eckert number $(0.2 \leq Ec \leq 0.8)$, *Chemical reaction parameter* $(0 \leq K_r \leq 3)$, *Schimdt number*$(0 \leq Sc \leq 1)$, *porosity parameter* $(0.1 \leq K_p \leq 1)$, *Prandtl number* $(0.71 \leq Pr \leq 7)$, *heat source parameter* $(0 \leq Q \leq 3)$ *and magnetic field parameter* $(0 \leq M \leq 3)$ *on heat and mass transfer have been discussed thoroughly. The present results are compared with the earlier results which gives a good agreement. Some important findings are: porosity acts as aiding force whereas magnetic parameter as resistive force for fluid velocity, larger values of chemical reaction parameter result lower velocity and concentration. The study is relevant in polymer processing, food processing industries and chemical industries.*

Keywords: Chemical reaction, mass transfer, MHD, nanofluids, viscous dissipation.

NOMENCLATURE

u	velocity components along the x -direction
A	viscosity ratio
B_0	magnetic field of uniform strength
C_p	specific heat at constant pressure
C_w	specific heat at constant pressure
C_∞	species concentration far away from the plate
D	chemical molecular diffusivity
Ec	eckert number
g	acceleration due to gravity
k	thermal conductivity
K_p	porosity parameter
K_r	chemical reaction parameter
M	magnetic field parameter
N	radiation parameter
Pr	prandtl number
Q	heat source parameter
Q_0	heat generation constant
Sc	schimdt number
T	temperature of the nanofluid
T_∞	the free stream dimensional temperature
T_w	the temperature at the wall

Greek symbols

β	coefficient of volume expansion for heat transfer
ν	kinematic viscosity
α	fluid thermal diffusivity
θ	dimensionless fluid temperature
ρ	density of the fluid
σ	electrically conductivity of the fluid
η	dimensionless normal distance
ϕ	dimensionless fluid concentration
λ_1, λ_2	buoyancy parameters

 Received on: March 2020

1. Introduction

Now-a-days, there are situations in which different types of nano particles are used. These nano sized particles play a vital role in handling the various thermo physical properties of different flows involved. Most of the fluids in practical such as water, ethylene, glycol, kerosene oil, engine oil, are the poor conductor of heat. Low thermal conductivity and other thermal properties are the major factors for this. To tackle this problem and to improve the thermal properties of these flows, nano particles are added to the base fluids. The commonly used materials for nanoparticles are made up of chemically stable metals (Al, Au, Ag, Cu), oxide ceramics (Al_2O_3, CuO, TiO_2, SiO_2) etc. The choice of the base fluid-particle combination depends on the application for which the nanofluid is intended. Nanofluids with their various potential applications in industrial, engineering and biomedicine have recently attracted intensive studies [Oztop et al. (2008), Abu-Nada et al. (2010), Xuan (2000)]. Mebarek-Oudina (2018) has also discussed the convective heat transfer of Titania nanofluids filling a cylindrical annulus using ethylene glycol, engine oil and water as base fluids.

Presence of external magnetic field in various MHD flow problems is considerably vital. When those flows are considered trough a porous medium, then study becomes more alluring. These forms of engineering issues are more admissible in energy extractions, oil exploration and also the physical phenomenon management within the field of aeromechanics. So, several investigations are explored by the celebrated researchers. A computational analysis of heat transport irreversibility has been performed by Marzougui et al. (2020). The whole phenomenon is considered in a magnetized porous channel taking care of the model of generalized Brinkman-extended Darcy with the Boussinesq approximation. Makinde et al. (2016), Makinde et al. (2018) and Mutuku-Njane et al. (2013) illuminated numerically the MHD radiating fluid flow over a slippery stretching sheet embedded in a porous medium. They also considered nanofluid flow over a convectively heated permeable vertical plate in a porous medium and gave some remarkable results. Swain and Senapati (2015) inspected aftermath of mass transfer on free convective flow in a porous medium. They used Laplace transform method to get the exact results for velocity, temperature and concentration. Swain et al. (2020) conjointly administered their analysis on MHD flow and gradient heat transport of a Newtonian fluid through a Porous Medium. They analysed the effects of viscous dissipation and joule heating by solving the governing equations numerically. Mebarek-Oudina et al. (2020) have investigated the stability of natural convection in an inclined ring filled with molten potassium under the influence of a radial magnetism. Abo-Dahab et.al. (2021) have investigated MHD casson nanofluid flow over nonlinearly heated porous medium under transversal magnetized field and along with provision of suction/injection to surface. Reddy et al. (2017) elucidated MHD boundary layer flow of nanofluid and heat transfer over a porous exponentially stretching sheet. They studied the effects of Brownian motion and thermophoresis on heat transfer and nano-particle volume fraction on mass transfer.

Furthermore, due to the action of the shear forces, viscous dissipation process transforms the kinetic energy to internal energy which helps in heating up the fluid. So, viscous dissipation has significant effect on heat transfer and therefore on flow velocity as well. Thus, it cannot be neglected. Applications of viscous dissipation may be observed in polymer processing flows, Aerodynamic heating in thin boundary layer around high speed aircraft etc. Many researchers (Chen, 2004, Abo-Eldahab et al., 2005, Bhargava and Singh, 2012, Parida et al., 2020) have nicely elucidated the behavior of dissipative flows in their researches under different circumstances. Chemical reaction is also of considerable importance in fluid flow along with the combined effect of heat and mass transfer. These types of studies have various applications in industrial processes and also in physiological flows. Reaction is said to be homogeneous when it occurs uniformly through a given phase. Otherwise, reaction is called heterogeneous. Again, chemical reaction is of first order if rate of reaction is directly proportional to the concentration. Many researchers have been discussed about this topic. Chamber and Young (1958) described the effects of homogeneous 1st order chemical reactions in the neighborhood of a flat plate for destructive and generative reactions. Muthucumaraswamy et al. (2008) and Swain et al. (2014) have studied the effect of chemical reaction in MHD boundary layer flow past a vertical plate. Ahmed (2014), Ahmed and Kalita (2014) solved numerically the Magnetohydrodynamics flow problems in the presence of chemical reaction. In both the studies, flow over an impulsively started vertical plate has been considered. Swain (2021) has also shown his interest on solving the problem of second order chemical reaction effect on MHD convective flow. He explained the consequences of the second order chemical reaction after solving the pertinent equations by finite difference method. Goud et al. (2020) have explored MHD Casson nanofluid flow in the presence of chemical reaction through a nonlinear inclined porous stretching sheet analyzing thermal radiation and joule heating effects.

In practical applications, the process of heating and cooling of fluid is very sensitive and sometimes requires very quick response of heating as well as cooling. Clear fluid like water and ethylene glycol fails this quick response property to heating and cooling due to which the nanoparticles with relatively high thermal conductivity such as Cu, Al_2O_3, and TiO_2 are required to be suspended into the base fluid. The term 'nanofluid' was first coined by Choi and Eastman (1997) and a number of experimental as well as numerical studies have been conducted by considering the thermal conductivity of nanofluids till date. Numerical study of mixed convection of Cu-water nanofluid in a two-sided lid-driven square enclosure is conducted by Malik and Nayak (2016) using two discrete heat sources along the bottom wall. They concluded that the heat transfer rate is sensitive to nanoparticle volume fraction and it increases with increase in nanoparticle volume fraction. The effect of magnetic field on heat and fluid flow in a CuO-water nanofluid filled enclosure heated from below is studied by Sheikholeslami et al. (2014). They found that heat transfer is increasing with the increment of heat source length and Hartmann number, and decreases with increment in Rayleigh number. Kumar and Kumar (2017) presented a comparative study of MHD flow of nanofluid and heat transfer over a stretching sheet considering the thermal radiation effects. They reported that velocity and temperature of the nanofluid decrease with increasing of radiation parameter.

From above discussion and literature review, it is noticed that MHD flow through a porous medium significantly affects the heat and mass transfer. It has drawn the interest of several researchers because of its importance in many industrial applications such as MHD power generator designing, control of chemical waste and pollutants dissemination. Further, viscous dissipation viscous dissipation alters the temperature distributions which affect heat transfer rates. The merit of the consequence of viscous dissipation depends on whether the sheet is being cooled or heated. On the other hand, it is needed for paying importance to the presence of chemical reaction during various physiological functions and in biochemical industries. The above discussion motivates the authors to analyze the mass transfer effect on MHD flow of nanofluid through a porous medium.

The objective of the paper is to study the mass transfer effect on the MHD flow of nanofluid over a stretching sheet embedded in a porous regime incorporating viscous dissipation and chemical reaction. Here, water and copper are taken as the base fluid and nano particle respectively. The governing equations are solved by shooting technique along with Runge-Kutta method of 4th order. The results are in good agreement with earlier published results Kumar and Kumar (2017).

The novelties of the study are given as follows:

(i) it gives the mass transfer analysis of nanofluid in presence of chemical reaction which was not considered in previous study Kumar and Kumar (2017).
(ii) Heat transfer is analyzed incorporating viscous dissipation which has a significant contribution.
(iii) The flow is considered through a porous medium. As a result, velocity is significantly affected by the corresponding porosity parameter K_p.

Study of the effect of higher order chemical reaction and entropy generation may be taken care of as future works.

2. Mathematical Formulation

The physical model and geometry of the problem are shown in Fig. 1. Here, we consider the two-dimensional steady MHD boundary layer flow of an incompressible nanofluid over a stretching sheet in the presence of a uniform transverse magnetic field. It is considered that the magnetic Reynolds number insignificant in the free convection flow, hence the induced magnetic field is ignored. The x -axis is taken along the stretching surface in the upward direction, y - axis is normal to it. A uniform magnetic field of strength B_0 is applied normal to the fluid flow direction. The nanofluid consist of water as base fluid and copper (Cu) as nanoparticle. It is assumed that the base fluid and the nanoparticle are in thermal equilibrium and no slip occurs between them. The thermophysical properties of water and copper are given in Table1.1.

The flow is governed by the following equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

Table 1.1 Thermophysical properties of water and copper at T=300K (Hakan and Eiyad (2008), Sekulic et al. (2005)).

Physical properties	**Water**	**Cu**
$\rho(kg/m^3)$	997.1	8933
$C_p(J/kg.K)$	4179	385
$k(W/m.K)$K)	0.613	401
$\beta * 10^5 K^{-1}$)	21	1.67
$\mu_f(Ns/m^2)$	0.001003	-
d_f or d_s (nm)	0.24	30-60

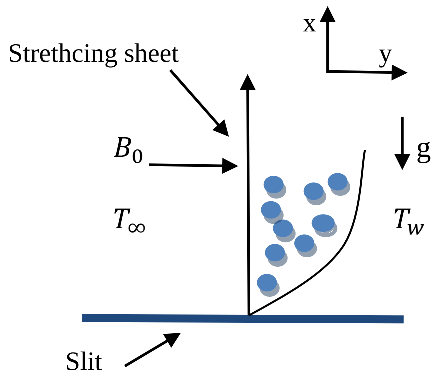

Fig. 1: Physical flow

$$\rho_{nf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu_{nf}\frac{\partial^2 u}{\partial y^2}+g(\rho\beta)_{nf}(T-T_\infty)+g(\rho\beta)_{nf}(C-C_\infty)-\sigma {B_0}^2 u-\mu_{nf}\frac{u}{K'}, \tag{2}$$

$$(\rho C_p)_{nf}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k_{nf}\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}+Q_0(T-T_\infty)+\mu_{nf}(\frac{\partial u}{\partial y})^2, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-K_c(C-C_\infty). \tag{4}$$

The boundary conditions of equations (1) to (3) are as follows

$$u=u_w(x)=ax, v=0, T=T_w, C=C_w \quad at\ \ y=0,$$
$$u\to 0,\ T\to T_\infty, C\to C_w \quad as\ \ y\to\ \infty. \tag{5}$$

u is the velocity components along the x -direction, T the temperature of the nanofluid, μ_{nf} the dynamic viscosity of the nanofluid, β_{nf} the thermal expansion coefficient of the nanofluid, $\rho_{nf}$the density of the nanofluid, k_{nf} the thermal conductivity of the nanofluid, $(\rho C_p)_{nf}$ theheat capacitance of the nanofluid, Q_0 heat generation constant, g the acceleration due to gravity and q_r the radiative heat flux. In Ref. (Choi (2001), Pak and Young (1998), Govindaraju et al. (2015)) these are given as:

$$\mu_{nf}=\mu_f(1+39.11S+533.9S^2), \rho_{nf}=(1-S)\rho_f+S\rho_s, (\rho C_p)_{nf}=(1-S)(\rho C_p)_f+S(\rho C_p)_s,$$
$$(\rho\beta)_{nf}=(1-S)(\rho\beta)_f+S(\rho\beta)_s. \tag{6}$$

where, S is the solid volume fraction of the nanoparticle. The effective thermal conductivity of nanofluid is calculated by Patel et al. (2005) model as follows:

$$\frac{k_{nf}}{k_f}=1+\frac{k_s A_s}{k_f A_f}+cP_e\frac{k_s A_s}{k_f A_f}, \frac{A_s}{A_f}=\frac{d_f}{d_s}\frac{S}{1-S}, P_e=\frac{u_s d_s}{\alpha_f}, u_s=\frac{2k_b T}{\pi\mu_f {d_s}^2}, c=25000,$$

where,ρ_f the density of the base fluid,$\rho_s$the density of the nanoparticle,$k_f$the thermal conductivity of the base fluid, k_s the thermal conductivity of the nanoparticle, P_e the Peclet number, α_f the thermal diffusivity of liquid, μ_f the viscosity of the base fluid, $(\rho C_p)_f$ the heat capacitance of the base fluid and $(\rho C_p)_s$ the heat capacitance of nanoparticles and c is constant.

The radiative heat flux for an optically thick fluid can be found from Rosseland (1931) approximation and its formula is derived from the diffusion concept of radiative heat transfer given by; $q_r=\frac{-4\sigma^*}{{k_{nf}}^*}\frac{\partial T^4}{\partial y}$,

where $\sigma^*(=\ 5.67\times 10^{-8}\ W/m^2K^4)$, ${k_{nf}}^*(m^{-1})$ states 'Stefan-Boltzmann' constant, the Rosseland mean absorption coefficient respectively. It is assumed that due to variation in temperature in the fluid flow domain are adequately very small and that T^4 may well derive as a linear function of temperature. Hence, this attained by elucidating T^4 in a Taylor series about T_∞, thus:

$$T^4=T_\infty^4+4T_\infty^3(T-T_\infty)+6T_\infty^2(T-T_\infty)^2+\cdots. \tag{7}$$

Ignoring higher order terms in Eq. (6) beyond the first order in $(T-T_\infty)$, we get

$$T^4\cong T_\infty^4+4T_\infty^3(T-T_\infty),$$
$$T^4\cong T_\infty^4+4TT_\infty^3-4T_\infty^4,$$
$$T^4\cong 4TT_\infty^3-3T_\infty^4.$$

Considering the above equation, Eq. (3) becomes

$$\left(\rho C_p\right)_{nf}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\left(k_{nf}+\frac{16\sigma^{*}T_{\infty}^{3}}{3k_{nf}{}^{*}}\right)\frac{\partial^2 T}{\partial y^2}-Q_0(T-T_{\infty})+\mu_{nf}(\frac{\partial u}{\partial y})^2 \quad . \tag{8}$$

Introducing the following similarity variables,

$$u=axf'(\eta),\ v=-\sqrt{av_f}f(\eta),\ \theta(\eta)=\frac{T-T_{\infty}}{T_w-T_{\infty}},\ \eta=\sqrt{a/v_f}\,y,\ ,\ \phi=\frac{C-C_{\infty}}{C_w-C_{\infty}},$$

the equations (2), (4) and (7) reduce to the dimensionless equations as;

$$f'''-Aa_1((f')^2-ff'')-\left(M+\frac{1}{K_p}\right)Af'+\lambda_1 Aa_2\theta+\lambda_2 Aa_2\phi=0, \tag{9}$$

$$\theta''+\frac{3NP_rk_fa_3}{k_{nf}(3N+4)}f\theta'+\frac{3NP_rk_fQ}{k_{nf}(3N+4)}\theta+\frac{1}{A}Ec\frac{3NP_rk_f}{k_{nf}(3N+4)}f''^2=0, \tag{10}$$

$$\frac{1}{Sc}\phi''+f\phi'-K_r\phi=0. \tag{11}$$

The corresponding boundary conditions become

$$f=0\ ,f'=1\ ,\quad \theta=1,\phi=1 \text{ at } \eta=0,$$
$$f'\rightarrow\infty\ ,\theta=00,\phi=0\ \text{ as}\eta\rightarrow\infty. \tag{12}$$

where $a_1=\left[(1-S)+S\left(\frac{\rho_s}{\rho_f}\right)\right],\ a_2=\left[(1-S)+S\frac{(\rho\beta)_s}{(\rho\beta)_f}\right],\ a_3=\left[(1-S)+S\frac{(\rho C_p)_s}{(\rho C_p)_f}\right].$

The dimensionless constants appearing in equations (9)-(11) are defined as follows:

$M=\frac{\sigma B_0^2}{a\rho_f}$, $\lambda_1=\frac{g\beta_f((T-T_{\infty}))}{au_w}$,$\lambda_2=\frac{g\beta_c((T-T_{\infty}))}{au_w}$,$A=\frac{\mu_f}{\mu_{nf}}$, $Pr=\frac{\nu_f(\rho C_p)_f}{k_f}$, $Q=\frac{Q_0}{a(\rho C_p)_f}$,$Ec=\frac{a^2x^2}{C_p(T_w-T_{\infty})}$,$N=\frac{k_{nf}k_{nf}^{*}}{4\sigma^{*}T_{\infty}^{3}}$, $Sc=\frac{\nu_f}{D}$, $K_r=\frac{K_c}{a}$. $K_p=\frac{\rho_f K' a}{\mu_{nf}}$.

Skin-friction: From velocity field, we study the skin-friction which is given in dimensionless form as follows:

$$C_fRe_x^{\frac{1}{2}}=-\frac{1}{A}\left[\frac{\partial u}{\partial\eta}\right]_{\eta=0}=-\frac{1}{A}f''(0). \tag{13}$$

Nusselt Number: From temperature field, we study the Nusselt number which is given in dimensionless form as follows:

$$Nu_xRe_x^{-\frac{1}{2}}=-\frac{k_{nf}}{k_f}\left[\frac{\partial\theta}{\partial\eta}\right]_{\eta=0}=-\frac{k_{nf}}{k_f}\theta'(0). \tag{14}$$

3. Solution of the Problem

The coupled nonlinear equations (9), (10) and (11) subject to the boundary conditions (12) are solved using the fourth order Runge-Kutta method with shooting technique. Equations (9), (10) and (11) are reduced to a set of first order differential equations. For this we make the following substitutions:

$$f=y_1,f'=y_2,f''=y_3,\theta=y_4,\theta'=y_5,\phi=y_6,\phi'=y_7.$$

Now the equations reduced to;

$$y_3{}'=(y_2{}^2-y_1y_3)Aa_1+\left(M+\frac{1}{K_p}\right)Ay_2-A\lambda_1a_2y_4-A\lambda_2a_2y_6, \tag{15}$$

$$y_5{}'=-\frac{3NPrk_f}{k_{nf}(3N+4)}y_1y_5-\frac{3NPrk_f}{k_{nf}(3N+4)}Qy_4-\frac{3NPrk_f}{k_{nf}(3N+4)}\frac{Ec}{A}y_3{}^2 \quad , \tag{16}$$

$$y_7{}'=-Scy_1y_7+ScK_ry_6. \tag{17}$$

The corresponding boundary conditions reduces to

$$y_1=0,y_2=?,y_4=1,y_6=?\ at\ \eta=0\ ,$$
$$y_2\rightarrow\infty\ ,y_4\rightarrow 0,y_6\rightarrow 0\text{as }\eta\rightarrow\infty. \tag{18}$$

To start the integration, the values of y_2 and y_6 at $\eta = 0$ are provided as guess values and the step by step integration is carried out with step length 0.01 using shooting technique with MATLAB code having error bound 10^{-3}.

Table 3.1: Comparison between the present solutions for various values of S with previously published results when Pr= 6.2, N=Q = $\lambda_1 = \lambda_2 = 0$.

M	S	**Hamad (2011) (Cu-water)**		**Kumar and Kumar (2017) (Cu-water)**		**Present Results (Cu-water)**	
		$-f''(0)$	$-\theta'(0)$	$-f''(0)$	$-\theta'(0)$	$-f''(0)$	$-\theta'(0)$
1	0.05	1.4524	1.5237	1.45243	1.5236	1.45258	1.52468
	0.1	1.4657	1.3884	1.45842	1.3882	1.46601	1.38738
	0.2	1.4331	1.1670	1.43301	1.1653	1.43427	1.43201

4. Results and Discussion

Here, we study the mass transfer effect on the MHD flow of nanofluid over a stretching sheet embedded in a porous regime incorporating viscous dissipation and chemical reaction. The governing equations are solved by shooting technique along with Runge-Kutta method of 4th order. The effects of pertinent parameters are discussed below by fixing the other parameters considering the values as Pr=24.4, M=1, Q=2, N=0.2, S=0.2, Ec=0.2, Sc=0.2, Kp=0.5, Kr=1. The results we found are compare with the previous results and shown in the Table 3.1.

It is observed in Fig. 2 that an increase in the Schmidt number suppresses the velocity. It is the ratio of momentum diffusivity to species diffusivity. For $Sc < 1$, the momentum diffusivity is lower than the species diffusivity and the species diffusion rate exceeds the momentum diffusion rate. For Sc >1, the reverse phenomenon is obtained. Increasing Sc lowers the chemical molecular diffusivity of the species. As Sc is increased the concentration boundary layer will become relatively thinner than the momentum boundary layer. Thus, velocity is reduced.

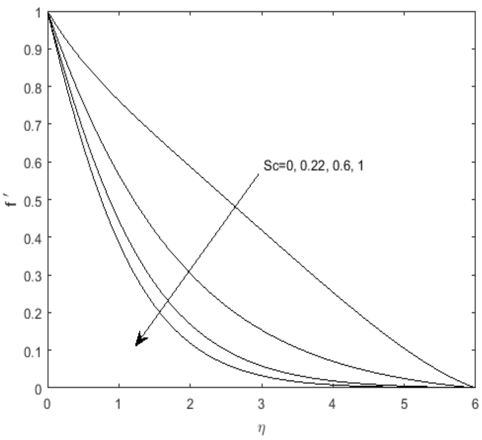

Fig. 2: Velocity profiles for Sc

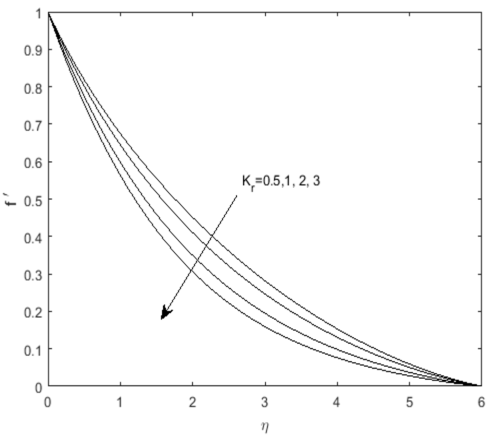

Fig. 3: Velocity profiles for K_r

From Fig. 3, it observed that that an increasing value of K_r reduce the velocity. The decrease may be attributed to the absorption of heat energy due to endothermic chemical reaction (K_r>0). The presence of heavier diffusing species causes decreases in velocity which leads to thinning of boundary layer thickness.

Fig. 4 shows that increasing values of K_p enhances the velocity. This is because with a rise in permeability of the medium, the regime becomes more porous. As a consequence, the Darcian body force decreases the magnitude (as it is increasing proportional to the permeability). The Darcian resistance acts to decelerate the fluid particles in continuous. This resistance diminishes as permeability of the medium increases. So progressively less drag is experienced by the flow and flow retardation is there by decreased. Hence, the velocity of the fluid increases as the permeability parameter increases.

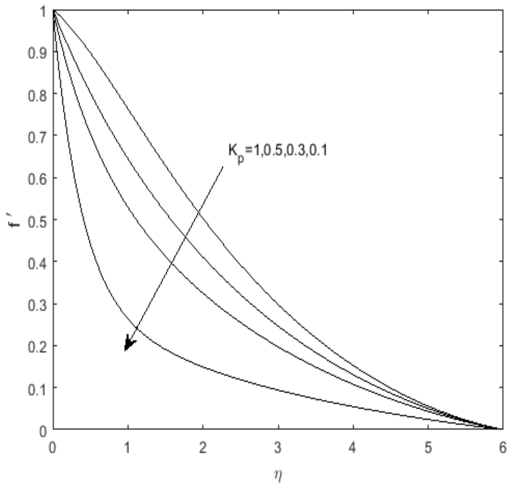

Fig. 4: Velocity profiles for K_p

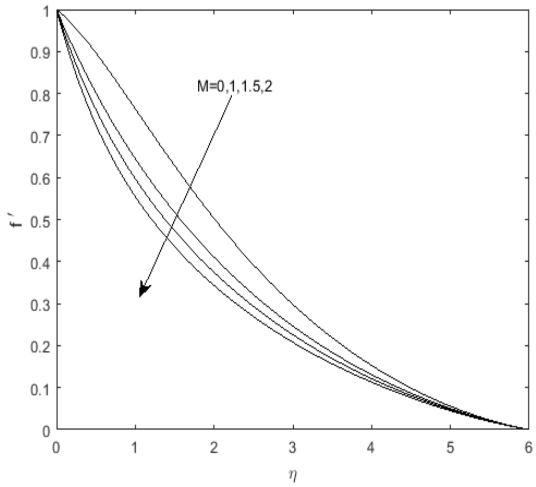

Fig. 5: Velocity profiles for M

Fig. 5 elucidates that the dimensionless velocity decreases with increasing values of M, this happens due to Lorentz force arising from the interaction of magnetic and electric fluids during the motion of the electrically conducting fluid. This force acts on the fluid creating a dry-like effect that slows down the motion of the fluid in the boundary layer.

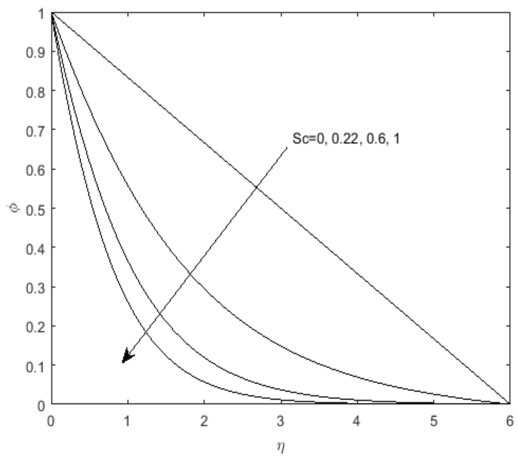

Fig. 6: Concentration profiles for *Sc*

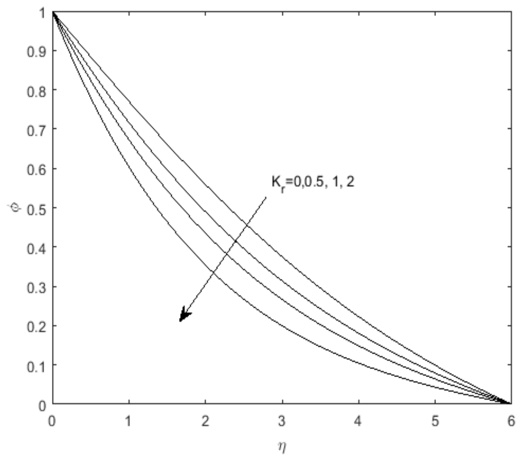

Fig. 7: Concentration profiles for K_r

The Schmidt number (*Sc*) represents the relative case of the molecular momentum and mass transfer. It is observed in the figure.6 that an increase in the Schmidt number suppresses the concentration boundary layer thickness which is associated with the reduction in the concentration. Physically, the increase of *Sc* means decrease of molecular diffusion. Hence, the concentration of the species is higher for smaller values of *Sc* and lower for larger values of *Sc*.

Fig. 7 shows the increasing values of K_r decreases species concentration in the boundary layer, this is due to the fact that destructive chemical reduces the solutal boundary layer thickness and increases the mass transfer. Physical point of view chemical reaction for destructive case is very large. Because of this fact molecular motion is quite higher which enhances the transport phenomenon, thus suppressing the concentration field in the fluid flow.

Fig. 8 shows the temperature profiles for various values of Eckert number (Ec). It is observed that the temperature rises for larger values of Ec due to viscous heating. Thermal boundary layer thickness is also enhanced by increasing Ec. As Eckert number comes from kinetic energy of flow and heat enthalpy difference, so improve in Eckert number enhances kinetic energy. Again, it is known that temperature is considered as average kinetic energy. Greater viscous dissipative heat rises the temperature. Thus, we can say that temperature of the fluid rises.

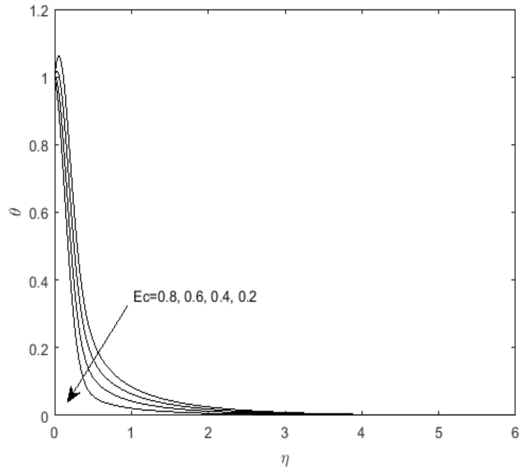

Fig. 8: Temperature profiles for Ec

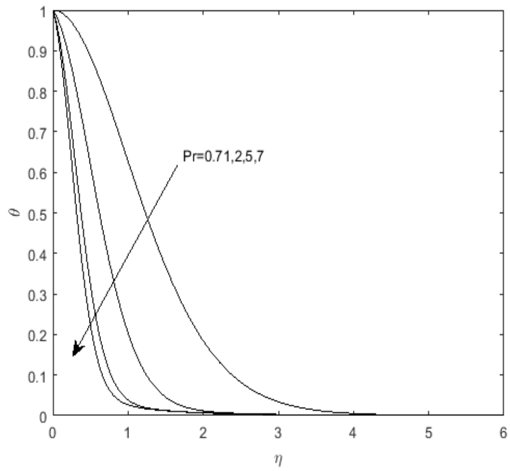

Fig. 9: Temperature profiles for Pr

It is clear from Fig. 9 that higher values of Prandtl number (Pr) reduce the temperature and thickness of thermal boundary layer as well. The lower values of Pr increase the thermal conductivity of the nanofluid due to which the heat is rapidly diffused away compared to the higher values of Pr from the heated plate. Hence, the rate of heat-transfer decreases.

Fig. 10 illustrates that more is the heat source (Q) more is the temperature which is obvious. Furthermore, an enhanced thermal boundary layer is obtained due to increase in heat source.

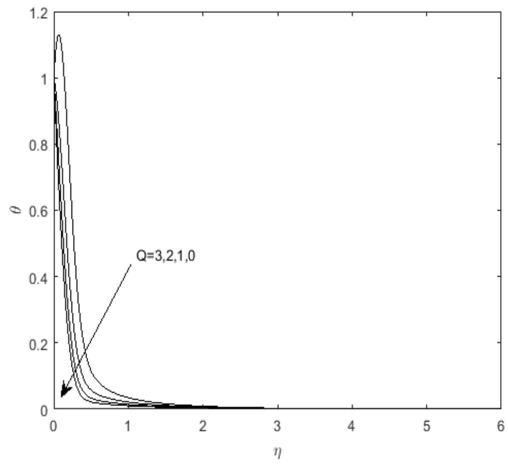

Fig. 10: Temperature profiles for Q

5. Conclusion

The steady free convective flow of nanofluid over a stretching sheet embedded in a porous medium has been studied in this research. From the above discussion, some of the important conclusions are given as follows:

- An increase in the values of K_r decreases the velocity.
- Higher values of Prandtl number (Pr) reduce the temperature.

- The increasing values of K_r decrease species concentration in the boundary layer.
- Increase in the Schmidt number decreases the velocity.
- Thermal boundary layer thickness is also enhanced by increasing *Ec*.
- The velocity of the fluid increases as the permeability parameter increases.
- An enhanced thermal boundary layer is obtained due to increase in heat source.

References

Abu-Nada, E., Chamkha, A. J. (2010): Effect of nanofluid variable properties on natural convection in enclosures filled with a CuO-EG-Water nanofluid, International Journal of Thermal Sciences, 49, 2339-2352. https://doi.org/10.1016/j.ijthermalsci.2010.07.006

Abo-Dahab, S. M., Abdelhafez, M. A., Bilal, S. M. (2021): MHD Casson nanofluid flow over nonlinearly heated porous medium under transversal magnetized field and along with provision of suction/injection, Indian Journal of Physics, published online, January, 2021. https://doi.org/10.1007/s12648-020-01923-z

Abo-Eldahab, E.M., EL Aziz, M.A. (2005): Viscous dissipation and joules heating effects on MHD free convection from a vertical flat plate with power-law variation in surface temperature in the presence of Hall and non-slip currents, Appl. Math. Model, 29, 579-595. https://doi.org/10.1016/j.apm.2004.10.005

Ahmed, S., Kalita, K. (2014): Unsteady MHD chemically reacting fluid through a porous medium bounded by a non-isothermal impulsively-started vertical plate: A numerical technique. Journal of Naval Architecture and Marine Engineering 11(1), 39–54. https://doi.org/10.3329/jname.v11i1.10269

Ahmed, S. (2014): Numerical analysis for magneto hydrodynamic chemically reacting and radiating fluid past a non-isothermal impulsively started vertical surface adjacent to a porous regime. Ain Shams Engineering Journal 5, 923-933. https://doi.org/10.1016/j.asej.2014.02.005

Bhargava, R., Singh, S. (2012): Numerical simulation of unsteady MHD flow and heat transfer of a second-grade fluid with viscous dissipation and Joule heating using mesh free approach, World Academy of Science, Eng. And Tech, 67, 1043-1049.

Chamber, P. L. and Young, J. D. (1958): The effects of homogeneous 1st order chemical reactions in the neighborhood of a flat plate for destructive and generative reactions. Physics of Fluids 1: 48-54.

Chen, C.H. (2004): Combined heat and mass transfer in MHD free convection from a vertical surface with ohmic heating and viscous dissipation, Int. J. Eng. Sci., 42, 699-713. https://doi.org/10.1016/j.ijengsci.2003.09.002

Choi, S. U. S., Zhang, Z. G., Yu, W., Lockwood, F. E., Grulke, E. A. (2001): Anomalous Thermal Conductivity Enhancement in Nanotube Suspensions, Applied Physics Letters, 79, 2252. http://dx.doi.org/10.1063/1.1408272

Choi, S.U.S. (1995): Enhancing thermal conductivity of fluids with nanoparticles, ASME-Publications-Fed 231, 99–106.

Goud, B. S., Reddy, Y. D., Rao, V. S. (2020): Thermal radiation and Joule heating effects on a magnetohydrodynamic Casson nanofluid flow in the presence of chemical reaction through a nonlinear inclined porous stretching sheet, Journal of Naval Architecture and Marine Engineering, Volume 17, No. 2. https://doi.org/10.3329/jname.v17i2.49978

Govindaraju, M., Vishnu Ganesh, N., Ganga, B., Abdul Hakeem, A. K. (2015): Entropy Generation Analysis of Magneto Hydrodynamic Flow of a Nanofluid over a Stretching Sheet, Journal of the Egyptian Mathematical Society, 23(2), 429-434. https://doi.org/10.1016/j.joems.2014.04.005

Hakan, F., Eiyad Abu-Nada, O. (2008): Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids, International Journal of Heat and Fluid Flow, 29(5), 1326- 1336. https://doi.org/10.1016/j.ijheatfluidflow.2008.04.009

Hamad, M.A.A. (2011): Analytical Solution of Natural Convection Flow of a Nanofluid over a linearly Stretching Sheet in the Presence of Magnetic Field, International Communications in Heat and Mass Transfer, 38(4), 487-492. https://doi.org/10.1016/j.icheatmasstransfer.2010.12.042

Kumar, S. T., Kumar, B. R. (2017): A comparative study of thermal radiation effects on MHD flow of nanofluids and heat transfer over a stretching sheet, FHMT, 9, 1-7.

Makinde, O. D., Khan Z.H., Ahmad R., Khan W. A. (2018): Numerical study of unsteady hydromagnetic radiating fluid flow past a slippery stretching sheet embedded in a porous medium. Physics of Fluids, 30, 083601-083607. https://doi.org/10.1063/1.5046331

Makinde, O. D., Khan, W.A., Khan, Z.H. (2016): Analysis of MHD nanofluid flow over a convectively heated permeable vertical plate embedded in a porous medium: Journal of Nanofluids 5, 574-580.

https://doi.org/10.1166/jon.2016.1236
Malik, S., Nayak A.K. (2016): A comparative study of mixed convection and its effect on partially active thermal zones in a two-sided lid-driven cavity filled with nanofluid, Engineering Science Technology, International Journal, 19, 1283-1298. https://doi.org/10.1016/j.jestch.2016.02.008
Marzougui, S., Bouabid, M., Mebarek-Oudina, F., Abu-Hamdeh, N., Ramesh, K. (2020): A computational analysis of heat transport irreversibility phenomenon in a magnetized porous channel, International Journal of Numerical Methods for Heat and Fluid Flow, https://doi.org/10.1108/HFF-07-2020-0418
Mebarek-Oudina, F. (2018): Convective heat transfer of Titania nanofluids of different base fluids in cylindrical annulus with discrete heat source, Heat Transfer-Asian Research, Volume 48, Issue 1. https://doi.org/10.1002/htj.21375
Mebarek-Oudina, F., Bessaih, R., Mahanthesh, B., Chamkha, J. R. (2020): Magneto-thermal-convection stability in an inclined cylindrical annulus filled with a molten metal, International Journal of Numerical Methods for Heat & Fluid flow, ISSN: 0961-5539, Vol. 31, No. 4. https://doi.org/10.1108/HFF-05-2020-0321
Muthucumaraswamy, R., Janakiramana (2008): Mass transfer effect on isothermal vertical oscillating plate in presence of chemical reaction. B. International Journal of Applied Mathematics and Mechanics, 4, 59-65.
Mutuku-Njane, W. N., Makinde, O. D. (2013): Combined effect of buoyancy force and Navier Slip on MHD flow of a nanofluid over a convectively heated vertical porous plate, The Scientific World Journal, 1-8. https://doi.org/10.1155/2013/725643
Oztop, H.F., Abu-Nada, E. (2008): Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids, International Journal of Heat Fluid Flow, 29, 1326-1336. https://doi.org/10.1016/j.ijheatfluidflow.2008.04.009
Pak, B. C., Young, I. C. (1998): Hydrodynamic and Heat Transfer Study of Dispersed Fluids with Submicron metallic Oxide Particles, Experimental Heat Transfer, 11(2), 151-170. https://doi.org/10.1080/08916159808946559
Parida, B. C., Swain, B. K., Senapati, N., Sahoo, S. (2020): Viscous dissipation effect on MHD free convective flow in the presence of thermal radiation and chemical reaction, 7, 387-394. https://doi.org/1018280/mmep.070308
Patel, H. E., Sundararajan, T., Pradeep, T., Dasgupta, A., Dasgupta, N., Das S. K. (2005): A Micro-Convection Model for Thermal Conductivity of Nanofluids, Pramana journal of physics, 65(5), 863-869. https://doi.org/10.1007/bf02704086
Reddy, Y. D., Rao, V. S., Babu, L. (2017): MHD boundary layer flow of nanofluid and heat transfer over a porous exponentially stretching sheet in presence of thermal radiation and chemical reaction with suction, International Journal of Mathematics Trends and Technology. https://doi.org/10.14445/22315373/IJMTT-V47P512
Rosseland, S. (1931): Astrophysik und atom-theoretische Grundlagen, Springer-Verlag, Berlin.
Sekuli ć, J. E. ten Elshof, Dave H. A. B. (2005): Selective Pervaporation of Water through a Nonselective Micro-porous Titania Membrane by a Dynamically Induced Molecular Sieving Mechanism, Langmuir, 21(2), 508 -510. https://doi.org/10.1021/la047458p
Swain, B. K., Senapati, N. (2015): The effect of mass transfer on MHD free convective radiating flow over an impulsively started vertical plate embedded in a porous medium, Journal of Applied Analysis and Computation, 5, 18–27. https://doi.org/10.11948/2015002
Swain, B.K., Parida, B.C., Kar, S., Senapati, N. (2020): Viscous dissipation and joule heating effect on MHD flow and heat transfer past a stretching sheet embedded in a porous medium, 6, 1-8. https://doi.org/10.1016/j.heliyon.2020.e05338
Swain, B. K., Senapati, N., Dash, M. (2014): The effect of chemical reaction and thermal radiation on the hydromagnetic free convective rotating flow past an accelerated vertical plate in the presence variable heat and mass diffusion, Der Chemica Sinica 5, 56-66.
Swain, B. K. (2021): Effect of second order chemical reaction on MHD free convective radiating flow over an impulsively started vertical plate, JNMA, 3, 167-178.
Xuan, Y.M., Li, Q. (2000): Heat transfer enhancement of nanofluids, International Journal of Heat Fluid Flow, 21, 58–64. https://doi.org/10.1016/S0142-727X(99)00067-3