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EFFECTS OF VISCOUS DISSIPATION ON MHD FLOW WITH HEAT AND MASS TRANSFER OVER A STRETCHING SURFACE WITH HEAT SOURCE, THERMAL STRATIFICATION AND CHEMICAL REACTION

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Abstract:

This paper deals with the study of nonlinear MHD flow, with heat and mass transfer characteristics of an incompressible, viscous, electrically conducting and Boussinesq fluid on a vertical stretching surface with thermal stratification and chemical reaction by taking in to account the viscous dissipation effects. Adopting the similarity transformation, governing nonlinear partial differential equations of the problem are transformed to nonlinear ordinary differential equations. The Quasi-linearization technique is used for the non-linear momentum equation and then the numerical solution of the problem is derived using implicit finite difference technique, for different values of the dimensionless parameters. The numerical values obtained for velocity profiles, temperature profiles and concentration profiles are represent graphically in figures. The results obtained show that the flow field is influenced appreciably by the presence of viscous dissipation, thermal stratification, chemical reaction and magnetic field.

Keywords: MHD, non-linear, quasi-linearization, finite-difference scheme, chemical reaction, thermal stratification, viscous dissipation.

NOMENCLATURE

B_0	magnetic field of strength
C_p	the specific heat at constant pressure
Pr	Prandtl number
Re_x	Reynolds number
M	magnetic field parameter
u ,v	velocity components in x ,y- directions respectively
Ec	Eckert number
S	Heat Source parameter
T	the temperature of the fluid
T_∞	the free stream temperature
T_w	the surface temperature of the plate
T_0	constant reference temperature
C	the concentration of the species
C_w	the concentration of the species at the plate
C_∞	the concentration of the species far from the wall
n_1	the consistency index
D	the molecular diffusion coefficient
f	dimensionless stream function
g	acceleration due to gravity
k	thermal conductivity of the fluid
Gr_x	Grashof Number
Gc_x	Modified Grashof Number
Sc	Schimdt number

Greek symbols

α	Thermal diffusivity
μ	Coefficient of viscosity
ν	Kinematics viscosity
ρ	Density of the fluid
θ	Dimensionless temperature
ϕ	Dimensionless concentration
β	coefficient of thermal expansion
β^*	coefficient of concentration expansion
η	Similarity parameter
ψ	Stream function
γ	Chemical reaction parameter

Subscripts:

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n	thermal stratification parameter	w	condition at wall
m_1, N, N_1	constants	∞	condition at infinity

1. Introduction

Combined heat and mass transfer problems with chemical reaction are importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Heat and mass transfer for an electrically conducting fluid flow under the influence of magnetic field are considered to be significant importance due to its applications in many engineering problems such as nuclear reactors and those dealing with liquid metals. In many mixed flows of practical importance as well as in many engineering devices, the environment in thermal stratified. The thermal stratification effects of heat transfer over a stretching surface are of interest in polymer extrusion process where the object, after passing through a die, enters the fluid of cooling below a certain temperature. A large amount of research work has been done in the field of chemical reaction, heat and mass transfer. The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many braches of science and engineering.

In the fast decades, the penetration theory of Highie (1935) had been widely applied to unsteady state diffusion problems with and without chemical reaction. As far as we can ascertain, all the solutions with chemical reactions were obtained for the case of a semi-infinite body of liquid, although physical absorption into a finite film was considered. Among some of the interesting problems which were studied is the analysis of laminar forced convection mass transfer with homogeneous chemical reaction.

A study has been carried out to obtain the nonlinear MHD flow with heat and mass transfer characteristics of an incompressible viscous, electrically conducting and Boussinesq fluid on a vertical isothermal, cone and heat generation/absorption by EL-Kabeir et al (2007). Kandasamy and Devi (2004) studied effects of chemical reaction, heat and mass transfer on non-linear laminar boundary layer flow over a wedge with suction and injection. Takhar et al (2000) investigated the flow and mass diffusion of chemical species with first order and higher order reactions over a continuous stretching sheet with an applied magnetic field.

A study on non-linear hydro magnetic flow, heat and mass transfer over an accelerating vertical surface with internal heat generation and stratification effects is carried by Kandasamy and Periasamy (2005). Singh (2001) analyzed the MHD free convection and mass transfer flow with heat source and thermal diffusion. The paper deals with the study of free convection and mass transfer flow of an incompressible, viscous and electrically conducting fluid past a continuously moving infinite vertical plate in the presence of large suction and under the influence of uniform magnetic field considering heat source and thermal diffusion. A study on MHD free convective flow of an incompressible viscous dissipative fluid in an infinite vertical oscillating plate with constant heat flux has been carried by Kishan et al (2006).

The problem of stretching surface with constant surface temperature was analyzed by Afzal (1993). The process involving the mass transfer effect has long been recognized as important principality in chemical processing equipment. Recently, the non linear MHD flow with heat and mass transfer characteristic of an incompressible, viscous, electrically conducting and Boussinesq fluid on a vertical stretching surface with chemical reaction and thermal stratification effects are studied by Kandasamy and Periasamy and Periasamy(2005).

In all the above papers the viscous dissipation has been neglected, but in practical point of view it has to be considered. The present paper deals with the two-dimensional steady nonlinear MHD boundary layer flow of an incompressible, viscous, electrically conductive and Boussinesq fluid flowing over a vertical stretching surface in the presence of uniform magnetic field by taking into account the viscous dissipation with heat, mass transfer chemical reaction and thermal stratification effects.

2. Mathematical analysis

According to the coordinate system, the x-axis is chosen parallel to the vertical surface and the y-axis is taken normal to it. A transverse magnetic field of strength Bo is applied parallel to y-axis. The fluid properties are assumed to be constant in a limited temperature range. The value of C_∞ is set zero in the problem as the concentration of species far from the wall C_∞ is infinitesimally small (Bird et al. 1992) and hence the Soret and Dufour effects are negligible. The chemical reactions are taking place in the flow and the physical properties μ, ρ, D and the rate of chemical reaction, k_1 are constant throughout the fluid. It is assumed that the induced magnetic field, the external electric field and the electric field due to the polarization of charges are negligible. Under these conditions, the governing boundary layer equations of momentum, energy and diffusion under Boussinesq approximation are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-(\frac{\sigma Bo^2}{\rho})u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\ Q(T_\infty-T)+\mu(\frac{\partial u}{\partial y})^2 \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_1 C \tag{4}$$

The boundary conditions are

$$\begin{aligned}
&u = U(x) = \mathbf{a}x, \quad v = 0.\\
&T = T_w(x), \quad C = C_w(x) \quad \text{at} \quad y = 0\\
&u = 0, \; T = T_\infty(x) = (1-n)T_0 + nT_w(x),\\
&C = C_\infty \quad \text{as } y \to \infty
\end{aligned} \tag{5}$$

where **a** is dimensional constant and **n** is a constant which is the thermal stratification parameter and is such that $0 \le n < 1$. The n defined as thermal stratification parameter is equal to $m_1/(1+m_1)$ of Nakayama and Koyama (1989), where m_1 is constant. T_0 is constant reference temperature say, $T_\infty(0)$. The suffix w and ∞ denote surface and ambient conditions respectively.

Now we introduce similarity variables as follows:

$$\psi = (\nu x U(x))^{\frac{1}{2}} f(\eta) \tag{6}$$

$$\eta = (\frac{U(x)}{\nu x})^{\frac{1}{2}} y \tag{7}$$

The velocity components are given by:

$$u=\frac{\partial\psi}{\partial y}, \qquad v=-\frac{\partial\psi}{\partial x} \tag{8}$$

It can be easily verified that the continuity Equation (1) is identically satisfied. Similarity solutions exist if we assume that U(x) = **a**x and introduce the non-dimensional form of temperature and concentration as:

$$\theta=\frac{(T-T_\infty)}{(T_w-T_\infty)}; \qquad \phi=\frac{(C-C_\infty)}{(C_w-C_\infty)}$$

$$Ec = \frac{U^3}{vx(T_w - T_\infty)}$$ (Eckert number)

$$Re_x = \frac{Ux}{v}$$ (Reynolds Number)

$$Gr_x = vg\beta\frac{(T_w - T_\infty)}{U^3}$$ (Grashof number)

$$Gc_x = vg\beta^*\frac{(C_w - C_\infty)}{U^3}$$ (Modified Grashof number)

$$\Pr = \mu\frac{C_p}{K}$$ (Prandtl Number)

$$Sc = \frac{v}{D}$$ (Schimdt number)

$$M^2 = \frac{\sigma Bo^2}{\rho a}$$ (Magnetic parameter)

$$\gamma = \frac{vk_1}{U^2}$$ (Chemical reaction parameter)

$$S = \frac{2XQ}{U}$$ (Heat source parameter)

In this work, temperature variation of the surface is taken into account and is also given by the power-law temperature, $T_w - T_\infty = Nx^n$ where N and n are constants. Also concentration variation is given by $C_w - C_\infty = N_1 x^{n_1}$ where N_1 and n_1 are constants. In terms of the above non-dimensional variables and using the Equations (6) – (8), the non-linear equations (2) – (4) written as

$$f''' + Gc\,\mathrm{Re}\,\phi + Gr\,\mathrm{Re}\,\theta - (f')^2 - (\frac{M^2}{\mathrm{Re}})f' + ff'' = 0 \quad (9)$$

$$\theta'' - \Pr\ f'(\theta + n/(1-n)) + \Pr\ f\theta' - S\Pr\theta + Ec(f'')^2 = 0 \quad (10)$$

$$\phi'' - Sc(\phi\gamma\,\mathrm{Re} + f'\phi) + Scf\phi' = 0 \quad (11)$$

The boundary condition equation (5) are given by

$$f(0) = 0,\quad f'(0) = 1,\quad \theta(0) = 1,\quad \phi(0) = 1$$
$$f'(\infty) = 0,\quad \theta(\infty) = 0,\quad \phi(\infty) = 0 \quad (12)$$

Applying the Quasi–linearization technique [12] to the non-linear equation (9), we obtain as

$$f''' + A_1 f'' - A_2 f' + A_3 f = D_1 \quad (13)$$

Where $A_1[i] = F[i]$; $A_2[i] = 2F'[i] + \frac{M^2}{\mathrm{Re}})$; $A_3[i] = f''[i];$

$D_1[i] = -(F')^2 + FF'' - Gc\,\mathrm{Re}\,\phi - Gr\,\mathrm{Re}\,\theta$

Where F is assumed to be known function. Using an implicit finite difference scheme for the Equation (13), (10) and (11) we get the following

$$f[i+2] + a_1[i]f[i+1] + b_1[i]f[i] + c_1[i]f[i-1] = d_1[i] \quad (14)$$

$$a_2[i]\theta[i+1] + b_2[i]\theta[i] + c_2[i]\theta[i-1] = d_2[i] \quad (15)$$

$$a_3[i]\phi[i+1]+b_3[i]\phi[i]+c_3[i]\phi[i-1]=0 \quad (16)$$

Where

$$a_1[i]=-3-0.5*h^2(2F_1[i]+\frac{M^2}{\mathrm{Re}})+F[i]h$$

$$c_1[i]=-1+0.5*h^2(2F_1[i]+\frac{M^2}{\mathrm{Re}})+F[i]h$$

$$d_1[i]=h^3(F[i]F_2[i]-Gc\,\mathrm{Re}\,\phi-Gr\,\mathrm{Re}\,\theta-(F_1[i])^2)$$

$$a_2[i]=1+0.5*h\Pr f[i]; \quad b_2[i]=(-2-h^2\Pr(f_1[i]+S));$$

$$c_2[i]=(1-0.5*h\Pr f[i])$$

$$d_2[i]=\frac{n}{(1-n)}\Pr f_1[i]h^2-h^2Ec(F_2[i])^2$$

$$a_3[i]=1+0.5*hScf[i]; \quad b_3[i]=-2-h^2Sc(\gamma\,\mathrm{Re}+f_1[i]); \quad c_3[i]=1-0.5*hScf[i]$$

To obtain the numerical solutions the system of Equations (14) – (16) with boundary conditions (12) are solved by using the Gauss-Siedel iterative method. Here the numerical solutions of f are considered as the j^{th} order iterative solution and F as the $(j-1)^{th}$ iterative solution. Here h represents the mesh size in η direction. For convergence of a solution, after each cycle of iteration, the tolerance is set at 10^{-6} i.e., $|F-f|<10^{-6}$ is satisfied at all points.

3. Results and Discussion

In order to get a clear insight of the physical problem, numerical results are displayed with the help of graphical illustrations. The velocity, temperature and concentration of the fluid are shown graphically to observe the effects of parameter entering in the problem. The dimensionless velocity profiles for different values of magnetic field parameter *M*, thermal stratification parameter n, Chemical reaction parameter γ, Eckert number *Ec*, Heat source parameter *S* and Schmidt number Sc are presented in Figures 1-5. In Fig.1 it is observed that the velocity of the fluid decreases with the increase of magnetic field parameter *M* for constant values of γ, n and *Ec*. The dimensionless velocity profiles for a different values of thermal stratification for constant values of *M*, γ, and *Ec* are depicted in Fig.2. It is seen that the velocity of the fluid decreases with increase in thermal stratification.

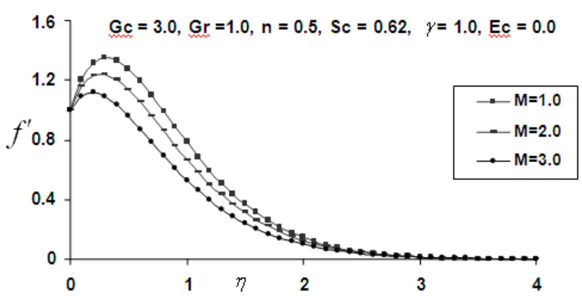

Fig. 1: Velocity profiles for different magnetic parameters

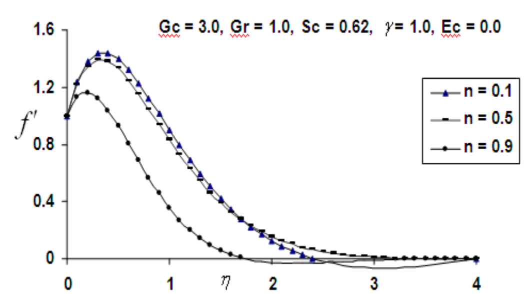

Fig. 2: Velocity profiles for different thermal stratification

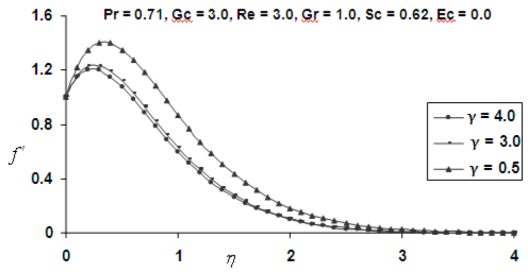

Fig. 3: Velocity profiles for different chemical reaction parameters

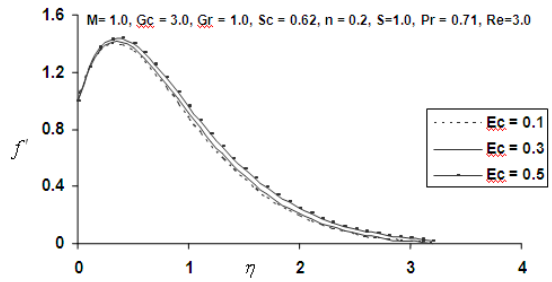

Fig. 4: Velocity profikes for different values of Eckert number

Fig.3 presents the velocity profiles for different values of chemical reaction parameter for fixed values of *M*, *n* and *Ec*. It is clear that the velocity of the fluid decreases with the increase of chemical reaction. Fig. 4 demonstrates the effect of viscous dissipation on velocity profiles for fixed values of *M*, *n* and γ . It is seen that the increase of viscous dissipation increases the velocity of the fluid. The increase of Schmidt number decreases the velocity of the fluid for fixed values of *M*, *n* and *Ec* is observed in Fig. 5.

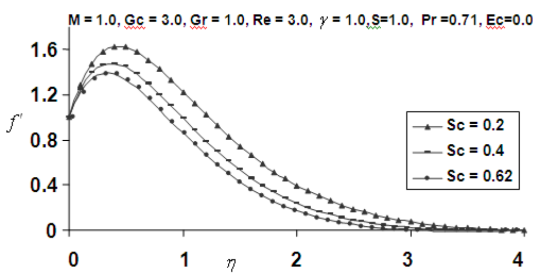

Fig. 5: Velocity profiles for different valus of Schmidt number

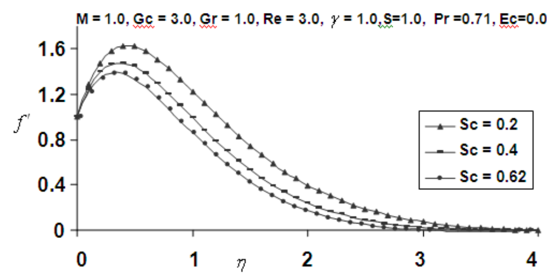

Fig. 6: Temperature profiles for different magnetic parameters

The dimensionless temperature profiles for different values of magnetic field parameter *M*, thermal stratification parameter n, Chemical reaction parameter γ, Eckert number *Ec* and Schmidt number *Sc* are presented in Figures 6-10. The effect of magnetic field on temperature profiles is shown in Fig. 6. It is noticed that the temperature of the fluid increases with increase of magnetic field for constant values of γ, *n* and *Ec*. The dimensionless temperature profiles for a different values of thermal stratification for constant values of *M*, γ, and *Ec* are depicted in Fig.7. It is seen that the temperature of the fluid decreases with increase in thermal stratification.

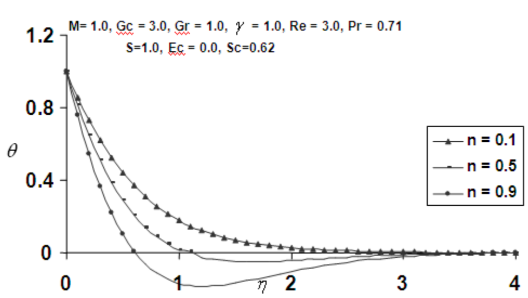

Fig. 7: Temperature profiles for different thermal stratification

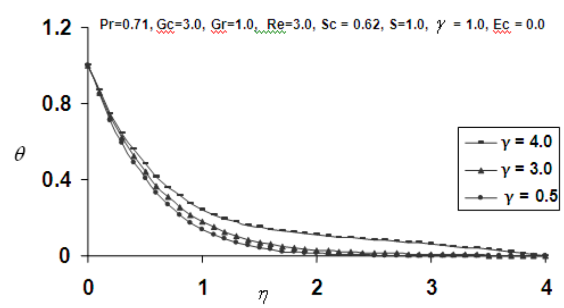

Fig. 8: Temperature Profiles for different chemical reaction parameters

From Fig. 8 it is clear that the temperature of the fluid increases with increase of chemical reaction parameter for fixed values of *M*, *n* and *Ec*. Whereas from Fig. 9, it is observed that the temperature of the fluid increases with increase of viscous dissipation parameter for fixed values of *n*, *M* and γ. The increase of Schmidt number increases the temperature of the fluid for fixed values of *M*, *n* and *Ec* it is seen in Fig.10.

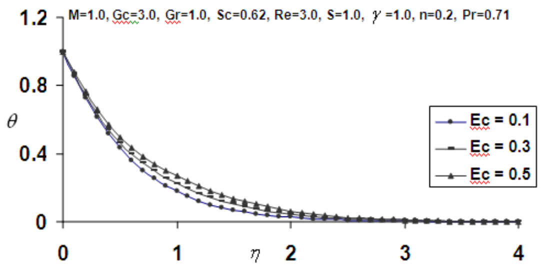

Fig. 9: Temperature profiles for different values of Eckert number

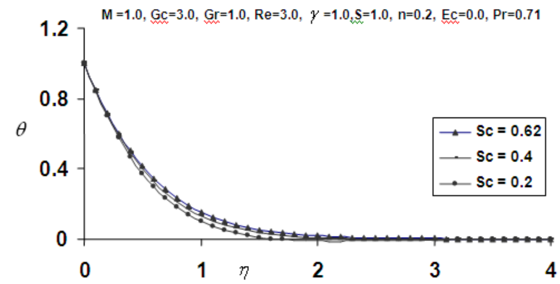

Fig.10: Temperature profiles for different values of Schmidt number

In Figures 11–14, the dimensionless concentration profiles for different values of magnetic field parameter *M*, thermal stratification parameter n, chemical reaction parameter γ and Schmidt number Sc are presented. From Fig.11 it is noticed that with the increase of magnetic field the concentration of the fluid increases for constant values of γ, *n* and *Ec*. Fig.12 shows that the concentration of the fluid increases with the increase of thermal stratification for constant values of *M*, γ, and *Ec*. From fig.13 it is clear that with the increase of chemical reaction parameter the concentration of the fluid decreases for fixed values of *M*, *n* and *Ec*. The concentration of the fluid decreases with the increase of Schmidt number for fixed values of *M*, *n* and *Ec* is observed from Fig. 14.

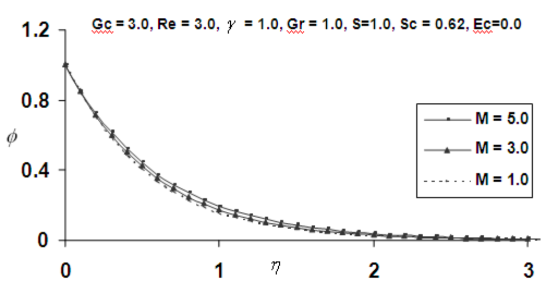

Fig. 11: Concentration profiles for different magnetic parameters

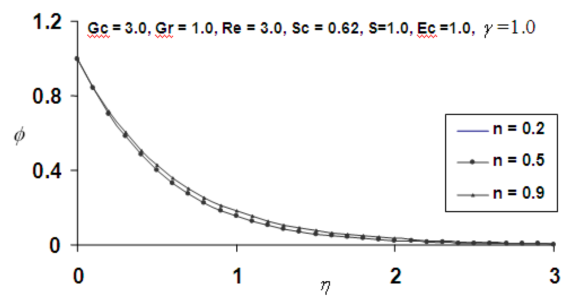

Fig. 12: Concentration profiles for different thermal stratification

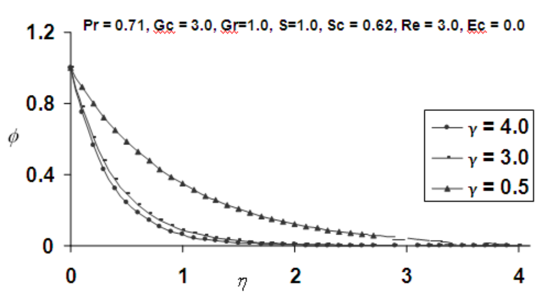

Fig. 13: Concentration profiles for different chemical reaction parameter

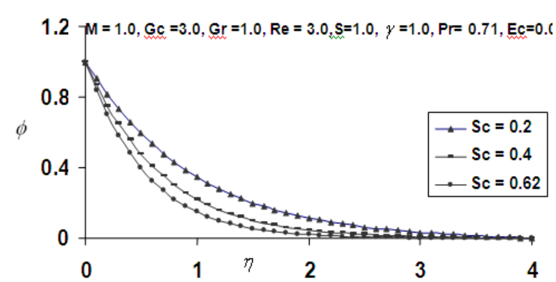

Fig. 14: Concentration profiles for different values of Schmidt number

4. Conclusion

In the presence of the uniform magnetic field, thermal stratification parameter and chemical reaction parameter, the velocity and the temperature of the fluid increase with the increase of viscous dissipation.

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