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HEAT AND MASS TRANSFER EFFECT ON A RADIATIVE SECOND GRADE MHD FLOW IN A POROUS MEDIUM OVER A STRETCHING SHEET

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Abstract:

The present problem deals with a radiative second grade fluid flow through a porous medium over a semi infinite stretching sheet. In the present study, governing equations for the third grade fluid has been formulated. However, computation has been made for a second grade fluid as a particular case of third grade of fluid. The bounding surface is subjected to power law temperature distribution and heat flux. Confluent hypergeometric function and Runge-Kutta method of fourth order are used to solve the transformed non-linear governing equations. The physical variables such as velocity, temperature and concentration are studied in response to radiative heat transfer, electromagnetic mechanical force and porosity of the medium. The important findings of the present study are: the applied transverse magnetic field prevents the growth of boundary layer but accelerates the mass transfer; the presence of porous medium in a higher Reynolds number-fluid reduces the skin friction which is desirable for maintaining laminarity of flow and also for reduction of heat transfer rate at the surface; the temperature distribution decreases with the thermal radiation for both PST and PHF cases. In asymptotic case, presence of thermal radiation improves thermal stability.

Keywords: Thermal radiation, MHD flow, second and third grade fluid, stretching sheet, porous medium, chemical reaction

NOMENCLATURE

x, y	coordinates
u, v	velocity components along x-axis and y-axis respectively
P	spherical pressure
I	identity matrix
A_1, A_2	Rivlin-Ericksen tensors
A	positive constant (stretching rate)
T	dimensional temperature
G	non-dimensional temperature (PHF)
C	dimensional concentration
T_w	temperature at the wall
C_w	concentration at the wall
C_p	specific heat at constant pressure
k^*	absorption coefficient
Kp^*	dimensional porosity parameter
Kp	non-dimensional porosity parameter
Pr	Prandtl number
Sc	Schmidt number
Rd	thermal radiation parameter
l	characteristic length
D	thermal diffusivity of the medium
Ec	Eckert number
B_1, B_2, B_3	Constants
Rc	elastic parameter

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T_∞	ambient temperature
C_∞	ambient concentration
Q	dimensional heat source parameter
B_0	external magnetic field
D	mass diffusion coefficient
k	fluid thermal conductivity
Kc^*	dimensional chemical reaction Parameter
Kc	non-dimensional chemical reaction Parameter
M	magnetic parameter

Greek symbols

θ	non-dimensional temperature (PST)
ϕ	non-dimensional concentration
ρ	fluid density
υ	kinematics viscosity ($\upsilon = \mu / \rho$)
σ	Stefan-Boltzmann constant
β	non-dimensional heat source parameter
μ	coefficient of viscosity
$\alpha_1, \alpha_2, \beta_3$	material constants

1. Introduction

The study of magneto-convective flow of visco-elastic fluids over a continuously moving wall has wide applications in technological and manufacturing processes. These include extrusion of plastic sheets, cooling of metallic plates, production of synthetic materials, aerodynamic and condensation processes etc. The present study finds numerous applications in problems of practical interest such as flows over stretching sheets (polymer extrusion), through porous medium (insulation and application in agriculture and biology), thermal radiation (a frequent occurrence in metallurgical processes) and electromagnetic application (electric motors, pumps). Therefore, it is an open problem of interest to many researchers. Further the spinning of fibers and glass blowing involve the flow due to the stretching surfaces. Most importantly, the qualities of final product depend upon the rate of heat and mass transfer at the stretching surface that warrants the present study. Crane (1970) presented a similarity analytical solution for two dimensional boundary layer flows due to stretching of a sheet. Carragher and Crane (1982) took up the heat transfer in the flow over a stretching sheet when the temperature difference between the surface and the ambient fluid is proportional to a power of spatial distance measured from the fixed point. Andersson and Dandapat (1991) studied the flow of a power law fluid past a stretching surface. Later, Chaim (1994) took up a stagnation point-flow of viscous fluid towards a stretching plate. An interesting result he arrived at, during the study that the flow near the plate is identical with the inviscid flow far off the plate and hence no boundary layer is formed near the plate but this result is an outcome of the restrictive assumption that the stretching velocity is proportional to the distance from the stagnation point such that this velocity is identical with the stagnation flow-velocity in the inviscid region. Abel et al. (2002) have considered the boundary layer flow and heat transfer of a visco-elastic fluid in a porous medium over a non-isothermal stretching sheet. Their study reveals that when the flow is through a porous medium, the viscosity parameter significantly decreases the surface temperature and the permeability parameter decreases the skin friction. Prasad et al. (2003) have studied the chemical reaction rate in a laminar visco-elastic fluid in presence of porous medium over a stretching sheet and have found that destructive chemical reaction reduces the thickness of concentration boundary layer and enhances the mass transfer rate from the sheet to the fluid. Liu (2005) has performed an analysis on the flow and heat transfer phenomena of second grade electrically conducting fluid in a porous medium over a stretching sheet with a transverse magnetic field and has found that for large Eckert number, the heat is transferred from the fluid to the surface but the reverse effect is observed for small Eckert number. Afzal (2010) has analyzed momentum and thermal boundary layer over a two-dimensional non-linear stretching surface in a stationary fluid and has presented the thermal boundary layer closed form solution, series solution and asymptotic solution for very large and small values of Prandtl number. Further, Prasad et al. (2010), Cortell (2011), Nandeppanavar et al. (2011), Rashidi et al. (2014) and Kar et al. (2014) enriched the literature on flow past a stretching surface due to their significant contributions. Sahoo et al. have studied the effect of heat source and chemical reaction on MHD flow past a vertical plate subject to a constant motion with variable temperature and concentration. Rajput and Kumar (2019) have analysed theoretically the effects of radiation, chemical reaction and porosity of the medium on unsteady flow of a viscous, incompressible and electrically conducting fluid past an exponentially accelerated vertical plate with variable wall temperature and mass diffusion in the presence of transversely applied uniform magnetic field. They have obtained the results which claim to have applications in the research related to the solar physics dealing with the sunspot development, the structure of rotating magnetic stars, cooling of electronic components of a nuclear reactors etc.

The effect of thermal radiation on mixed convection flow past a stretching sheet in a porous medium has attracted a lot of research interests in the past few decades. In modern metallurgical and continuous casting of metals, the study of MHD flow of an electrically conducting fluid in the boundary layer flow due to stretching of sheet is of considerable interest. Many researchers such as Abel et al. (2005), Sidheswar and Mahabaleswar (2005) and Singh (2008) have studied the flow problems where the flow is caused due to stretching of sheets. Pal (2011) has studied the unsteady laminar boundary layer flow of a viscous incompressible fluid and heat transfer phenomenon with thermal radiation over continuously stretching permeable surface in the presence of a non-uniform heat source/sink. The recent works of Panda et al. (2012), (2011) and (2010) have contributed to heat and mass transfer on MFD flow through porous media over an accelerating surface in the presence of constant and oscillatory suction/injection on viscous and visco-elastic fluids subject to volumetric variable/constant heat sources. In contrast to heat conduction, in case of mass diffusion, the average velocity of the particles of the individual materials in a volume element can be different from each other, so that the relative movement of individual particle to each other is macroscopically perceptible. Hence, consideration of mass diffusion is taken care of in the present analysis. Thermal radiation is released from all bodies and is dependent on their material properties and temperature. The radiative energy may be transferred or absorbed in the body, i.e., flow domain. It plays a vital role in modifying the flow and heat transfer processes.

The following lines describe Rheological equations of the third grade fluid. The Rivlin-Ericksen fluids are acceptable from theoretical and experimental point of view. The special cases of the model are the fluids of second grade and third grade. Following Fosdick and Rajagopal (1980), the stress tensor for third grade fluid is given by

$$\tau = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_3 (trA_1^2) A_1 \tag{1}$$

$$\left.\begin{aligned} A_1 &= L + L^T \\ A_2 &= \frac{dA_1}{dt} + A_1 L + L^T A_1 \end{aligned}\right\} \tag{2}$$

where $L = \nabla V$ and d/dt is the material derivative and the material constants should satisfy the relations

$$\mu \geq 0,\ \alpha_1 \leq 0,\ \beta_3 \geq 0,\ |\alpha_1 + \alpha_2| \leq \sqrt{24\mu\beta_3} \tag{3}$$

Hence Equation (3) gives $\mu \geq 0,\ \alpha_1 \leq 0, \text{ and } \alpha_1 + \alpha_2 = 0$ (4)

Further Rajagopal (1979) showed that unlike second grade fluids, third grade fluids provide additional drag which is of the same sign as that of viscous terms. Fosdick and Satrughan (1981) showed that when $\alpha_1 < 0$, non physical results occur. Further Pakdemirli (1992) derived boundary layer equations of third grade fluids for steady two dimensional fluids using a special co-ordinate system (orthogonal curvilinear set of co-ordinates). Then it is shown that the boundary layer equations do not have similarity solutions. Further Pakdemirli calculated the shear stress on the boundary for the co-ordinate system. Hayat (2007) studied the steady flow of third grade fluid in a porous half space using Homotopy Analysis Method (HAM). Moreover, Sahoo and Poncet (2011) studied third grade fluid flow past an exponentially stretching sheet with partial slip boundary condition. The Prandtl number Pr, signifies the ratio of momentum diffusivity to thermal diffusivity. Fluid with lower Pr possesses higher thermal conductivity and gives rise to thicker boundary layer structure, so that, heat can diffuse from the bounding surface faster than higher Prandtl number fluid. In order to obtain thinner boundary layer as well as to reduce the rate of thermal conductivity as per the requirement, the analysis for large values of Prandtl number $(Pr \to \infty)$ is motivated.

Liu (2005) has confined his discussion to flow and heat transfer aspects leaving aside the heat losses due to radiation which is an important criteria for radiating surfaces and high temperature flow phenomena. He has not considered mass transfer ignoring diffusing species. Kar et al. (2014) have studied the visco-elastic fluid model and mass transfer in a chemically reactive species. But they have considered a permeable surface with a cross flow. In the present study we have an impermeable surface having no suction/injection. Therefore, the case of Kar et al. (2014) cannot be discussed as a particular case of the present study.

To obviate the limitations of the earlier works, we have incorporated thermal radiation and mass transport aspects of a chemically reactive diffusing species in the present study. Our study is confined to an incompressible, electrically conducting, steady and laminar flow of a second grade fluid past a vertical stretching porous wall in the presence of thermal radiation and chemical reaction. The work is more amenable to industrial application because corrosive/reactive species are abundant in industrial wastes. The industrial liquids are also conducting in nature. Further, the inclusion of first order chemical reaction in mass transport equation which generates/absorbs heat, justifies the inclusion of heat source in the heat equation. The analytical solution of the study is accomplished by hypergeometric function (Kummer's function) and the numerical solution by Runge-Kutta method provides the consistency and reliability. An interesting outcome is to note that, the inclusion of thermal radiation in asymptotic case indicates thermal stability which is desirable in the thermal transport processes.

2. Flow Analysis

Consider a two dimensional steady convective laminar flow of an electrically conducting incompressible grade fluid past a stretching sheet with the plane y=0. Due to the linear stretching of the bounding surface, the flow is generated. A uniform transverse magnetic field B_0 of moderate strength is applied to the flow field. Let u and v be the velocity components along x-axis and y-axis respectively. From a thin slit at the origin, the stretching sheet is originated as shown in Fig.1. Due to moderate magnetic field strength, the induced magnetic field is neglected. The heat transfer and mass transfer phenomena are subjected to volumetric heat source, dissipative heat energy, thermal radiation and chemical reaction.

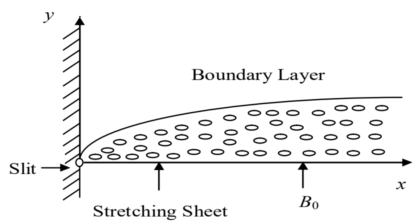


Figure1: Flow geometry

The governing boundary layer equations of third grade fluid following Pakdemirli (1992) are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{5}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+\frac{\alpha_1}{\rho}\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+3\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}+v\frac{\partial^3 u}{\partial y^3}\right]+\frac{2\alpha_2}{\rho}\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}+\frac{6\beta_3}{\rho}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho}-\frac{\upsilon}{Kp^*}u \tag{6}$$

If we put $\beta_3=0$ and $\alpha_2=-\alpha_1$ Equation (6) coincides with Equation (6) of second grade fluid of Liu (2005) which is given by

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+\frac{\alpha_1}{\rho}\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}+v\frac{\partial^3 u}{\partial y^3}\right]-\frac{\sigma B_0^2 u}{\rho}-\frac{\upsilon}{Kp^*}u \tag{7}$$

The appropriate boundary conditions are

$$\left.\begin{array}{l} u=Ax, v=0 \ \text{ at } \ y=0 \\ u\to 0, \partial u/\partial y\to 0 \ \text{ as } \ y\to\infty \end{array}\right\} \tag{8}$$

The condition $\partial u/\partial y\to 0$ as $y\to\infty$ has been discussed by Garg and Rajagopal (1991). We introduce the following similarity transformations satisfying Equation (5) as discussed by Rajagopal et al. (1984).

$$u=Axf'(\eta),\ v=-(A\upsilon)^{1/2}f(\eta),\quad \eta=(A/\upsilon)^{1/2}y \tag{9}$$

where f is the dimensionless stream function and the prime denotes the differentiation with respect to the similarity variable η.

With the help of Equation (9), Equations (7) and (8) are reduced to

$$f'^2-ff''=f'''+Rc(2f'f'''-f''^2-ff^{iv})-\left(M+1/Kp\right)f' \tag{10}$$

$$\text{and}\quad \left.\begin{aligned} &f=0, f'=1 \ \text{ at } \eta=0\\ &f'\to 0, f''\to 0 \ \text{ as } \eta\to\infty \end{aligned}\right\} \tag{11}$$

where $Rc=\alpha_1 A/\mu$, $M = \sigma B_0{}^2/\rho A$ and $Kp = AKp^*/\upsilon$.

An exact solution to (10) is given by

$$f(\eta)=\frac{1}{m}\left(1-exp(-m\eta)\right) \tag{12}$$

where $m=\sqrt{(1+(M+1/Kp))/(1+Rc)}$.

The dimensionless wall shear stress at the stretching sheet $(\eta=0)$, the skin friction coefficient C_f is given by

$$C_f=\frac{\tau_{12}(0)}{\rho u_w^2/2}=\frac{2}{Re_x^{1/2}}(1+3Rc)f''(0)=-2m(1+3Rc)/Re_x^{1/2} \tag{13}$$

where $Re_x=B_0x^2/\upsilon$ is the local magnetic Reynolds number.

3. Heat Transfer Analysis

The heat transfer in a flow of a visco-elastic electrically conducting fluid is obtained as the balance of energy for a fluid element in motion in conjunction with viscous dissipation. It is also to be noted that during the motion of a visco-elastic second grade fluid, a certain amount of energy is stored up in the flow as strain energy and some energy is lost due to thermal radiation, viscous as well as Ohmic dissipation. In the present study we have neglected the Ohmic dissipation as the applied magnetic field is of low magnetic induction (small local magnetic Reynolds number). On the other hand, we have accounted for an internal volumetric temperature dependent heat source which is of frEquent occurrence along with thermal radiation. Using the boundary layer approximations, the heat transfer Equation for third grade fluid is given by

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\alpha_1\frac{\partial u}{\partial y}\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)+2\beta_3\left(\frac{\partial u}{\partial y}\right)^4-\frac{\partial q_r}{\partial y}+Q\left(T-T_\infty\right) \tag{14}$$

When $\beta_3=0$, Equation(14) reduces to

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\alpha_1\frac{\partial u}{\partial y}\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)-\frac{\partial q_r}{\partial y}+Q\left(T-T_\infty\right) \tag{15}$$

Using Rosseland approximation for radiation as outlined by Brewstar (1972), we have

$$q_r=-\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y}, T^4\equiv 4T_\infty{}^3T-3T_\infty{}^4 \tag{16}$$

Therefore, Eq. (15) is reduced to

$$\begin{aligned}\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\alpha_1\frac{\partial u}{\partial y}\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)\\ +\frac{16\sigma T_\infty{}^3}{3k^*}\frac{\partial^2 T}{\partial y^2}+Q\left(T-T_\infty\right)\end{aligned} \tag{17}$$

We have discussed the temperature distribution in two forms such as (i) Prescribed Surface Temperature and (ii) Prescribed Surface Heat Flux.

3.1 The prescribed surface temperature (PST):

The boundary conditions are

$$y=0: T=T_w=T_\infty+B_1(x/l)^2 \ ; \ y\to\infty: T\to T_\infty \tag{18}$$

According to Nield and Bejan (1998), the temperature is now expressed as

$$T(\eta)=T_\infty+B_1(x/l)^2\theta(\eta), \text{ where } \theta(\eta)=(T-T_\infty)/(T_w-T_\infty) \tag{19}$$

Using Eqs. (9), (12) and (19) in Eqs. (17) and (18), we obtain

$$\theta''+\frac{3RdPr}{3Rd+4}f\theta'-\frac{3RdPr}{3Rd+4}(2f'-\beta)\theta=-\frac{3RdPrEc}{3Rd+4}\left[(f'')^2+Rc\,f''(f'\,f''-ff''')\right] \tag{20}$$

and $\eta=0:\theta=1\ ;\ \eta\to\infty:\theta\to 0,$ (21)

where $Pr=\mu C_p/k$, $\beta=Q/A\rho C_p$, $Rd=kk^*/4\sigma T_\infty^3$ and $Ec=A^2l^2/B_1C_p$.

Introducing the new variable $\xi=-re^{-m\eta}$ with $r=\dfrac{3PrRd}{m^2(3Rd+4)}$, Eqs. (20) and (21) reduce to

$$\xi\frac{d^2\theta}{d\xi^2}+(1-r-\xi)\frac{d\theta}{d\xi}+(2+\frac{\beta r}{\xi})\theta=-\frac{3RdPrEc(1+Rc)\xi}{m^2(3Rd+4)r^2}, \tag{22}$$

$\theta(-r)=1$ and $\theta(0)=0.$ (23)

The solution of (22) in view of the boundary conditions (23) in terms of confluent hypergeometric function (Kummer's function) $F(a,b,\xi)$ is as follows

$$\theta(\xi)=(1+H)(-\xi/r)^{(r+s)/2}\left(\frac{F\left(\frac{r+s-4}{2},s+1,\xi\right)}{F\left(\frac{r+s-4}{2},s+1,-r\right)}\right)-H(-\xi/r)^2 \tag{24}$$

where $s=\sqrt{r(r-4\beta)}$, $H=\dfrac{3EcPrRd(1+Rc)}{(3Rd+4)(4-2r+r\beta)}$, $F(a,b,\xi)=1+\sum_{n=1}^{\infty}\dfrac{(a)_n}{(b)_n}\xi^n$,

$(a)_n=a(a+1)(a+2)L(a+n-1)$ and $(b)_n=b(b+1)(b+2)\cdots(b+n-1)$

The solution (24) in terms of η is given by

$$\theta(\eta)=\left((1+H)exp(-(r+s)m\eta)/2\right)\frac{F\left(\frac{r+s}{2}-2,s+1,-re^{-m\eta}\right)}{F\left(\frac{r+s}{2}-2,s+1,-r\right)}-He^{-2m\eta}, \tag{25}$$

The rate of heat transfer at the wall, i.e., Nusselt number, Nu is given by

$$Nu=\frac{-k\left.\frac{\partial T}{\partial y}\right|_{y=0}}{k(T_w-T_\infty)}x=-Re_x^{1/2}\theta'(0) \tag{26}$$

where $\theta'(0)=(1+H)\left[\left(mr\dfrac{r+s-4}{2(s+1)}\dfrac{F\left(\frac{r+s}{2}-2,s+2,-r\right)}{F\left(\frac{r+s}{2}-2,s+1,-r\right)}\right)-\dfrac{m}{2}(r+s)\right]+2mH$

3.2 The prescribed surface heat flux (PHF):

The boundary conditions for this case are:

$$y=0:-k\frac{\partial T}{\partial y}=q_w=B_2\left(\frac{x}{l}\right)^2\ ;\ y\to\infty:T\to T_\infty \tag{27}$$

Now, we set $T-T_\infty=\dfrac{B_2}{k}\left(\dfrac{x}{l}\right)^2\left(\dfrac{\upsilon}{A}\right)^{1/2}g(\eta)$ (28)

Substituting Eqs. (28) and (9) in Eq. (17), we get

$$g''+\frac{3RdPr}{3Rd+4}fg'+\frac{3RdPr}{3Rd+4}(\beta-2f')g=-\frac{3RdPrEc}{3Rd+4}\left[(f'')^2+Rcf''(f'\,f''-ff''')\right] \tag{29}$$

and $\eta = 0 : g' = -1;\ \eta \to \infty : g = 0$, (30)

where $Ec = kA^2 l^2 (A/\upsilon)^{1/2} \big/ B_2 C_p$ which differs from the Eckert number in previous section and all other parameters are the same as before.

Using the same transformation $\xi = -re^{-m\eta}$, Eqs. (29) and (30) reduce to

$$\xi \frac{d^2 g}{d\xi^2} + (1 - r - \xi)\frac{dg}{d\xi} + (2 + \frac{\beta r}{\xi})g = -\frac{3RdPrEc(1+Rc)\xi}{m^2(3Rd+4)r^2} \tag{31}$$

$$\frac{dg}{d\xi} = \frac{-1}{rm} \text{ for } \xi = -r \text{ and } g = 0 \text{ for } \xi = 0 \tag{32}$$

The solution of Equation (31) satisfying the boundary conditions (32) is given by

$$g(\xi) = \left(\frac{1}{m} + 2H\right)\left(\frac{\xi}{-r}\right)^{r+s/2} \times \left[\frac{r+s}{2}F\left(\frac{r+s}{2} - 1, s+1, -r\right) - r\frac{(r+s-4)/2}{(s+1)}F\left(\frac{r+s}{2} - 1, s+2, -r\right)\right]^{-1}$$
$$\times F\left(\frac{r+s}{2} - 2, s+1, \xi\right) - H\left(\frac{\xi}{-r}\right)^2 , \tag{33}$$

The following solution $g(\eta)$ is obtained by substituting $\xi = -re^{-m\eta}$ in Eq. (33)

$$g(\eta) = -He^{-2m\eta} + \left(\frac{1}{m} + 2H\right)exp\left((r+s)m\eta/2\right) \times \left[\frac{r+s}{2}F\left(\frac{r+s}{2} - 2, s+1, -r\right) - r\frac{(r+s-4)/2}{s+1}F\left(\frac{r+s}{2} - 1, s+2, -r\right)\right]^{-1}$$
$$\times F\left(\frac{r+s}{2} - 2, s+1, -re^{-m\eta}\right) \tag{34}$$

From Eq. (28) the dimensionless wall temperature is obtained as

$$\left(T - T_\infty\right) / \left(\frac{B_2}{k}\left(\frac{x}{l}\right)^2 \left(\frac{\upsilon}{A}\right)^{1/2}\right) = g(\eta)\Big|_{\eta=0} , \tag{35}$$

where $g(0) = -H + \left(2H + \frac{1}{m}\right)F\left(\frac{r+s}{2} - 2, s+1, -r\right) \times \left[\frac{r+s}{2}F\left(\frac{r+s}{2} - 2, s+1, -r\right)\right.$
$$\left. - \frac{r(r+s-4)/2}{s+1}F\left(\frac{r+s}{2} - 1, s+2, -r\right)\right]^{-1} \tag{36}$$

Asymptotic analysis is important as it is valid for the region with very low thermal conductivity, analogous to potential flow $\left(Re \to \infty\right)$ following Schlichting and Gersten (1996). For large values of Pr ($Pr \to \infty$), the asymptotic solutions of (20) and (29) are as follows:

Prescribed Surface Temperature: Using Eq. (12) in Eq. (20), we have

$$\theta'' + \frac{3PrRd}{m(3Rd+4)}\left(1 - e^{-m\eta}\right)\theta' - \frac{3PrRd}{(3Rd+4)}\left(2e^{-m\eta} - \beta\right)\theta = -\frac{3PrRdEc}{(3Rd+4)}\left(1+Rc\right)m^2 e^{-2m\eta} \tag{37}$$

$$\eta = 0 : \theta = 1;\ \eta \to \infty : \theta \to 0 \tag{38}$$

Let $w = \theta + He^{-2m\eta}$. Then we have

$$\frac{3Rd+4}{3PrRd}w'' + \frac{1}{m}\left(1 - e^{-m\eta}\right)w' - (2e^{-m\eta} - \beta)w = 0 \tag{39}$$

The boundary conditions are

$$\eta = 0 \ : \ w = 1 + H \ ; \eta \to \infty : \ w = 0 \tag{40}$$

As the thickness of the thermal boundary layer is of the order $1/\sqrt{RePr}$, using the coordinate transformation $\varsigma=\sqrt{\left(\frac{3PrRd}{3Rd+4}\right)^{\eta}}$ and putting $w=e^{(-1/4)\varsigma^2}\psi$, Equations (39) and (40) become

$$\frac{d^2\psi}{d\varsigma^2}-\left(\frac{1}{4}\varsigma^2+\frac{5}{2}-\beta\right)\psi=0 \tag{41}$$

$$\varsigma=0 \quad :\psi=1+H\ ;\ \varsigma\to\infty:\psi\to 0 \tag{42}$$

In the derivation of the parabolic cylindrical equation (41), the limiting case $Pr\to\infty$ has been considered. From the solution of (41) satisfying boundary conditions (42), we get

$$\theta(\eta)=-He^{-2m\eta}+\left(1+H\right)e^{\frac{-3}{2}\left(\frac{PrRd}{3Rd+4}\right)\eta^2}\left[F\left(\frac{3-\beta}{2},\frac{1}{2},\frac{1}{2}\left(\frac{3PrRd}{3Rd+4}\right)\eta^2\right)-\sqrt{\frac{6PrRd}{3Rd+4}}\frac{\Gamma\left(\frac{4-\beta}{2}\right)}{\Gamma\left(\frac{3-\beta}{2}\right)}\eta F\left(\frac{4-\beta}{2},\frac{3}{2},\frac{1}{2}\left(\frac{3PrRd}{3Rd+4}\right)\eta^2\right)\right]. \tag{43}$$

$$\theta'(0)=2mH-(1+H)\sqrt{\frac{6PrRd}{3Rd+4}}\frac{\Gamma\left(\frac{4-\beta}{2}\right)}{\Gamma\left(\frac{3-\beta}{2}\right)}. \tag{44}$$

Prescribed surface heat flux: From the solution of Equation (29) as $Pr\to\infty$, we get

$$g(\eta)=-He^{-2m\eta}+\left(1+2mH\right)e^{\frac{-3}{2}\left(\frac{PrRd}{3Rd+4}\right)\eta^2}\left[\sqrt{\frac{3Rd+4}{6PrRd}}\frac{\Gamma\left(\frac{3-\beta}{2}\right)}{\Gamma\left(\frac{4-\beta}{2}\right)}F\left(\frac{3-\beta}{2},\frac{1}{2},\frac{1}{2}\left(\frac{3PrRd}{3Rd+4}\right)\eta^2\right)-\eta F\left(\frac{4-\beta}{2},\frac{3}{2},\frac{1}{2}\left(\frac{3PrRd}{3Rd+4}\right)\eta^2\right)\right] \tag{45}$$

$$g(0)=-H+(1+2mH)\sqrt{\frac{3Rd+4}{6PrRd}}\frac{\Gamma\left(\frac{3-\beta}{2}\right)}{\Gamma\left(\frac{4-\beta}{2}\right)} \tag{46}$$

4. Mass Transfer Analysis

The species concentration equation and the relevant boundary condition are given by

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-Kc^*(C-C_\infty) \tag{47}$$

$$y=0:C=C_w=C_\infty+B_3(x/l)^2\ ;\ y\to\infty:C\to C_\infty \tag{48}$$

Assuming the concentration profile as

$$C(\eta)=C_\infty+B_3(x/l)^2\phi(\eta),\text{ where }\phi(\eta)=(C-C_\infty)/(C_w-C_\infty), \tag{49}$$

the mass transfer Eq. (47) and the boundary condition (48) reduce to

$$\phi''+Scf\phi'-(2Scf'+KcSc)\phi=0, \tag{50}$$

and $\eta=0\ :\ \phi=1\ ;\ \eta\to\infty\ :\ \phi\to 0$, (51) where $Sc = \upsilon / D$ and $Kc = Kc^*/A$.

Now, introducing $\zeta = -\frac{Sc}{m^2} e^{-m\eta}$, Eqs. (50) and (51) reduce to

$$\zeta \frac{d^2\phi}{d\zeta^2} + (1 - \frac{Sc}{m^2} - \zeta)\frac{d\phi}{d\zeta} + \left(2 - \frac{ScKc}{m^2\zeta}\right)\phi = 0, \tag{52}$$

$$\phi\left(\zeta = -Sc/m^2\right) = 1, \ \phi\left(\zeta = 0\right) = 0. \tag{53}$$

Using the transformation $\phi = \zeta^{\delta} h(\zeta)$, Eqs. (52) and (53) reduce to

$$\zeta \frac{d^2 h}{d\zeta^2} + (1 - d - \zeta)\frac{dh}{d\zeta} - \left(\frac{c + d - 4}{2}\right)h = 0, \tag{54}$$

$$h\left(\zeta = -Sc/m^2\right) = (-Sc/m^2)^{-\delta}, \ h\left(\zeta = 0\right) = 0, \tag{55}$$

where $c = \frac{Sc}{m^2}$, $d = \sqrt{c - \frac{4ScKc}{m^2}}$ and $\delta = \frac{c+d}{2}$.

Eq. (54) is a confluent hypergeometric differential equation and its solution satisfying Equation (55) and on back substitution, ζ in terms of η , is obtained as

$$\phi(\eta) = exp\left(-\frac{c+d}{2} m\eta\right) \frac{F\left(\frac{c+d}{2} - 2, 1 + d, -\frac{Sc}{m^2} e^{-m\eta}\right)}{F\left(\frac{c+d}{2} - 2, 1 + d, -\frac{Sc}{m^2}\right)} \tag{56}$$

The rate of mass transfer in terms of Sherwood number (Sh) is given by

$$Sh = \phi'(0) = \frac{Sc}{2m}\left(\frac{c + d - 4}{1 + d}\right) \frac{F\left(\frac{c+d-2}{2}, 2 + d, -\frac{Sc}{m^2}\right)}{F\left(\frac{c+d}{2} - 2, 1 + d, -\frac{Sc}{m^2}\right)} - \frac{m(c+d)}{2} \tag{57}$$

5. Results and Discussion

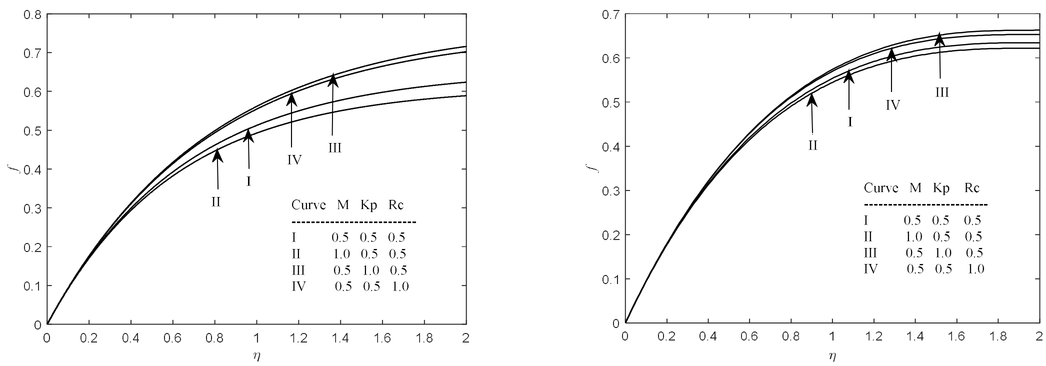
The solution of the velocity boundary layer equation has been obtained in a closed form whereas the confluent hypergeometric function has been applied to obtain the solution of heat transfer as well as mass transfer equations. In order to establish the reliability of the solutions, we have also applied Runge-Kutta method of fourth order with shooting technique for solution. The governing equations are formulated for third grade fluid, however, the computations and discussions are carried out for the second grade fluid. Also we have mentioned the relation between two fluid models in the present study.

5.1 Effect of flow parameters on velocity profile

Figs. 2(a), 2(b) and 3(a), 3(b) depict the transverse and longitudinal components of the velocity profiles for both the methods. It is observed that the resistive electromagnetic force decreases the primary as well as transverse velocity. This observation is similar to the effect of suction at the plate which reduces the boundary layer thickness. But the elasticity and permeability of the medium enhance both the components of velocity. Further, it is to note that the asymptotic variation of the primary velocity component aids to the laminarity of the flow pattern. Thus, it is concluded that the transverse magnetic field prevents the growth of boundary layer. Figures 2(b) and 3(b), based upon numerical method show a good agreement.

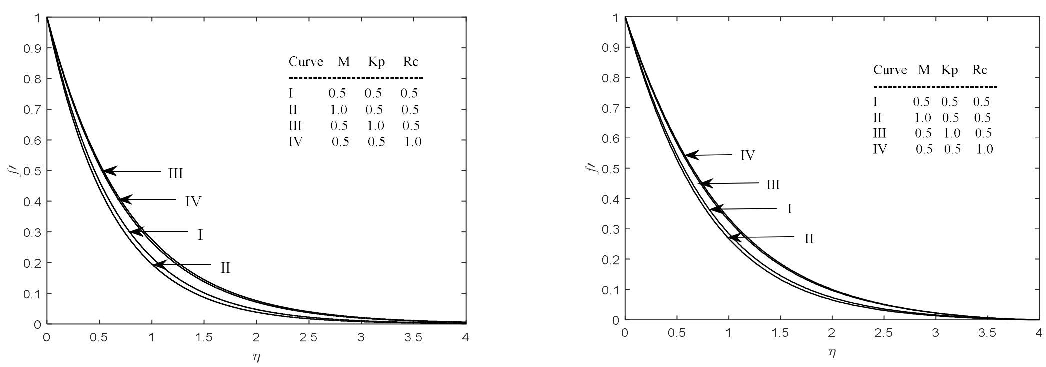
5.2 Effect of flow parameters on temperature profile

It is observed from Figs. 4(a) and 4(b) (PST case) that the force generated due to interaction of applied magnetic field and conducting flowing fluid, enhances the thermal energy in the flow domain but the permeability of the medium absorbs the thermal energy by reducing the temperature. The observation is compatible to physical property of the parameters since the resistive force generated due to transverse magnetic field, decelerates the fluid motion generating heat energy, consequently, enhancing the temperature. Further, the increase in Pr, reduces the temperature distribution of the flow region. The increase in Pr means slow rate of thermal diffusion, which causes a fall in temperature. The similar explanation can be attributed to permeability of the medium also. The Fig 4(b) presents the consistency of the two methods in temperature distribution also.



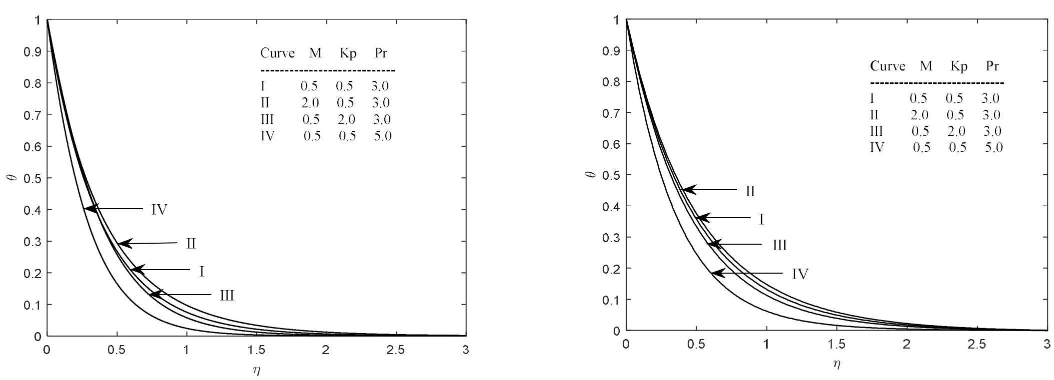
(a) Analytical method (b) Numerical method

Fig. 2: Transverse velocity profile for M, Kp and Rc



(a) Analytical method (b) Numerical method

Fig. 3: Longitudinal velocity profile for M, Kp and Rc



(a) Analytical method (b) Numerical method

Fig. 4: Temperature profile for M, Kp and Pr in case of PST.

Fig. 5 (PST case) depicts the effect of elastic parameter (Rc), radiation parameter (Rd), Eckert number (Ec) and the heat source parameter (β). The increase in visco-elastic parameter leads to increase in temperature. This is in conformity with the facts that the increase in elastic property results into stored up higher energy, so as to increase the thermal energy. Further, the figure reveals that the effect of increasing Ec, increases slightly the temperature distribution due to viscous dissipative heat. Moreover, the heat source parameter (β) also increases the temperature distribution.

Figs. 6 and 7 depict the temperature profiles in case of prescribed heat flux (PHF). The effect of all the parameters (Fig. 6) on temperature distribution remains same as that of PST case (Fig. 4). Only difference in temperature is marked at the bounding surface. The variations depend on the power of heat flux applied to the flow which acts as an embedded volumetric heat source in the flow domain. The Fig.7 has been compared with Fig. 5 (PST case). It is remarked that the effects of all the parameters remain same except the elastic parameter (Rc), where the opposite effect is observed (decrease in temperature) in the presence of heat flux. The higher elastic property (increasing Rc) leads to more amount of stored up energy, resulting lower down of temperature in the entire fluid mass.

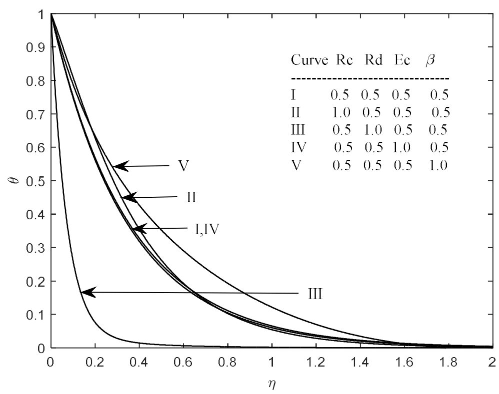


Fig.5: Temperature profile for Rc, Rd, Ec and β in case of PST

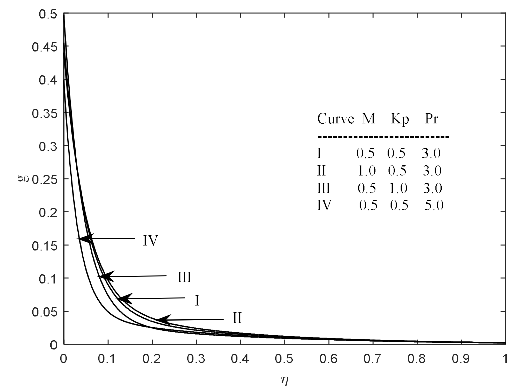


Fig.6: Temperature profile for M, Kp and Pr in case of PHF

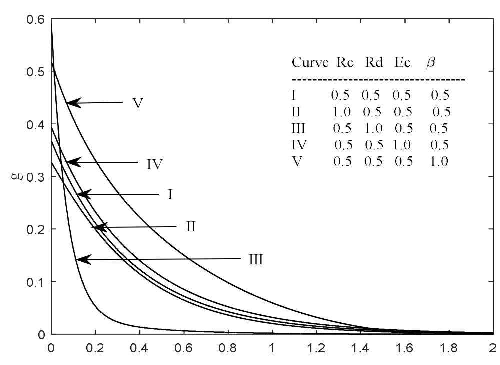


Fig.7: Temperature profile for Rc, Rd, Ec and β in case of PHF

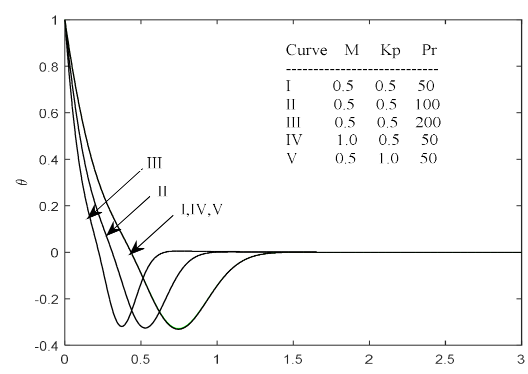


Fig. 8: Temperature profile for M, Kp and Pr in case of asymptotic PST

Fig. 8 shows the effect of magnetic parameter (M), permeability parameter (Kp) and Prandtl number (Pr) on temperature profile in case of asymptotic surface temperature ($Pr \to \infty$, low thermal conductivity). The temperature distribution exhibits the negative temperature within a few layers close to the plate indicating absorption of thermal energy resulting fall in temperature and then rises due to prescribed surface plate temperature. Further, it is seen that, the fall of temperature is accelerated for higher value of Pr (curves III and IV). Due to the low thermal conductivity of the fluid, the thermal energy fails to navigate within the layers close to the plate resulting negative temperature distribution which is not for the non-asymptotic case. Hence, the asymptotic analysis reveals the temperature fluctuation near the wall showing thermal instability due to the appearance of point of inflexion. This information may have industrial bearing for the right choice of fluid with required thermal conductivity. On careful observations it is also found that variation of other parameters, fails to affect the temperature distribution.

5.3 Effect of flow parameters on concentration profile

Fig. 9 shows the variation of solutal concentration across the flow field. On careful observation it is seen that permeability of the medium (Kp), impacts proactively by reducing the concentration distribution (curve III, IV) in comparison with magnetic field parameter (M). However, an increase in M, increases the concentration level slightly, since it reduces the velocity across flow domain due to the low momentum diffusivity.

Fig. 10 indicates some fluctuations due to higher value of Sc i.e. for heavier species combined with higher rate of chemical reaction. The similar fluctuation is indicated in case of temperature distribution also. The striking results indicating the fluctuations in thermal diffusivity (Fig. 8) and mass diffusivity (Fig. 10) are of utmost importance in industrial applications.

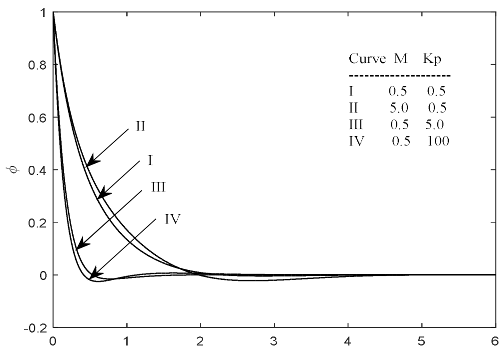


Fig.9 Concentration distribution for M and Kp

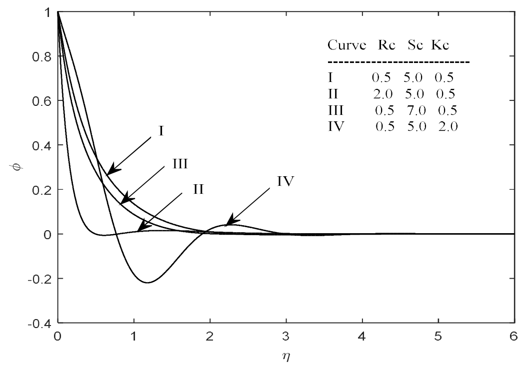


Fig.10 Concentration distribution for Rc, Sc and Kc

Table 1: Skin friction coefficient, C_f (wall shear stress)

M	***Kp***	***Rc***	***Re***	C_f
0.5	**0.5**	**0.5**	**0.5**	**-10.8012**
1.0	**0.5**	**0.5**	**0.5**	**-11.5470**
0.5	**1.0**	**0.5**	**0.5**	**-9.1287**
0.5	**0.5**	**1.0**	**0.5**	**-14.9666**
0.5	**0.5**	**0.5**	**1.0**	**-7.6376**
0.0	**0.5**	**0.5**	**0.5**	**-10.0000**
0.5	**0.5**	**0.0**	**0.5**	**-5.2915**
0.5	**100**	**0.5**	**0.5**	**-7.0946**

5.4 Effect of flow parameters on skin friction, C_f

Table 1 presents the values of skin friction for different values of M, Kp, Rc and Re. It is interesting to note that all the entries are negative. It is further observed that $\left|C_f\right|$ increases with the increase in magnetic and elastic parameters but decreases with the increase in porosity parameter and Reynolds number. For increasing magnetic Reynolds number, the inertia force is predominant and the effect of viscosity is considered to be confined within the boundary layer adjacent to the solid surface. Thus, presence of porous medium in a higher Reynolds number-flow is favorable for reducing skin friction which is desirable for maintaining laminarity of flow.

5.5 Effect of flow parameters on Nusselt number, *Nu*

Table 2 presents the values of Nusselt number, measure of the rate of heat transfer coefficient at the solid surface for different values of M, Kp, Pr, Rc, Re, Rd, Ec and β. It is observed that Nusselt number increases with the increase in magnetic parameter, Prandtl number, Reynolds number and thermal radiation but decreases with the increase in porosity, elasticity, heat source and higher value of Eckert number. Therefore, it is concluded that, for reduction of heat transfer rate at the wall, it is recommended to reduce the strength of applied magnetic field, Prandtl number as well as Reynolds number. It is interesting to note that rate of heat transfer is quite sensitive to higher value of Reynolds number (Re), Eckert number (Ec) and heat source parameter (β) which contribute significantly to increase/decrease the rate of heat transfer.

Table 2: Nusselt number, Nu (heat flux at the wall)

M	*Kp*	*Pr*	*Rc*	*Re*	*Rd*	*Ec*	β	*Nu*
0.5	0.5	5	0.5	0.5	0.5	0.5	0.5	4.5293
1.0	0.5	5	0.5	0.5	0.5	0.5	0.5	4.7241
0.5	1.0	5	0.5	0.5	0.5	0.5	0.5	4.0222
0.5	0.5	10	0.5	0.5	0.5	0.5	0.5	4.8813
0.5	0.5	5	1.0	0.5	0.5	0.5	0.5	4.0710
0.5	0.5	5	0.5	1.0	0.5	0.5	0.5	6.4054
0.5	0.5	5	0.5	0.5	1.0	0.5	0.5	5.0075
0.5	0.5	5	0.5	0.5	0.5	3.0	0.5	3.9657
0.5	0.5	5	0.5	0.5	0.5	0.5	1.0	3.6815
0.5	0.5	5	0.5	0.5	0.5	0.5	0.0	5.2993
0.5	0.5	5	0.0	0.5	0.5	0.5	0.5	5.1546

5.6 Effect of flow parameters on Sherwood number, *Sh*

Table 3 presents the values of Sherwood number measuring the rate of mass transfer at the solid surface for different values of M, Kp, Rc, Sc and Kc ($Kc > 0$, exothermic). It is observed that Sherwood number increases with the increasing permeability, elasticity and higher rate of chemical reaction but decreases with an increasing magnetic field as well as heavier diffusing species. Thus, the decrease in mass transfer at the surface is well supported by the physical property of diffusing species as well as exothermic chemical reaction.

Table 3: Sherwood number, *Sh* (mass flux at the wall)

M	*Kp*	*Rc*	*Sc*	*Kc*	*Sh*
0.5	0.5	0.5	0.5	0.5	-1.1939
1.0	0.5	0.5	0.5	0.5	-1.2993
0.5	1.0	0.5	0.5	0.5	-0.9657
0.5	0.5	1.0	0.5	0.5	-0.9956
0.5	0.5	0.5	1.0	0.5	-1.8734
0.5	0.5	0.5	0.5	1.0	-0.9113

5.7 Effect of flow parameters on temperature gradient, $-\theta'(0)$

Table 4 presents the temperature gradients in the presence/absence of thermal radiation. The positive values of $-\theta'(0)$ indicate the heat flows from the plate to the fluid. In spite of the non-availability of values of all the parameters of Table 2 of Liu (2005), we have compared the physical aspects of the parameters. In case of β, "the temperature gradient $\theta'(0)$ should increase, $|\theta'(0)|$ decrease with β". The same observation is made from table 4(1st line and last line) in absence of thermal radiation. The other parameters also show good agreement.

Table 4: Comparison of temperature gradient ($-\theta'(0)$)

M	***Kp***	***Pr***	***Rc***	***Ec***	β	**In presence of *Rd* (*Rd*=0.5)**	**In absence of *Rd* (*Rd*=0.0)**
						$-\theta'(0)$	$-\theta'(0)$
0.5	**0.5**	**5**	**0.5**	**0.5**	**0.5**	**6.4054**	**3.9102**
1.0	**0.5**	**5**	**0.5**	**0.5**	**0.5**	**6.6809**	**4.8008**
0.5	**1.0**	**5**	**0.5**	**0.5**	**0.5**	**5.6882**	**3.2269**
0.5	**0.5**	**10**	**0.5**	**0.5**	**0.5**	**6.9031**	**1.4928**
0.5	**0.5**	**5**	**1.0**	**0.5**	**0.5**	**5.7573**	**3.4458**
0.5	**0.5**	**5**	**0.5**	**1.0**	**0.5**	**6.2460**	**4.6172**
0.5	**0.5**	**5**	**0.5**	**0.5**	**1.0**	**5.2064**	**3.0922**

It is further observed that an increase in M, increases the surface temperature gradient in the presence as well as absence of radiation indicating more amount of heat energy flows from the plate to the fluid. It is interesting to note that permeability of the medium has reverse impacts as compared to magnetic strength in both the cases.

5.8 Effect of flow parameters on temperature gradient, $-\theta'(0)$ in asymptotic case

Table 5 shows that $-\theta'(0) > 0 \Rightarrow \theta'(0) < 0$, i.e. the wall is warmer than the fluid outside it in the presence of thermal radiation. It is interesting to note that for higher value of Ec (Ec=1) and $\beta(\beta = 1)$, $\theta'(0) > 0$ which shows that heat flows from the boundary layer to the plate (heating of the plate). It is also seen that effect of all the parameters, except Pr, has the same effect irrespective of the flow phenomena exposed to thermal radiation or not. The rate of heat transfer increases with higher value of Pr but the reverse effect is observed in the absence of thermal radiation.

Table 5: Comparison of temperature gradient ($-\theta'(0)$) in asymptotic case

M	***Kp***	***Pr***	***Rc***	***Ec***	β	**In presence of *Rd* (*Rd*=0.5)**	**In absence of *Rd* (*Rd*=0.0)**
						$-\theta'(0)$	$-\theta'(0)$
0.5	**0.5**	**100.0**	**0.5**	**0.5**	**0.5**	**7.3164**	**1.9729**
1.0	**0.5**	**100.0**	**0.5**	**0.5**	**0.5**	**7.3009**	**1.4274**
0.5	**1.0**	**100.0**	**0.5**	**0.5**	**0.5**	**7.3527**	**3.2096**
0.5	**0.5**	**150.0**	**0.5**	**0.5**	**0.5**	**8.9346**	**1.7268**
0.5	**0.5**	**100.0**	**1.0**	**0.5**	**0.5**	**7.3008**	**1.5620**
0.5	**0.5**	**100.0**	**0.5**	**1.0**	**0.5**	**7.1441**	**-3.5428**
0.5	**0.5**	**100.0**	**0.5**	**0.5**	**1.0**	**6.3410**	**-0.1913**

6. Conclusion

Both analytical and numerical methods provide consistency of the solutions. The applied transverse magnetic field prevents the growth of boundary layer. The fluid with higher visco-elastic property along with volumetric heat source and viscous dissipative heat enhance the fluid temperature, consequently, heat flows from the fluid to the bounding surface (PST Case). Most importantly, thermal instability is marked in a few layers close to the bounding surface due to low thermal conductivity of the fluid. The permeability of the porous medium impacts proactively on the reduction of concentration distribution in comparison with imposed magnetic field. The higher value of Schmidt number combined with high rate of chemical reaction gives rise to fluctuation in concentration profiles indicating solutal instability. Thus, thermal instability and solutal instability brings analogy between high Prandtl number as well as high Schmidt number. The rate of heat transfer at the plate is quite sensitive to high value of Prandtl number, Eckert number and heat source parameter. The presence of heat source in the present study reduces the heat transfer at the plate slowing down the cooling of the bounding surface. The present analysis includes asymptotic as well as non-asymptotic cases representing high and low conductivity fluid properties to encounter cooling and heating of the bounding surface. In metallurgical industries, cooling and heating processes are taken care for extraction and processing of metals.

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