



FINITE ELEMENT ANALYSIS OF SORET AND DUFOUR EFFECTS ON UNSTEADY MHD FREE CONVECTION FLOW PAST AN IMPULSIVELY STARTED VERTICAL POROUS PLATE WITH VISCOUS DISSIPATION

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Abstract:

The Soret and Dufour effects of an unsteady free convection flow magnetohydrodynamic and viscous dissipating fluid past an impulsively started vertical plate has been analyzed. The local unsteady similarity solutions of the transformed dimensionless equations for the flow. With appropriate transformations the boundary layer equations are transformed into nonlinear ordinary differential equations. These equations are solved numerically by a finite element method. The effects of the various parameters on the velocity, temperature and concentration profiles as well as the skin-friction coefficient, Nusselt number and Sherwood number are presented graphically and in tabulated form.

Keywords: MHD, Vertical plate, Viscous dissipation, Finite element method.

NOMENCLATURE

U_0	the uniform velocity
$v(t)$	the suction velocity at the plate
c_p	specific heat at constant pressure
f	dimensionless stream function
g	acceleration due to gravity
Gr	Grashof number
Du	Dufour number
D_m	the mass diffusivity
T_m	the mean fluid temperature
M	magnetic parameter
Gm	solubility Grashof number
B_0	the magnetic induction
Pr	Prandtl number
T	temperature of the boundary layer
T_∞	temperature in the fluid far away from plate
C	the species concentration
C_∞	Concentration in the fluid far away from plate
Nu	Nusselt number
C_f	skin-friction coefficient

Greek symbols

α	the thermal diffusivity
u	dimensionless velocity component
ϕ	dimensionless concentration
x, y	dimensionless cartesian coordinates
β	co-efficient of thermal expansion
θ	dimensionless temperature
η	dimensionless similarity variable
k_T	the thermal diffusion ratio
Ec	Eckert number
β^*	co-efficient of concentration expansion
Sc	Schmidt number
μ, ν	dynamic and kinematic viscosities
ρ	density of the fluid
σ	electrical conductivity
c_s	the concentration susceptibility
ψ	stream function
Sr	Soret number
Sh	Sherwood number

1. Introduction

Coupled heat and mass transfer finds applications in a variety of engineering applications, such as the migration of moisture through the air contained in fibrous insulation and grain storage installations, filtration, chemical catalytic reactors and processes, spreading of chemical pollutants in plants and diffusion of medicine in blood veins. Free convection flow of an incompressible viscous fluid past an infinite or semi-infinite vertical plate has been studied since long because of its technological importance. Callahan and Marner (1976) solved the problem of transient free convection with mass transfer on an isothermal vertical plate using an explicit finite difference scheme. Unsteady free convective flow on taking into account the mass transfer phenomenon past an infinite vertical porous plate with constant suction was studied by Soundalgekar and Wavre (1977). Soundalgekar (1979) studied the effects of mass transfer and free convection currents on the flow past an impulsively started vertical plate. In these studies the magnetohydrodynamic phenomena is ignored. However in metallurgical transport systems, by drawing plates in an electrically conducting fluid subjected to a transverse magnetic field, the rate of cooling can be controlled and the final desired characteristics can be further refined. Magnetohydrodynamic flows has applications in meteorology, solar physics, cosmic fluid dynamics, astrophysics, geophysics and in the motion of earth's core. Shanker and Kishan (1997) presented the effect of mass transfer on the MHD flow past an impulsively started infinite vertical plate. Elabashbeshy (1997) studied heat and mass transfer along a vertical plate in the presence of magnetic field.

In all these studies Soret / Dufour effects are assumed to be negligible. Such effects are significant when density differences exist in the flow regime. For example when species are introduced at a surface in fluid domain, with different (lower) density than the surrounding fluid, both Soret and Dufour effects can be significant. Also, when heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of more intricate nature. It has been found that an energy flux can be generated not only by temperature gradients but by composition gradients as well. The energy flux caused by a composition gradient is called the Dufour or diffusion-thermo effect. On the other hand, mass fluxes can also be created by temperature gradients and this is the Soret or thermal-diffusion effect. The thermal-diffusion (Soret) effect, for instance, has been utilized for isotope separation, and in mixture between gases with very light molecular weight (H_2 , He) and of medium molecular weight (N_2 , air), the diffusion-thermo (Dufour) effect was found to be of a considerable magnitude such that it cannot be ignored (Eckert and Drake (1972)). In view of the importance of these above mentioned effects, Dursunkaya and Worek (1992) studied diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface, whereas Kafoussias and Williams (1995) studied the same effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity. Recently, Anghel et al. (2000) investigated the Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in a porous medium. Very recently, Postelnicu (2004) studied numerically the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects.

Unsteady free convection flows of dissipative fluids past an infinite plate have received a little attention because of non-linearity of the governing equations. Gebhart (1962) stated that "a significant viscous dissipation may occur in natural convection in various devices which are subject to large deceleration or which operate at high rotational speeds. Viscous dissipation effects may also be present in stronger gravitational fields and in processes wherein the scale of the process is very large, e.g., on larger planets, in large masses of gas in space, and in geological processes in fluids internal to viscous bodies". Gebhart and Mollendorf (1969) considered the effects of viscous dissipation for an external natural convection flow over a surface. Soundalgekar (1972) analyzed the effect of viscous dissipative heat on the two-dimensional unsteady, free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate.

However, the study of Soret and Dufour effects on unsteady MHD free convection flow of a dissipative fluid has received a little attention. Hence, the objective of the present paper is to analyze the Soret and Dufour effects on unsteady MHD free convection flow of a viscous incompressible fluid past an impulsively started vertical porous plate by taking viscous dissipation into account. The governing equations are transformed by using similarity transformation and the resultant dimensionless equations are solved by using the finite element method. The effects of various governing parameters on the velocity, temperature, concentration, skin-friction coefficient, Nusselt number and Sherwood number are shown in figures and tables and analyzed in detail.

2. Mathematical Analysis

An unsteady two-dimensional free convection flow of a viscous incompressible electrically conducting and dissipating fluid along an impulsively started vertical porous plate is considered. The x - axis is taken along the plate in the upward direction and the y - axis is taken normal to the plate. Initially, the plate and the fluid are at the same temperature T_∞ and the concentration C_∞ . At time $t > 0$, the plate starts moving impulsively in its own plane with a velocity U_0 and the plate temperature and concentration are raised to T_w and C_w respectively and are maintained constantly thereafter. A uniform magnetic field is applied in the direction perpendicular to the plate. The fluid is assumed to be slightly conducting, and hence the magnetic Reynolds number is much less than unity and the induced magnetic field is negligible in comparison with the applied magnetic field. It is further assumed that there is no applied voltage, so that electric field is absent. It is assumed that all the fluid properties are constant except that of the influence of the density variation with temperature and concentration in the body force term (Boussinesq's approximation). Then, under the above assumptions, the governing boundary layer equations of the flow field are

Mass conservation

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum conservation

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho} u \tag{2}$$

Thermal energy conservation

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{c_s c_p} \frac{\partial^2 C}{\partial y^2} + \frac{\nu}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 \tag{3}$$

Species conservation

$$\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial y^2} \tag{4}$$

The initial and boundary conditions are

$$\begin{aligned} t \leq 0 : u = 0, \quad v = 0, \quad T = T_\infty, \quad C = C_\infty \\ t > 0 : u = U_0, \quad v = v(t), \quad T = T_w, \quad C = C_w \quad \text{at } y = 0 \\ u \rightarrow 0, \quad v \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty \end{aligned} \tag{5}$$

In order to obtain a local similarity solution in the time dependent problem under consideration, a time dependent length scale δ is taken as

$$\delta = \delta(t) \tag{6}$$

In terms of this length scale, a convenient solution of Equation (1) is considered to be in the following form

$$v = v(t) = -v_0 \frac{y}{\delta} \tag{7}$$

where v_0 is the suction parameter.

In order to write the governing Equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\begin{aligned} \eta = \frac{y}{\delta}, \quad u = U_0 f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad Gr = \frac{g\beta(T_w - T_\infty)\delta^2}{\nu U_0}, \\ Gm = \frac{g\beta^*(C_w - C_\infty)\delta^2}{\nu U_0}, \quad M = \frac{\sigma B_0^2 \delta^2}{\nu \rho}, \quad Pr = \frac{\nu}{\alpha}, \quad Ec = \frac{U_0^2}{c_p(T_w - T_\infty)}, \\ Sc = \frac{\nu}{D_m}, \quad Du = \frac{D_m k_T (C_w - C_\infty)}{c_s c_p (T_w - T_\infty)}, \quad Sr = \frac{D_m k_T (T_w - T_\infty)}{\nu T_m (C_w - C_\infty)} \end{aligned} \tag{8}$$

where f is the dimensionless velocity and η is the dimensionless variable.

In view of the Equations (6) - (8), the Equations (2), (3) and (4) reduce to the following dimensionless form

$$f'' + \eta \left(\frac{\delta}{\nu} \frac{d\delta}{dt} \right) f' + \nu_0 f' + Gr\theta + Gm\phi - Mf = 0 \tag{9}$$

$$-\eta \left(\frac{\delta}{\nu} \frac{d\delta}{dt} \right) \theta' - \nu_0 \theta' = \frac{1}{Pr} \theta'' + Du\phi'' + Ec f'^2 \tag{10}$$

$$-\eta \left(\frac{\delta}{\nu} \frac{d\delta}{dt} \right) \phi' - \nu_0 \phi' = \frac{1}{Sc} \phi'' + Sr\theta'' \tag{11}$$

The corresponding dimensionless boundary conditions are

$$\begin{aligned} f = 1, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad \eta = 0 \\ f \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty \end{aligned} \tag{12}$$

Now, the Equations (9) - (11) are locally similar except $\left(\frac{\delta}{\nu} \frac{d\delta}{dt} \right)$, where t appears explicitly. Thus, the local similarity condition requires the form $\left(\frac{\delta}{\nu} \frac{d\delta}{dt} \right)$ in the Equations (9) - (11) must be a constant quantity.

Hence, following Hasimoto (1956), Sattar and Hossain (1992) and Sattar et al. (2000), one can try a class of solutions of the equations (9) - (11) by assuming that

$$\left(\frac{\delta}{\nu} \frac{d\delta}{dt} \right) = \lambda \text{ (a constant)} \tag{13}$$

Integrating equation (13), we have

$$\delta = \sqrt{2\lambda \nu t} \tag{14}$$

where the constant of integration is determined through the condition that $\delta=0$ when T

$t=0$. Here the problem is considered for small time. In this case the normal velocity in Equation (7) will be large i.e., suction will be large, which can be applied to increase the lift of airfoils. From Equation (14), choosing $\lambda =2$, the length scale $\delta(t) = 2\sqrt{\nu t}$ exactly corresponds to the usual scaling factor for various unsteady boundary layer flows (Schlichting (1968)). Since δ is a scaling factor as well as a similarity parameter, any value of λ in Equation (13) would not change the nature of the solutions, except that the scale would be different.

In view of the Equation (13) (with $\lambda =2$), the Equations (9) - (11) reduce to the following dimensionless ordinary differential equations, which are locally similar in time but not explicitly time dependent.

$$f'' + (2\eta + \nu_0) f' + Gr\theta + Gm\phi - Mf = 0 \tag{15}$$

$$\theta'' + Pr(2\eta + \nu_0) \theta' + Pr Du \phi'' + Pr Ec f'^2 = 0 \tag{16}$$

$$\phi'' + Sc(2\eta + \nu_0) \phi' + Sc Sr \theta'' = 0 \tag{17}$$

The corresponding boundary conditions are same as (12).

For the type of flow under consideration, the physical quantities such as the wall shear stress, surface heat flux and the surface mass flux are very important, which are given by

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \tag{18}$$

$$q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0} \tag{19}$$

$$M_w = -D_m \left(\frac{\partial C}{\partial y} \right)_{y=0} \tag{20}$$

The wall shear stress, surface heat flux and the surface mass flux for an impulsively started plate are

$$\frac{\tau_w \delta}{\mu U_0} = f'(0) \tag{21}$$

$$\frac{q_w \delta}{k (T_w - T_\infty)} = -\theta'(0) \tag{22}$$

$$\frac{M_w \delta}{D_m (C_w - C_\infty)} = -\phi'(0) \tag{23}$$

Hence, the dimensionless the skin-friction coefficient, Nusselt number and Sherwood number for impulsively started plate are given by

$$C_f = \frac{2\tau_w}{\rho U_0^2} = 2 (\text{Re}_\delta)^{-1} f'(0) \tag{24}$$

$$Nu = \frac{q_w \delta}{k (T_w - T_\infty)} = -\theta'(0) \tag{25}$$

$$Sh = \frac{M_w \delta}{D_m (C_w - C_\infty)} = -\phi'(0) \tag{26}$$

where $\text{Re}_\delta = \frac{U_0 \delta}{\nu}$ is the Reynolds number.

3. Solution of the problem

The set of differential Equations (15) - (17) are highly nonlinear and therefore it cannot be solved analytically. Hence, following Reddy (1985) and Bathe (1996), the finite element method is used to obtain an accurate and efficient solution to the boundary value problem under consideration.

The convergence has been efficiently achieved by fixing infinity as 6. The whole domain is discretized into a set of 100 line elements of equal width, each element being three-noded.

3.1 Variational formulation

The variational form associated with Equations (18) - (20) over a typical three-noded linear element (η_e, η_{e+2}) is given by

$$\int_{\eta_e}^{\eta_{e+2}} w_1 \{ f'' + (2\eta + \nu_0) f' + Gr \theta + Gm \phi - Mf \} d\eta = 0 \tag{27}$$

$$\int_{\eta_e}^{\eta_{e+2}} w_2 \{ \theta'' + RPr (2\eta + \nu_0) \theta' + RPr Ec f'^2 \} d\eta = 0 \tag{28}$$

$$\int_{\eta_e}^{\eta_{e+2}} w_3 \{ \phi'' + Sc (2\eta + \nu_0) \phi' \} d\eta = 0 \tag{29}$$

Where w_1 , w_2 and w_3 are the arbitrary test functions which may be viewed as the variations in the functions f , θ and ϕ respectively.

3.2 Finite element formulation

The finite element method seeks an approximate solution to the differential equation over each element. The polynomial approximation of the solution within a three-noded element is of the form

$$\begin{aligned} f &= \sum_{i=1}^3 f_i^e \psi_i^e \\ \theta &= \sum_{i=1}^3 \theta_i^e \psi_i^e \\ \phi &= \sum_{i=1}^3 \phi_i^e \psi_i^e \end{aligned} \tag{30}$$

where f_i^e , θ_i^e and ϕ_i^e are the values of the solution at the i^{th} node of the element and ψ_i^e are the shape functions for a typical element (η_e, η_{e+2}) and are taken as

$$\psi_1^e = \frac{\left(\eta - \frac{(2e-1)s}{200}\right)\left(\eta - \frac{(2e)s}{200}\right)}{\left(\frac{(2e-2)s}{200} - \frac{(2e-1)s}{200}\right)\left(\frac{(2e-2)s}{200} - \frac{(2e)s}{200}\right)} \quad (e = 1, 2, 3, \dots, 100),$$

$$\psi_2^e = \frac{\left(\eta - \frac{(2e-2)s}{200}\right)\left(\eta - \frac{(2e)s}{200}\right)}{\left(\frac{(2e-1)s}{200} - \frac{(2e-2)s}{200}\right)\left(\frac{(2e-1)s}{200} - \frac{(2e)s}{200}\right)} \quad (e = 1, 2, 3, \dots, 100), \quad (31)$$

$$\psi_3^e = \frac{\left(\eta - \frac{(2e-2)s}{200}\right)\left(\eta - \frac{(2e-1)s}{200}\right)}{\left(\frac{(2e)s}{200} - \frac{(2e-2)s}{200}\right)\left(\frac{(2e)s}{200} - \frac{(2e-1)s}{200}\right)} \quad (e = 1, 2, 3, \dots, 100).$$

where $s = 6$.

The coefficients f_i^e , θ_i^e and ϕ_i^e are determined such that Equations (27) - (29) are satisfied in the weighted integral sense.

Taking $w_1 = w_2 = w_3 = \psi_i^e$ ($i = 1, 2, 3$), the finite element model of the equations thus formed.

The stiffness matrix for the Equation (27) is as follows.

$$[K_{e,j}^f]\{f\} = [B_{e,j}^f]$$

where

$$K_{e,j}^f = - \int_{\eta_e}^{\eta_{e+2}} \left(\sum_{i=1}^3 f_i^e \frac{\partial \psi_i^e}{\partial \eta} \frac{\partial \psi_j^e}{\partial \eta} \right) d\eta + \int_{\eta_e}^{\eta_{e+2}} \left((2\eta + v_0) \left(\sum_{i=1}^3 f_i^e \frac{\partial \psi_i^e}{\partial \eta} \right) \right) \psi_j^e d\eta$$

$$- M \int_{\eta_e}^{\eta_{e+2}} \left(\sum_{i=1}^3 f_i^e \psi_i^e \right) \psi_j^e d\eta,$$

$$[B_{e,j}^f] = -Gr \int_{\eta_e}^{\eta_{e+2}} \left(\sum_{i=1}^3 \theta_i^e \psi_i^e \right) \psi_j^e d\eta - Gm \int_{\eta_e}^{\eta_{e+2}} \left(\sum_{i=1}^3 \phi_i^e \psi_i^e \right) \psi_j^e d\eta,$$

$$(e = 1, 2, 3, \dots, 100), (j = 1, 2, 3)$$

and

$$\{f\} = [f_1, f_2, f_3, \dots, f_{201}]^T.$$

The stiffness matrix for Equation (28) is as follows.

$$[K_{e,j}^\theta]\{\theta\} = [B_{e,j}^\theta]$$

where

$$[K_{e,j}^\theta] = - \int_{\eta_e}^{\eta_{e+2}} \left(\sum_{i=1}^3 \theta_i^e \frac{\partial \psi_i^e}{\partial \eta} \frac{\partial \psi_j^e}{\partial \eta} \right) d\eta + R Pr \int_{\eta_e}^{\eta_{e+2}} \left((2\eta + v_0) \sum_{i=1}^3 \theta_i^e \frac{\partial \psi_i^e}{\partial \eta} \right) \psi_j^e d\eta,$$

$$[B_{e,j}^\theta] = R Pr Ec \int_{\eta_e}^{\eta_{e+2}} \left(\sum_{i=1}^3 f_i^e \frac{\partial \psi_i^e}{\partial \eta} \right)^2 \psi_j^e d\eta, \quad (e = 1, 2, 3, \dots, 100), (j = 1, 2, 3)$$

and

$$\{\theta\} = [\theta_1, \theta_2, \theta_3, \dots, \theta_{201}]^T.$$

The stiffness matrix for the Equation (29) is as follows.

$$[K_{e,j}^\phi]\{\phi\} = [B_{e,j}^\phi]$$

where

$$[K_{e,j}^\phi] = - \int_{\eta_e}^{\eta_{e+2}} \left(\sum_{i=1}^3 \phi_i^e \frac{\partial \psi_i^e}{\partial \eta} \frac{\partial \psi_j^e}{\partial \eta} \right) d\eta + Sc \int_{\eta_e}^{\eta_{e+2}} \left((2\eta + v_0) \sum_{i=1}^3 \phi_i^e \frac{\partial \psi_i^e}{\partial \eta} \right) \psi_j^e d\eta,$$

$$[B_{e,j}^\phi] = 0, \quad (e = 1, 2, 3, \dots, 100), (j = 1, 2, 3)$$

and

$$\{\phi\} = [\phi_1, \phi_2, \phi_3, \dots, \phi_{201}]^T$$

These system of 201x201 matrix equations are solved by the iteration scheme. Applying the given boundary conditions only a system of 195 equations remains to be solved. At the beginning of the first iteration the velocity is taken to be zero and the system of equations is solved using a Gaussian elimination method for the nodal temperature and concentration. Thus the values at the first iteration are obtained. Now, the nodal temperature and concentration values are used to get nodal velocity and then second iteration nodal temperature and concentration values are obtained. This process is repeated until the desired accuracy of 0.0005 is obtained.

4. Results and discussion

A representative set of numerical results is shown graphically in Figs.1-9, to illustrate the influence of physical parameters viz., the thermal Grashof number Gr , solutal Grashof number Gm , magnetic field parameter M , Prandtl number Pr , Eckert number Ec , Dufour number Du , Schmidt number Sc , Soret number Sr and the suction parameter v_0 on the velocity, temperature, concentration.

The effect of the thermal Grashof number Gr on the velocity is presented in Fig.1. The thermal Grashof number Gr signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. It is observed that an increase in Gr causes a significant increase in the velocity. Therefore, with an increase in buoyancy force due to temperature differences, the flow is accelerated in the boundary layer. Here the positive values of Gr correspond to cooling of the plate by natural convection.

The influence of the solutal Grashof number Gm on the velocity is presented in Fig.2. The solutal Grashof number Gm defines the ratio of the species buoyancy force to the viscous hydrodynamic force. It is found that the velocity increases considerably with a rise in Gm . The solutal Grashof number boosts velocity of the fluid indicating that species buoyancy has an accelerating effect on the flow field.

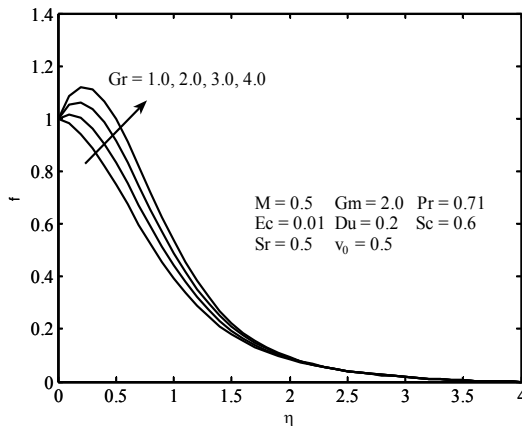


Fig.1: Velocity profiles for different values of Gr

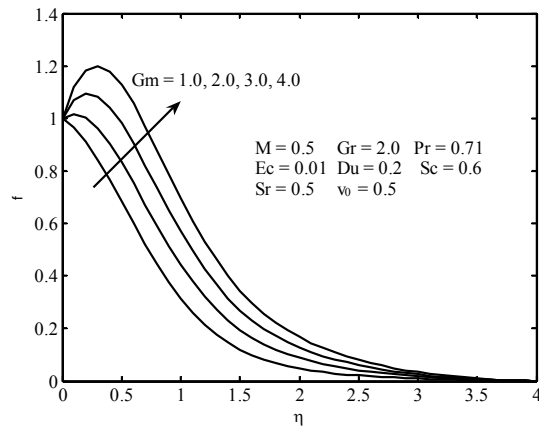


Fig.2: Velocity profiles for different values of Gm

Fig.3 depicts the velocity profiles for different values of the magnetic field parameter M . It is noticed that an increase in the magnetic parameter M leads to a decrease in the velocity. The application of a transverse magnetic field to an electrically conducting fluid gives rise to a resistive force called Lorentz force. This force has the tendency to slow down the motion of the fluid. Also, the boundary layer thickness decreases with an increase in the magnetic parameter.

For different values of the Prandtl number Pr , the velocity and temperature profiles are plotted in Figs. 4(a) and 4(b). The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. From Fig. 4 (a), it is clear that an increase in the Prandtl number leads to a fall in the velocity. From Fig.4 (b), it is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated surface more

rapidly than for higher values of Pr . Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.

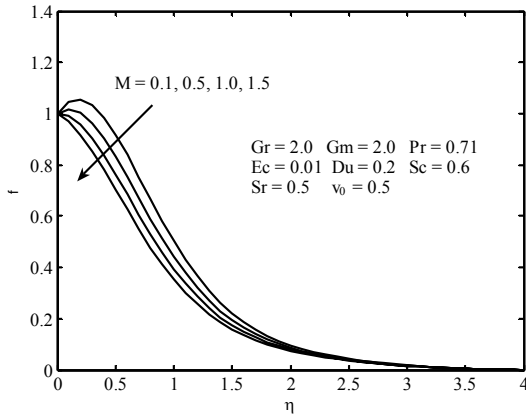


Fig.3: Velocity profiles for different values of M

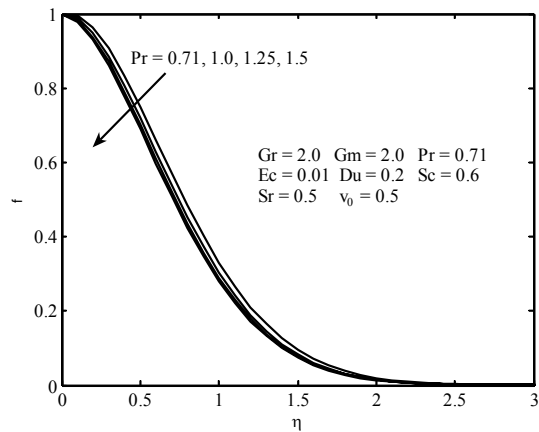


Fig.4(a): Velocity profiles for different values of Pr

The influences of Eckert number on the dimensionless velocity and temperature functions are shown in Figs 5(a) and 5(b) respectively. The Eckert number designates the ratio of the kinetic energy of the flow to the boundary layer enthalpy difference. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. The positive Eckert number implies cooling of the plate i.e., loss of heat from the plate to the fluid. Hence, greater viscous dissipative heat causes a rise in the temperature as well as the velocity, which is evident from Figs. 5 (a) and 5 (b).

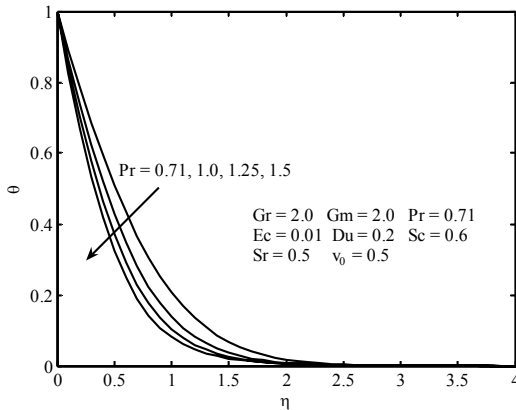


Fig.4(b): Temperature profiles for different values of Pr

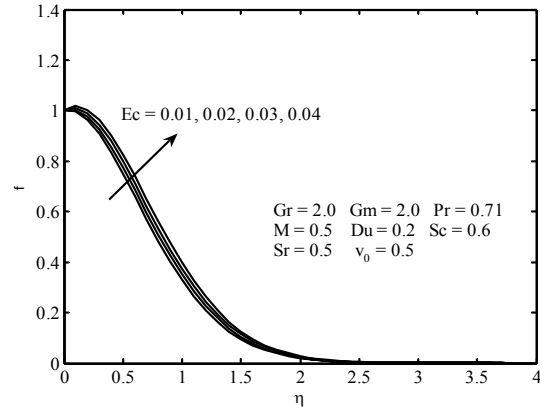


Fig.5(a): Velocity profiles for different values of Ec

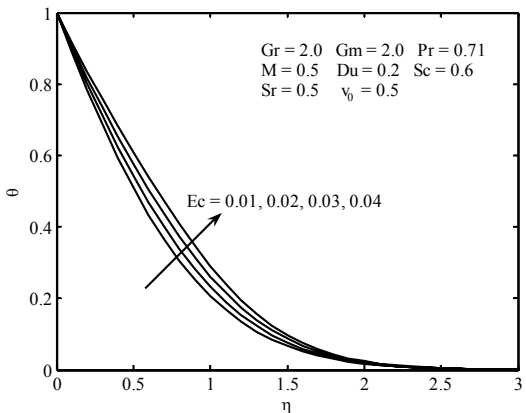


Fig.5(b): Temperature profiles for different values of Ec

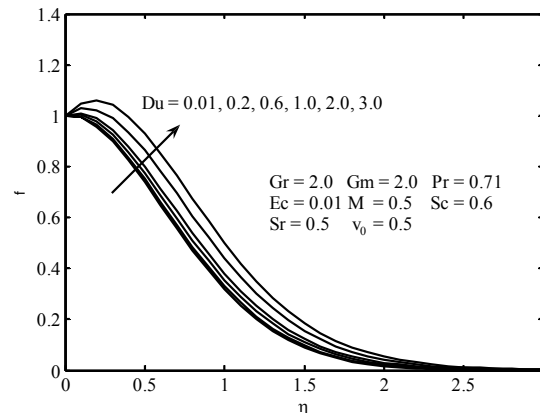


Fig.6(a): Velocity profiles for different values of Du

For different values of the Dufour number Du , the velocity and temperature profiles are plotted in Figs. 6(a) and 6(b) respectively. The Dufour number Du signifies the contribution of the concentration gradients to the thermal energy flux in the flow. It is observed that an increase in the Dufour number causes a rise in the velocity and temperature throughout the boundary layer. For $Du \leq 1$, the temperature profiles decay smoothly from the surface to the free stream value. However for $Du > 1$, a distinct velocity overshoot exists near the plate, and thereafter the profile falls to zero at the edge of the boundary layer.

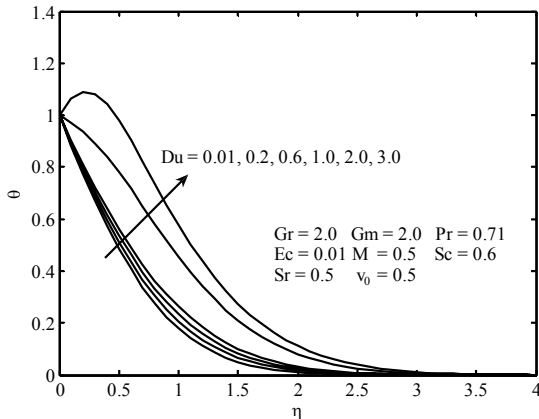


Fig.6(b): Temperature profiles for different values of Du

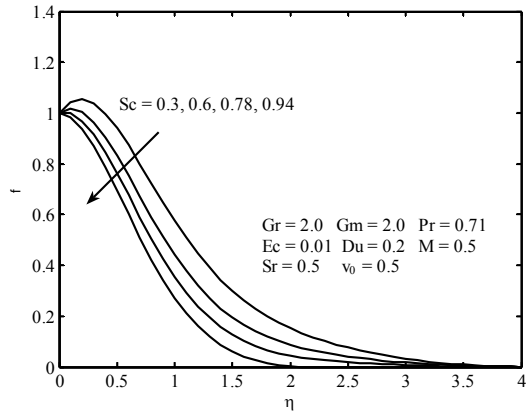


Fig.7(a): Velocity profiles for different values of Sc

Figs. 7(a) and 7(b) illustrate the velocity and concentration profiles for different values of Schmidt number Sc . The Schmidt number embodies the ratio of the momentum to the mass diffusivity. The Schmidt number therefore quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. It is observed that as the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers.

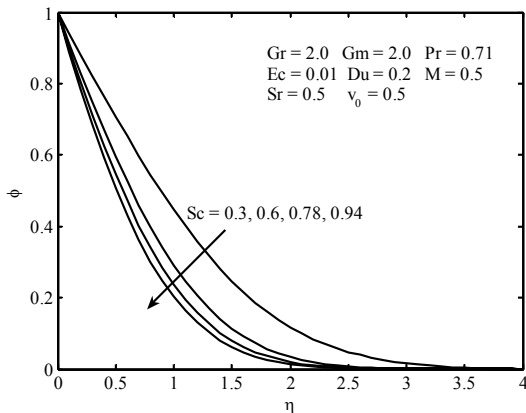


Fig.7(b): Concentration profiles for different values of Sc

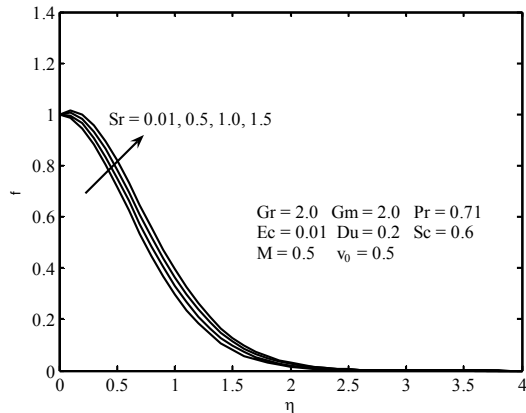


Fig.8(a): Velocity profiles for different values of Sr

The influence of Soret number Sr , on the dimensionless velocity and concentration are shown in Figs 8(a) and 8(b) respectively. The Soret number Sr defines the effect of the temperature gradients inducing significant mass diffusion effects. It is noticed that an increase in the Soret number Sr results an increase in the velocity and concentration within the boundary layer.

The effect of suction parameter v_0 on the velocity, temperature and concentration are shown in Figs. 9(a), 9(b) and 9(c) respectively. It is found that an increase in the suction parameter results a decrease in the velocity, temperature and concentration.

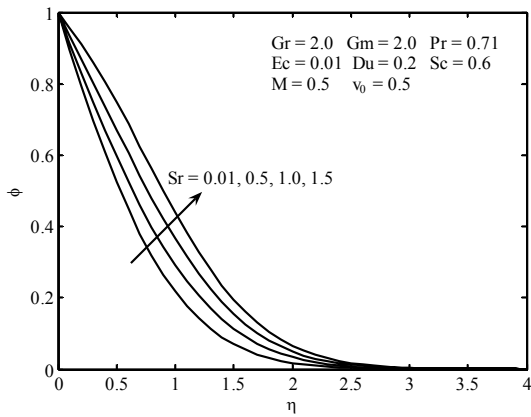


Fig.8(b): Concentration profiles for different values of Sr

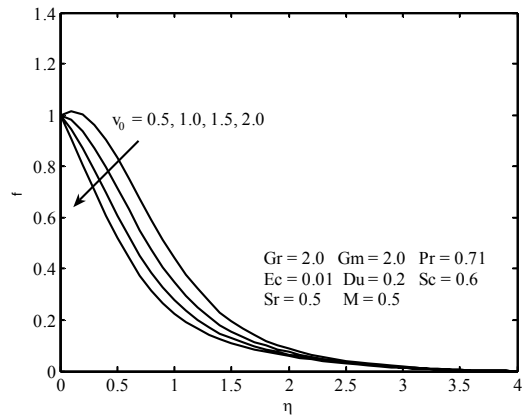


Fig.9(a): Velocity profiles for different values of v_0

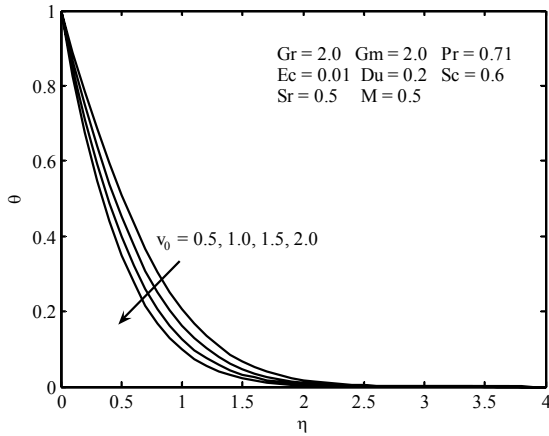


Fig.9(b): Temperature profiles for different values of v_0

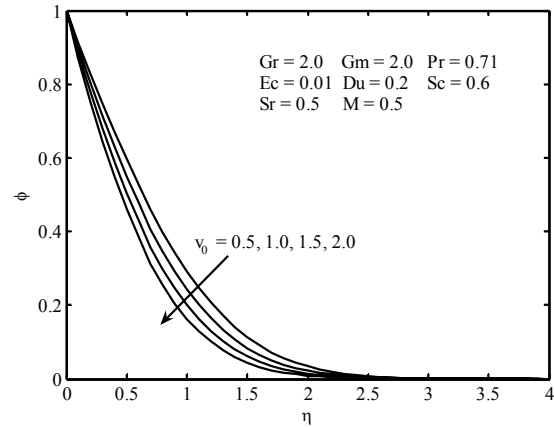


Fig.9(c): Concentration profiles for different values of v_0

Table 1 Numerical values of the skin-friction coefficient, Nusselt number and Sherwood number for $Pr = 0.71$, $Ec = 0.01$, $Du = 0.2$, $Sc = 0.6$, $Sr = 1.0$

Gr	Gm	M	v_0	C_f	Nu	Sh
2.0	2.0	0.5	0.5	0.13667	1.12041	0.86906
4.0	2.0	0.5	0.5	0.96562	1.12856	0.86928
2.0	4.0	0.5	0.5	1.08165	1.12939	0.86932
2.0	2.0	1.0	0.5	0.08308	1.12164	0.86908
2.0	2.0	0.5	0.5	0.05655	1.16602	0.89668

Table 2 Numerical values of the skin-friction coefficient, Nusselt number and Sherwood number for $Gr = 2.0$, $Gm = 2.0$, $M = 0.5$, $v_0 = 0.5$

Pr	Ec	Du	Sc	Sr	C_f	Nu	Sh
0.71	0.01	0.2	0.6	1.0	0.13657	1.12041	0.86906
1.0	0.01	0.2	0.6	1.0	0.04929	1.38856	0.79899
0.71	0.02	0.2	0.6	1.0	0.13768	1.11745	0.86782
0.71	0.01	0.4	0.6	1.0	0.16925	1.05172	0.88610
0.71	0.01	0.2	0.78	1.0	0.06718	1.10419	1.00834
0.71	0.01	0.2	0.6	2.0	0.23309	1.14636	0.65036

Tables 1-2 show the effects of the thermal Grashof Number, solutal Grashof number, magnetic field parameter, suction parameter, Prandtl number, Eckert number, Dufour number, Soret number and the Schmidt number on the skin-friction coefficient C_f , Nusselt number Nu , and Sherwood number Sh . From Table 1, it is noticed that as Gr or Gm increases, there is a rise in the skin-friction coefficient, Nusselt number and the Sherwood number. It is observed that as M increases, there is a fall in the skin-friction coefficient, the Nusselt number and the Sherwood number. It is noticed that as the suction parameter increases, the skin-friction coefficient decreases, while the Nusselt number and the Sherwood number increase. From Table 2, it is found that as Pr increases there is a decrease in the skin-friction coefficient and Sherwood number, while the Nusselt number increases. It is also observed that an increase in Ec or Du leads to an increase in the skin-friction coefficient and Sherwood number while a decrease in the Nusselt number. It is clear that as the Schmidt number increases the skin-friction coefficient and the Nusselt number decrease while the Sherwood number increases. It is also seen that an increase in Sr leads to an increase in the skin-friction coefficient and Nusselt number while a decrease in the Sherwood number.

5. Conclusions

The Soret and Dufour effects of an unsteady free convection flow magnetohydrodynamic and viscous dissipating fluid past an impulsively started vertical plate has been analyzed. With appropriate transformations the boundary layer equations are transformed into nonlinear ordinary differential equations. These equations are solved numerically by a finite element method. From the above mentioned discussion, following conclusions can be drawn:

1. The velocity increases with the increase thermal Grashof number and solutal Grashof number.
2. The velocity decreases with an increase in the magnetic parameter.
3. A positive increase in Eckert number is shown to reduce the velocity and temperatures in the flow.
4. Increasing the Prandtl number substantially decreases the translational velocity and the temperature .
5. An increase in the Dufour number causes a rise in the velocity and temperature throughout the boundary layer.
6. The velocity as well as concentration decreases with an increase in the Schmidt number.
7. An increase in the Soret number results an increase in the velocity and concentration with in the boundary layer.

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