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SEA WAVE MODELLING FOR MOTION CONTROL APPLICATIONS

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Abstract:

The modelling of sea environment is important in designing an effective motion control system for any marine vehicle. Inadequate representation of the components of a typical random sea might lead to poor performance of the control system. A multiple output system such as the one having components of wave elevation and slope, facilitates designing the control system taking into account the different degrees of freedom. The method of modelling the sea environment presented here provides the basis for the design of motion control systems for multiple degree of freedom cases, which give rise to excitation forces and moments acting on the marine vehicle. The method used here models the sea environment using Gaussian white noise and shaping filter to generate a multiple output form of the random sea state. In the first step a given standard wave spectrum is approximated using a rational polynomial, the coefficients of the polynomial are obtained by least square fitting method to best match the spectrum. The established rational polynomial is then decomposed to get the transfer function of the shaping filter. The wave slope spectrum is similarly approximated using the same rational polynomial. The transfer functions of the two components of amplitude and slope, representing the filters are combined to generate a state space form. Using the white noise as input, the state space form obtains the wave elevation and slope as outputs. By performing spectral analysis using Welch method (1967), the quality of the obtained output is checked against the targetted spectrum. The application of the simulated wave slope spectrum in a closed loop state space model is demonstrated as applied to the roll stabilization characteristics of a stationary ship using a passive tank.

Keywords: Shaping filter, white noise, rational polynomial, least square fitting, state space model.

NOMENCLATURE

$a_{\delta\delta}$	tank added mass coefficient	C_{44}	restoring moment coefficient
$b_{\delta\delta}$	tank damping coefficient	E_w	wave induced roll moment
$c_{\delta\delta}$	tank stiffness coefficient	h_r	height of the fluid in the reservoir
$a_{\delta4}$	tank moment due to unit roll acceleration	h_d	height of the horizontal duct
$c_{\delta4}$	tank moment due to unit roll displacement	r_d	Distance between ship CG and tank origin
$a_{4\delta}$	roll moment due to unit tank angle acceleration	Q_t	passive tank parameter
$c_{4\delta}$	roll moment due to unit tank angle displacement	q_v	tank coefficient of resistance
I_{44}	virtual mass moment of inertia	x_t	length of the tank
B_{44}	roll damping coefficient	w	length between reservoir
		w_r	width of vertical reservoir

1. Introduction

Modelling the stochastic nature of a seaway is important in designing and analyzing an effective motion control system for marine applications. The efficacy of any motion control system basically depends on its performance in the case of non-linearities and irregular changes in the environmental conditions. In ship hydrodynamics, the irregular environmental conditions are expressed in terms of sea state spectrum which describes the nature of a particular sea. Random seas are essentially composed of an infinite sum of sinusoids at various frequencies and magnitudes, combined to form with a uniform probability distribution, and random phase. It is necessary to have a meaningful mathematical description of the apparently random nature. Statistically the sea surface follows the well-known Gaussian or normal distribution. In probability theory, the central limit theorem justifies the use of normal distribution in many applications. By the central limit theorem a random variable formed by the sum of a large number of independent random variables, is characterized by an approximately normal distribution (Rice, 2007). Different methods are used to reproduce the characteristics of the sea and many of them which are used for numerical simulations, are not applicable for control system design strategy. In this context, the method of

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shaping filter has been cited as a valuable tool for modelling the stochastic nature of the linear dynamical systems (Martin and David, 2007).

The present paper describes the method of shaping filter for modelling a given sea state condition for motion control applications. The method of shaping filter is based on random process theory, established by filtered white noise. The output of a linear system having appropriate spectral density with an ergodic Guassian white noise input is also an ergodic Guassian random process possessing required spectral density. When the Guassian white noise is sent through the shaping filter, it shapes the white noise signal into the desired colored noise output, i.e., desired random waves of specified frequency. This is the theoretical basis of using shaping filter to generate random wave.

2. Methodology

The methodology used to generate the components of the sea state spectrum using the method of shaping filter through white noise filtering is depicted in Fig.1. The method has three parts namely: approximation of standard wave spectrum, parameter estimation and finally, simulation of the desired sea state. The standard wave spectrum is approximated for the desired sea state using a rational filter, which consists of a finite degree of polynomials in the numerator and denominator. The spectrum to be achieved is known and therefore the degree of polynomials can be selected to give the estimated shape of the desired spectrum. Secondly, the coefficients of the rational filter are determined by least square fitting to best match the targetted spectrum, named as rational spectrum. The established rational spectrum is then decomposed to get the transfer function of the shaping filter. This can be carried out through spectral factorization (Kallstrom, 1981). The power spectral density function can be related for a unit white noise by the equation:

$$\Phi_x(\omega) \;=\; |Z(j\omega)|^2 .1 \tag{1}$$

where, Z is the transfer function of the linear system. By finding Z from Eq. (1), the spectral density function can be modelled as a linear dynamical system with white noise as input signal. For an even rational polynomial, the power spectrum is decomposed into $Z(j\omega)\; Z(-j\omega)$ and the transfer function of shaping filter for the power spectrum is simply $Z(j\omega)$ (Xu et al., 2011).

The rational polynomial is also used to approximate the wave slope spectrum. The filter transfer function of standard wave spectrum with its slope component is represented into a state space form to obtain single-input and two-output system. By inputting a stationary Guassian white noise signal to the state space representation, the desired sea can be realized in the form of wave elevation and wave slope.

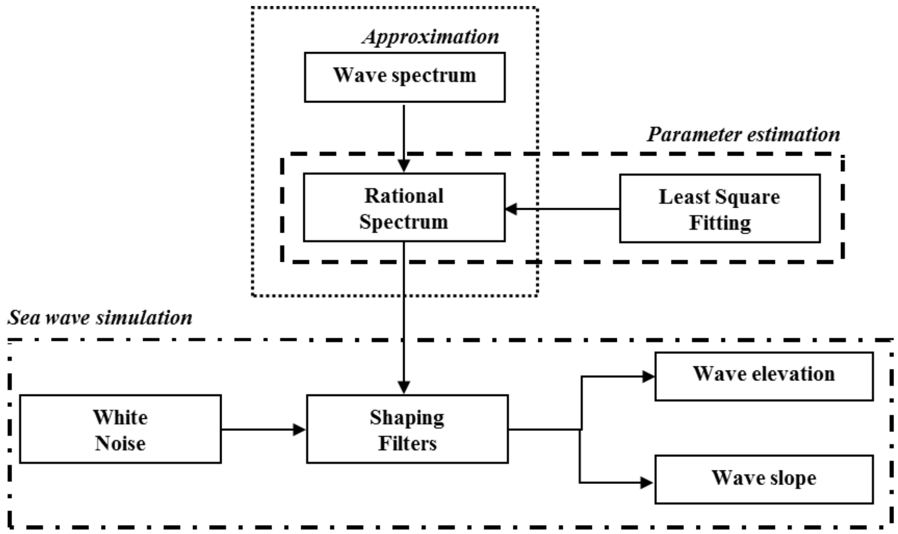

Fig. 1: Methodology for sea wave simulation

3. Sea Wave Modelling

The seastate spectrum used for the present study takes the form of ITTC 2-parameter wave spectrum that permits period and wave height to be assigned separately, defined by the following (Bhattacharya, 1978):

$$S(\omega) = \frac{173\,H_s^2}{T_1^4\,\omega^5}\exp\left(\frac{-691}{T_1^4\,\omega^4}\right) \tag{2}$$

where $S(\omega)$ is the wave amplitude spectral ordinate, H_s is the significant wave height in m, ω is the wave frequency in rad/sec and T_1 is the period corresponding to average frequency of the component waves. The definition Table for the sea state considered and the typical values associated with them are given in Table 1.

Table 1: Sea state parameter

Sea state	Significant wave height, H_s (m)	Characteristic period T_1(sec)
5	3.3	9.7

3.1 Rational spectrum approximation

The following rational polynomial was selected to approximate the standard ITTC spectrum (Kallstrom, 1981):

$$S_R(\omega) = \frac{b_2^2\omega^2}{\omega^6 + (a_1^2 - 2a_2)\omega^4 + (a_2^2 - 2a_1a_3)\omega^2 + a_3^2} \tag{3}$$

where, a_1, a_2, a_3, and b_2 are real numbers.

We now derive the solution for the coefficients in the rational polynomial as shown below:

Eq. (3) can be written in matrix form as:

$$\begin{bmatrix}\omega^2 & -S_R(\omega)\omega^4 & -S_R(\omega)\omega^2 & -S_R(\omega)\end{bmatrix}\begin{bmatrix} b_2^2 \\ a_1^2 - 2a_2 \\ a_2^2 - 2a_1a_3 \\ a_3^2 \end{bmatrix} = S_R(\omega)\,\omega^6 \tag{4}$$

The ITTC spectrum can be approximated using the rational function as,

$$S(\omega_i) = S_R(\omega_i)\ , \ i = 1, 2, ..., N \tag{5}$$

The parameters of the rational spectrum, Eq. (3) can be best approximated by establishing the N algebraic equations by least square method. In matrix notation, the equation for polynomial fit is given by:

$$H = X.A \tag{6}$$

where,
$$X = \begin{bmatrix} \omega_1^2 & -S(\omega_1)\,\omega_1^4 & -S(\omega_1)\,\omega_1^2 & -S(\omega_1) \\ \omega_2^2 & -S(\omega_2)\,\omega_2^4 & -S(\omega_2)\,\omega_2^2 & -S(\omega_2) \\ ... & ... & ... & ... \\ ... & ... & ... & ... \\ \omega_N^2 & -S(\omega_N)\,\omega_N^4 & -S(\omega_N)\,\omega_N^2 & -S(\omega_N) \end{bmatrix} \tag{7}$$

$$H' = [S(\omega_1)\omega_1^6 \quad S(\omega_2)\omega_2^6 \quad ... \quad S(\omega_N)\omega_N^6] \tag{8}$$

$$A' = [A(1) \quad A(2) \quad A(3) \quad A(4)] \tag{9}$$

where $A(1) = b_2^2$, $A(2) = a_1^2 - 2a_2$, $A(3) = a_2^2 - 2a_1a_3$ and $A(4) = a_3^2$

Eq. (6) can be solved by pre-multiplying by the transpose X^T,

$$X^T . H = X^T . X . A \tag{10}$$

The parameters of the rational spectrum are obtained by solving the matrix equation by the least square method:

$$A = (X^T . X)^{-1} . X^T . H \tag{11}$$

In order to know the coefficient values of the rational polynomial a_1, a_2, a_3, and b_2, A(3) is now written as:

$$A(3) = \left(\frac{a_1^2 - A(2)}{2}\right)^2 - 2a_1\sqrt{A(4)} \tag{12}$$

Eq. (12) is a fourth degree equation:

$$a_1^4 - 2A(2)a_1^2 - 8\sqrt{A(4)}\,a_1 + A(2)^2 - 4A(3) = 0 \tag{13}$$

The coefficient value for a_1 is obtained by solving Eq. (13) and from the expression for A(2), the coefficient value for a_2 is also obtained. The least square fitted values are substituted in Eq. (3) to get the approximated rational spectrum of the Standard ITTC spectrum, Eq. (2).

The comparisons of sea spectrum with the obtained rational spectrum are shown in Fig. (2). The coefficient values obtained are presented in Table 2.

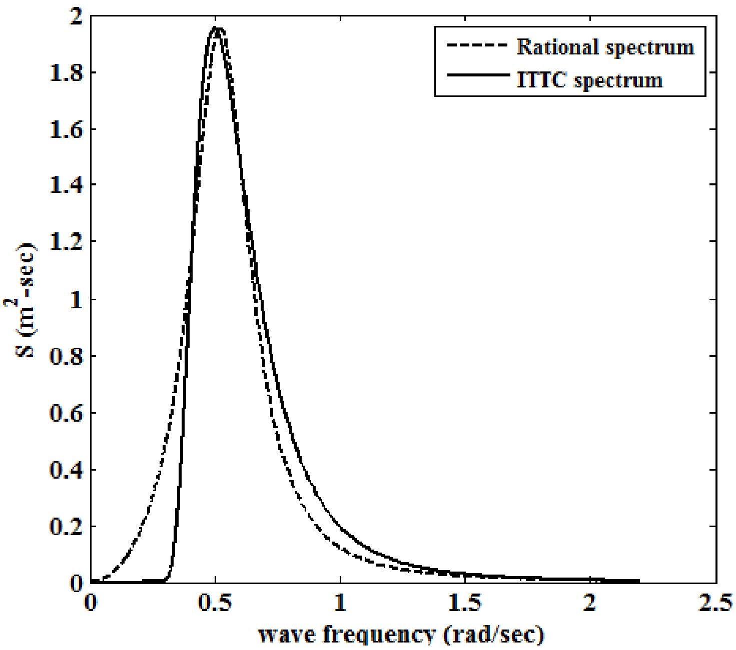

Fig. 2: Rational spectrum approximation for given sea state

Table 2: Coefficient values for rational spectrum

Sea state	b_2	a_1	a_2	a_3
5	0.298	0.792	0.442	0.148

3.2 Shaping filter

The spectral factorization on the rational function in Eq. (3) will have the form in S-domain (Kallstrom, 1981):

$$S_R(s) = \frac{-b_2 s}{-s^3 + a_1 s^2 - a_2 s + a_3} \cdot \frac{b_2 s}{s^3 + a_1 s^2 + a_2 s + a_3} \tag{14}$$

The zero points, pole points and gain in the left plane of S-domain constitute the right hand factor of Eq. (14). As mentioned earlier, the transfer function of shaping filter is,

$$S(s) = \frac{b_2 s}{s^3 + a_1 s^2 + a_2 s + a_3} \tag{15}$$

Oscillatory motion such as roll motion, is more sensitive to wave slope than the wave height. The same formulation can be used to define the wave slope realization by approximating the wave slope spectrum. The wave slope spectrum can be represented with respect to rational filter:

$$\Phi_s = \frac{\omega^4}{g^2} S_R(\omega) \tag{16}$$

The slope spectrum obtained in Eq. (16) can be approximated by the following filter:

$$\Phi_s(s) = \frac{c b_2 s^2}{s^3 + a_1 s^2 + a_2 s + a_3} \tag{17}$$

where, c is a constant. The value of c can be obtained in an analogous manner by approximating the slope spectrum in Eq. (16) with the slope filter, i.e., Eq. (17). Figure 3 shows the wave slope spectrum approximation for the given sea state with the filter Eq. (17). The comparison shows good match in the peak region of the slope spectrum. Thus, the values for c were obtained and estimated as -0.061 for generating the wave slope spectrum. By combining filter Eqs. (15) and (17), a state space representation for a single input and multiple output system is achieved which can be used as an input for control system design:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -a_1 & 1 & 0 \\ -a_2 & 0 & 1 \\ -a_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ b_2 \\ 0 \end{bmatrix} e \tag{18}$$

$$\begin{bmatrix} h \\ s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -a_1 c & c & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{19}$$

where, wave height (h) and wave slope (s) are the output of this system. The wave height and wave slope have been realized by passing the white noise signal through the shaping filter for standard deviation corresponding to 4.5 and time step of 0.1s. The MATLAB based simulations are shown as results in Fig. 4.

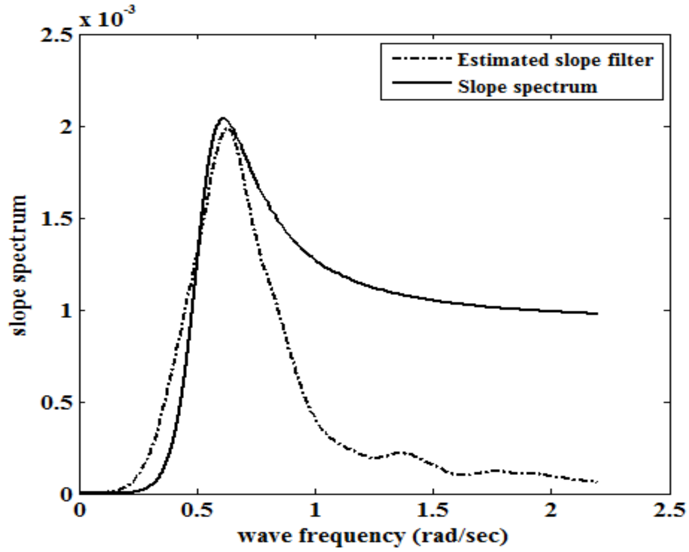

Fig. 3: Estimated filter with slope spectrum

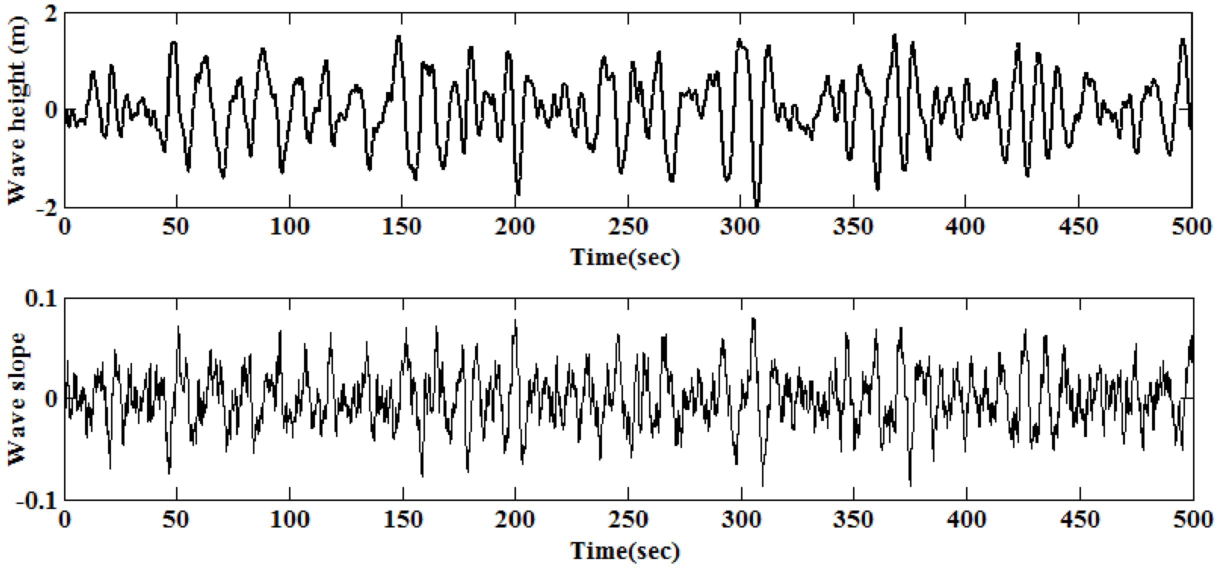

Fig. 4: Wave height and wave slope realization for sea state 5

4. Spectral Analysis

A spectral analysis of the output has been performed using Welch method (1967) to test the filter output against the desired sea state spectrum. The Welch method estimates the spectrum of a given signal by grouping the data into overlapping segments, computing a modified periodogram of each segment, and then averaging the spectrum over all the sets. Fast Fourier Transform (FFT) is used to estimate the spectrum of each set, which involves sectioning the record, taking modified periodograms of these sections, and averaging these modified periodograms. By permitting data sample overlap, the averaging of modified periodograms tends to decrease the variance of the estimated power spectral density.

In the method, for a given stochastic process, K segments are assumed which cover the entire record of the process, then for each segment of length L, a modified periodogram is calculated. A data window $\mathrm{W}(j)$, $j = 0,\ldots,\mathrm{L}\text{-}1$, is selected to form the segment sequence. Then finite Fourier transforms are taken, $\mathrm{A}_1(n), \ldots, \mathrm{A}_\mathrm{K}(n)$ to estimate these sequences. The spectral estimate is the average of these periodograms. The estimator (Welch, 1967) is given as follows:

$$\mathrm{P}(f_n) = \frac{1}{\mathrm{K}}\sum_{k=1}^{\mathrm{K}}\mathrm{I}_k(f_n) \;, \quad (n=0,1,\ldots,\mathrm{L}/2 \;\text{ and }\; k=1,2,\ldots,\mathrm{K}) \tag{20}$$

where, $f_n = \dfrac{n}{\mathrm{L}}$

$$\mathrm{I}_k(f_n) = \frac{\mathrm{L}}{\mathrm{U}}\left|\mathrm{A}_k(n)\right|^2 \;, \text{ and}$$

$$\mathrm{U} = \frac{1}{\mathrm{L}}\sum_{j=0}^{\mathrm{L}\text{-}1}\mathrm{W}^2(j)$$

A periodic Hamming window is taken as the window function. The method reduces computations and gives better control over the variance characteristics of the estimated power spectral density. A close correlation is achieved and the spectral analysis confirms the sea state spectra. The estimated spectrum is shown in Fig.5.

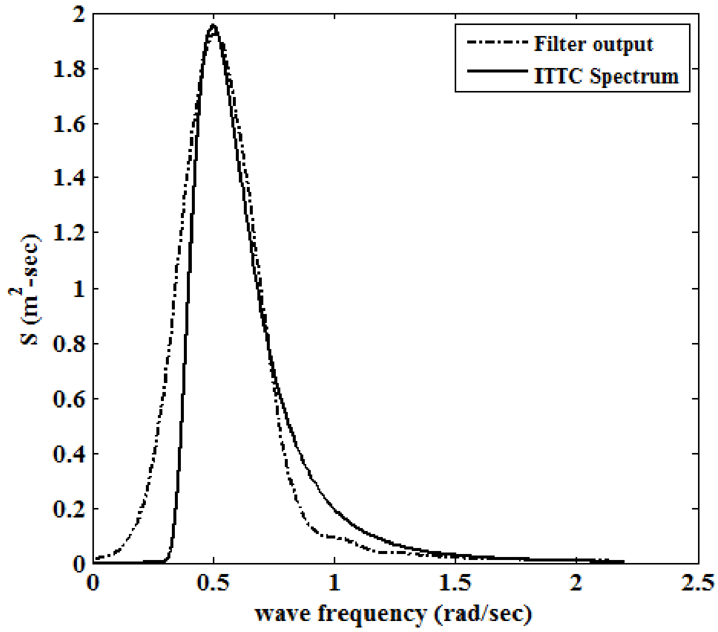

Fig. 5: Spectral estimation of wave spectrum for sea state 5

5. State Space Modelling of Passive Tank Stabilization

The generated wave slope components are used to demonstrate the effect of passive tank stabilization in irregular waves. Passive tanks are more effective in regular waves than random waves. Phan et al., (2008) have validated the performance of an anti-roll tank in irregular wave conditions. A state space modelling of ship system with passive tank has been presented here to demonstrate the effectiveness of the method and for design of such motion control systems.

The generated wave slope has been used to perturb the ship from its equilibrium position at zero speed in beam sea condition. The particulars of the ship and anti-roll tank are shown in Table 3 and 4. The geometric representation and the nomenclature used for the tank dimensions are followed as given in Gawad et al., (2001).

Table 3: Main particulars of the ship

Ship particulars	
Length, L_{PP}	123.20 m
Displacement	3990 tons
Max. beam	14.40 m
Draft	4.50 m
Metacentric height (GM_T)	1.30 m
Natural roll period	9.80 s
K_{XX}	5.40 m
K_{YY}	30.30 m
K_{ZZ}	30.30 m

Table 4: General characteristics of the tank

Tank particulars	
Length of the tank	3.74 m
Length between reservoir	11.00 m
Width of vertical reservoir	3.00 m
Fluid height in the reservoir	4.00 m
Height of horizontal duct	0.85 m
Distance between ship CG and tank origin	2.50 m
Fluid density	1025 kg/m^3
Tank fluid volume	127 m^3

The liquid level inside the tank is adjusted to get the highest effect, i.e., at the selected liquid level the natural frequency of the tank and ship are best matched. The coupled equation of motion of ship in the presence of a passive tank and fluid motion inside the tank is expressed as:

$$\ddot{\phi} = \frac{E_w}{I_{44}} - \frac{B_{44}}{I_{44}}\dot{\phi} - \frac{C_{44}}{I_{44}}\phi + \frac{a_{4\delta}}{I_{44}}\ddot{\delta} + \frac{c_{4\delta}}{I_{44}}\delta \tag{21}$$

$$\ddot{\delta} = -\frac{a_{\delta 4}}{a_{\delta\delta}}\ddot{\phi} - \frac{c_{\delta 4}}{a_{\delta\delta}}\phi - \frac{b_{\delta\delta}}{a_{\delta\delta}}\dot{\delta} - \frac{c_{\delta\delta}}{a_{\delta\delta}}\delta \tag{22}$$

The formula to obtain the tank coefficients are given below (Gawad et al., 2001):

$$a_{\delta\delta} = Q_t w_r \left(\frac{w}{2h_d}\frac{h_r}{w_r}\right) \tag{23}$$

$$b_{\delta\delta} = Q_t\, q_v\, w_r \left(\frac{w}{2h_d^2} + \frac{h_r}{w_r^2}\right) \tag{24}$$

$$c_{\delta\delta} = Q_t\, g \tag{25}$$

$$c_{\delta 4} = Q_t\, g \tag{26}$$

$$a_{\delta 4} = Q_t\,(r_d + h_r) \tag{27}$$

$$a_{4\delta} = a_{\delta 4} \tag{28}$$

$$c_{4\delta} = c_{\delta 4} \tag{29}$$

$$Q_t = \frac{1}{2}\,\rho_t w_r w^2 x_t \tag{30}$$

The ship motion coefficients have been obtained using standard strip theory program, SEAWAY (Journee, 2001). For modelling the anti-roll tank stabilization, a state space representation:

$$\dot{x} = Fx + Gu + \Pi E_W \tag{31}$$

where, F is the system matrix, G is the input matrix and Π is the disturbance distribution matrix. For this case, i.e., passive stabilization, input is zero and the stabilization is only due to the movement of water inside the tank. For a beam sea, the wave induced roll moment is calculated from the wave slope and the quasi-static excitation (Sgobbo and Parsons, 1999):

$$E_w = (\nabla\rho g\, GM_T).\ (\text{wave slope}) \tag{32}$$

Therefore, Eqs. (21) and (22) are represented in a state space form as follows:

$$\begin{bmatrix}\ddot{\phi}\\ \dot{\phi}\\ \ddot{\delta}\\ \dot{\delta}\end{bmatrix} = \begin{bmatrix} -\frac{B_{44}}{P} & -\frac{1}{P}\left(C_{44} + \frac{a_{4\delta}\, c_{\delta 4}}{a_{\delta\delta}}\right) & -\frac{a_{4\delta}\, b_{\delta\delta}}{P\, a_{\delta\delta}} & \frac{1}{P}\left(c_{4\delta} - \frac{a_{4\delta} c_{\delta\delta}}{a_{\delta\delta}}\right)\\ 1 & 0 & 0 & 0\\ \frac{a_{\delta 4} B_{44}}{Q\, I_{44}} & -\frac{1}{Q}\left(c_{\delta 4} - \frac{a_{\delta 4} C_{44}}{I_{44}}\right) & -\frac{b_{\delta\delta}}{Q} & -\frac{1}{Q}\left(c_{\delta\delta} + \frac{a_{\delta 4} c_{4\delta}}{I_{44}}\right)\\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix}\dot{\phi}\\ \phi\\ \dot{\delta}\\ \delta\end{bmatrix} + \begin{bmatrix}\frac{\nabla\rho g\, GM_T}{P}\\ 0\\ -\frac{a_{\delta 4}\,\nabla\rho g\, GM_T}{Q\, I_{44}}\\ 0\end{bmatrix}[\text{wave slope}] \tag{33}$$

where, $P = I_{44} + \frac{a_{4\delta} a_{\delta 4}}{a_{\delta\delta}}$ and $Q = a_{\delta\delta} + \frac{a_{\delta 4} a_{4\delta}}{I_{44}}$

Based on the above state space representation, a computational model is setup using MATLAB and Simulink. The corresponding mathematical expressions were specified through user defined function (Fcn block) in Simulink. The modelling of system dynamics along with the wave slope is shown in Fig.6. The performance of the passive tank in irregular sea is shown in Fig.7. The RMS value obtained for the simulation has been presented in Table 5.

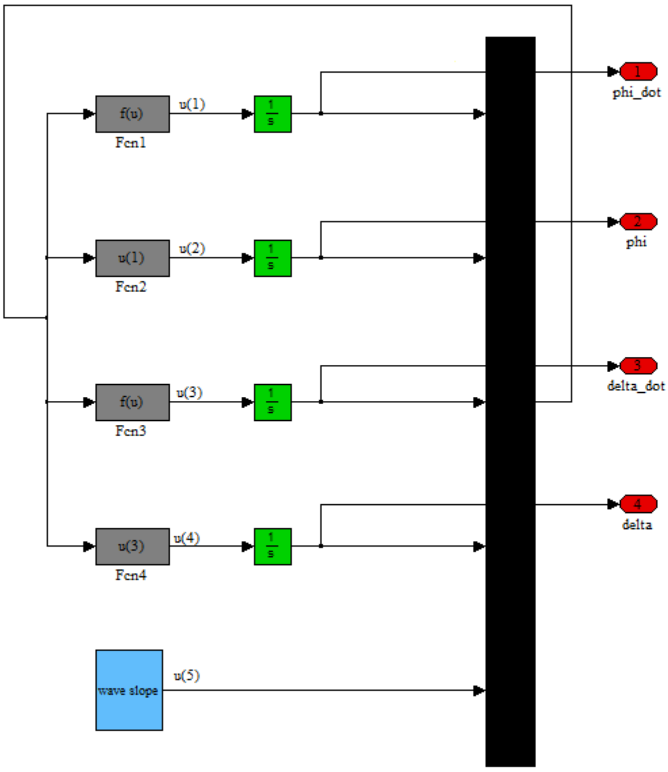

Fig. 6: Closed loop modelling of system matrix

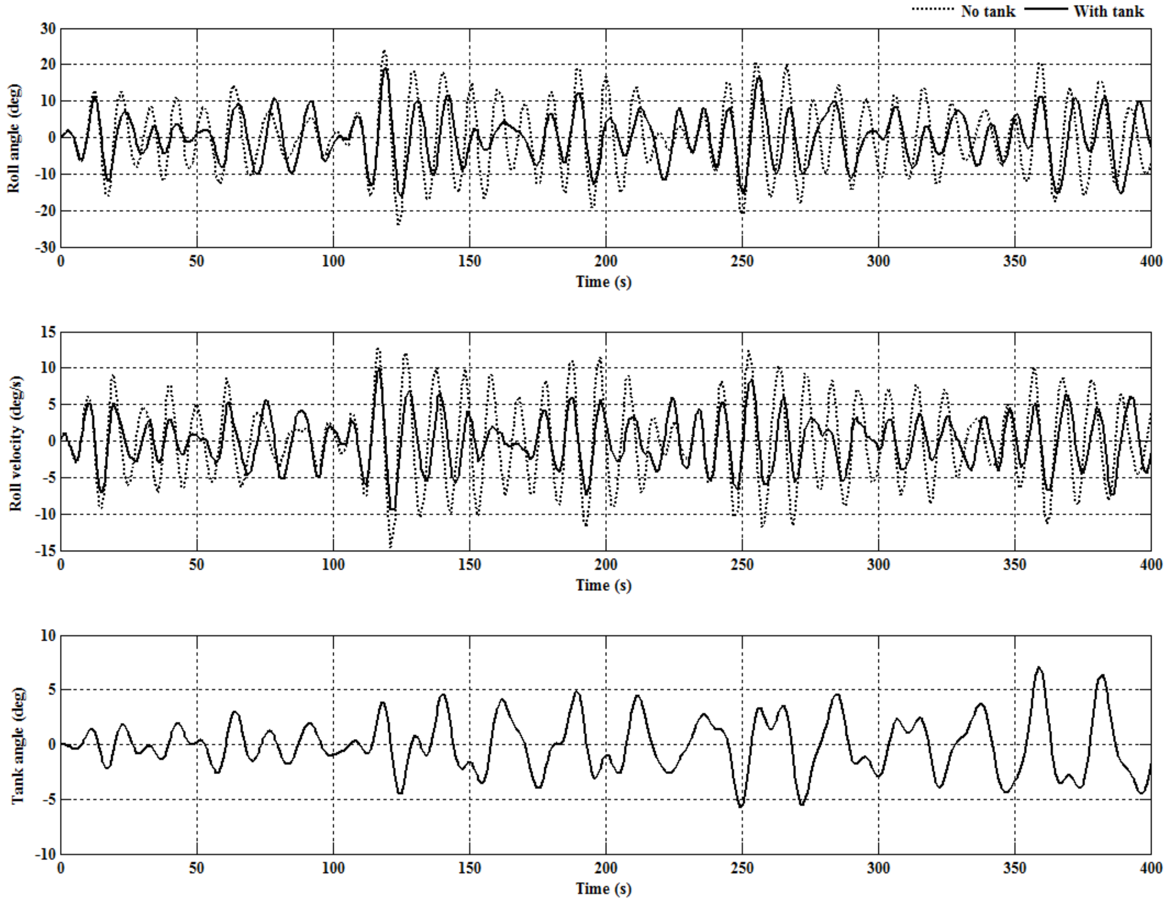

Fig. 7: Simulated results of roll reduction using passive tank in irregular sea

Table 5: Passive tank simulation report in sea state 5

Passive tank stabilization performance	RMS value	Units
Roll angle without passive tank	9.12	deg
Roll angle with passive tank	6.57	deg
Roll angle reduction percentage	28.03	%
Roll velocity without passive tank	5.49	deg/s
Roll velocity with passive tank	3.37	deg/s
Roll velocity reduction percentage	38.54	%
Tank angle	2.34	deg

5. Conclusion

The simulation of irregular sea in the form of a single-input, multiple-output system has been described in this paper. The sea wave simulated by feeding white noise to the shaping filter can be used to perturb the marine vehicle from its equilibrium position for control system design. The method can be extended for approximating the excitation forces and moments for analyzing the multiple degree problems in motion control. The least square fitting approach for the rational polynomial in this paper can be used for any other sea condition to get the coefficient values to best match the spectrum. Through this approach, close approximations of seas are obtained. The wave slope spectrum is matched in the main region of the spectrum. Roll reduction modelling a passive tank in a specific ship in irregular sea condition using this method has been successfully demonstrated. Appreciable roll reduction is reported.

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