EFFECTS OF RADIATION ON NATURAL CONVECTION FLOW AROUND A SPHERE WITH UNIFORM SURFACE HEAT FLUX

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Abstract: *The effects of radiation on natural convection flow around a sphere with uniform surface heat flux have been investigated in this paper. We have considered here a sphere with uniform surface heat flux immersed in a viscous incompressible optically thick fluid. The governing equations are first transformed into non-dimensional form and the resulting nonlinear systems of partial differential equations are then solved numerically using Finite-difference method with Keller-box scheme. We have focused our attention on the evolution of the shear stress in terms of local skin friction coefficient and the rate of heat transfer in terms of local Nusselt number. Also, velocity as well as temperature profiles are shown graphically for some selected values of radiation parameter (Rd), surface temperature parameter (Δ) and Prandtl number (Pr).*

Keywords: *Thermal radiation, Prandtl number, natural convection, uniform surface heat flux.*

INTRODUCTION

The effects of radiation on free convection flow are important in the context of space technology and very little is known about the effects of radiation on the boundary layer flow of a radiating fluid past a body. The inclusion of a radiation effects in the energy equation, however, leads to a highly non-linear partial differential equation. The problem of free convection boundary layer over or on various shapes such as vertical flat plate, cylinder, sphere, etc, have been studied by many investigators and it has been a very popular research topic for many years. It is readily recognized that a wealth of information is now available on convective heat and mass transfer for viscous fluids. Many researchers have studied the problems of free convection boundary layer flow over or on a various types of shapes. Amongst them Nazar *et al.* [1], Huang and Chen [2] considered the free convection boundary layer on an isothermal sphere and on an isothermal horizontal circular cylinder both in a micropolar fluid. Molla *et al.* [3] have studied the problem of natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation or absorption. But they were not concerned about the radiation effects. Soundalgekar *et al.* [4] have studied radiation effects on free convection flow of a gas past a semi-infinite flat plate using the Cogley-Vincenti-Giles equilibrium model [5]. Later Hossain and Takhar [6] have analyzed the effects of radiation using the Rosseland diffusion approximation which leads to non-similar solutions for free convection flow past a heated vertical plate. Limitations of this approximation are discussed briefly in Özisik [7]. The problem of the free convection boundary layer on a vertical plate with prescribed surface heat flux was studied by Merkin and Mahmood [8]. The above investigators were not concerned about the uniform heat flux.

In the present work, the effects of radiation with Rosseland diffusion approximation on free convection boundary layer flow around a sphere with uniform surface heat flux have been studied analytically and numerically. The governing partial differential equations are reduced to locally non-similar partial differential forms by adopting appropriate transformations. The transformed boundary layer equations are solved numerically using implicit finite difference method known as Keller box method [9] and later used by Cebeci and Bradshaw [10].

Numerical results have been shown in terms of local skin friction, rate of heat transfer, velocity profiles as well as temperature profiles for a selection of relevant physical parameters consisting of heat radiation parameter, Rd, Prandtl number, Pr and the wall temperature parameter, θ_w.

FORMULATION OF THE PROBLEM

Natural convection boundary layer flow around a sphere of radius *a* in a steady two-dimensional viscous incompressible fluid in the presence of radiation heat transfer and uniform surface heat flux q_w has been considered. It is assumed that the surface temperature of the sphere is T_w, where $T_w > T_\infty$. Here T_∞ is the ambient temperature of the fluid, T is the temperature of the fluid in the boundary layer, *g* is the acceleration due to gravity, r(x) is the radial distance from the symmetrical axis to the surface of the sphere and (u, v) are velocity components along the (x, y) axis. The physical configuration considered is as shown in Fig 1:

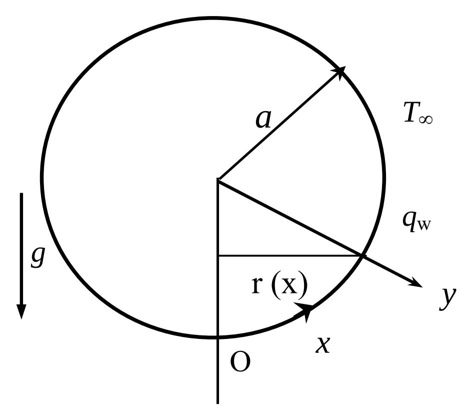

Figure 1: Physical model and coordinate system.

Under the usual Bousinesq approximation, the equations those govern the flow are

$$\frac{\partial}{\partial x}(r\hat{u})+\frac{\partial}{\partial y}(r\hat{v})=0 \quad (1)$$

$$\hat{u}\frac{\partial \hat{u}}{\partial x}+\hat{v}\frac{\partial \hat{u}}{\partial y}=\mu\frac{\partial^2 \hat{u}}{\partial y^2} +\rho g\beta(T-T_\infty)\ \sin\left(\frac{x}{a}\right) \quad (2)$$

$$\hat{u}\frac{\partial T}{\partial x}+\hat{v}\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \quad (3)$$

The boundary conditions of Equations (1) to (3) are

$$\hat{u}=\hat{v}=0,\ q_w=-k\frac{\partial T}{\partial y}\quad \text{at}\ \ y=0$$
$$\hat{u}\to 0,\ T\to T_\infty\ \ \text{as}\ \ y\to\infty \quad (4)$$

where ρ is the density, k is the thermal conductivity, β is the coefficient of thermal expansion, μ is the viscosity of the fluid, C_p is the specific heat at constant pressure and q_r is the radiative heat flux in the y direction. In order to reduce the complexity of the problem and to provide a means of comparison with future studies that will employ a more detail representation for the radiative heat flux, we have considered the optically dense radiation limit. Thus the Rosseland diffusion approximation proposed by Siegel and Howell [11] is given by simplifying radiation heat flux term as:

$$q_r=-\frac{4\sigma}{3(\alpha_r+\sigma_s)}\frac{\partial T^4}{\partial y} \quad (5)$$

We now introduce the following non-dimensional variables:

$$Gr=g\beta\left(\frac{aq_w}{k}\right)\frac{a^3}{\nu^2}$$
$$\xi=\frac{x}{a},\qquad \eta=Gr^{1/5}\left(\frac{y}{a}\right),$$
$$u=\frac{a}{\nu}Gr^{-2/5}\hat{u},\quad v=\frac{a}{\nu}Gr^{-1/5}\hat{v},$$
$$\theta=Gr^{1/5}\frac{T-T_\infty}{aq_w/k} \quad (6)$$

where ν (=μ/ρ) is the reference kinematic viscosity and Gr is the Grashof number, θ is the non-dimensional temperature function .

Substituting variables (6) into equations (1)-(3) leads to the following non-dimensional equations

$$\frac{\partial}{\partial \xi}(ru)+\frac{\partial}{\partial \eta}(rv)=0 \quad (7)$$

$$u\frac{\partial u}{\partial \xi}+v\frac{\partial u}{\partial \eta}=\frac{\partial^2 u}{\partial \eta^2}+\theta\sin\xi \quad (8)$$

$$u\frac{\partial \theta}{\partial \xi}+v\frac{\partial \theta}{\partial \eta} =\frac{1}{Pr}\frac{\partial}{\partial \eta}\left[\left\{1+\frac{4}{3}Rd\left(1+\Delta\theta\right)\right\}\frac{\partial \theta}{\partial \eta}\right] \quad (9)$$

Where $\Delta=\frac{aq_w/k}{T_\infty}$

With the boundary conditions (4) as

$$u=v=0,\ \frac{\partial \theta}{\partial \eta}=-1\ \ \text{at}\ \ \eta=0$$
$$u\to 0,\ \theta\to 0\ \ \text{as}\ \ \eta\to\infty \quad (10)$$

where Rd is the radiation parameter and Pr is the Prandtl number defined respectively as

$$Rd=\frac{4\sigma T_\infty^{\ 3}}{k\left(\alpha_r+\sigma_s\right)}\ \text{and}\ Pr=\frac{\mu C_p}{k} \quad (11)$$

To solve equations (8)-(9), subject to the boundary conditions (10), we assume the following variables

$$\psi=\xi\, r(\xi)\, f(\xi,\eta),\quad \theta=\theta(\xi,\eta) \quad (12)$$

where ψ is the non-dimensional stream function defined in the usual way as

$$u=\frac{1}{r}\frac{\partial \psi}{\partial \eta},\quad v=-\frac{1}{r}\frac{\partial \psi}{\partial \xi} \quad (13)$$

By substituting (13) in equations (8) and (9) we get the new form as (14) and (15):

$$\frac{\partial^3 f}{\partial \eta^3}+\left(1+\frac{\xi}{\sin\xi}\cos\xi\right)f\frac{\partial^2 f}{\partial \eta^2}-\left(\frac{\partial f}{\partial \eta}\right)^2 +\frac{\sin\xi}{\xi}\theta=\xi\left(\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial \xi\partial \eta}-\frac{\partial f}{\partial \xi}\frac{\partial^2 f}{\partial \eta^2}\right) \quad (14)$$

$$\frac{1}{Pr}\frac{\partial}{\partial \eta}\left[\left\{1+\frac{4}{3}Rd\left(1+\Delta\theta\right)^3\right\}\frac{\partial \theta}{\partial \eta}\right] +\left(1+\frac{\xi}{\sin\xi}\cos\xi\right)f\frac{\partial \theta}{\partial \eta} =\xi\left(\frac{\partial f}{\partial \eta}\frac{\partial \theta}{\partial \xi}-\frac{\partial \theta}{\partial \eta}\frac{\partial f}{\partial \xi}\right) \quad (15)$$

Along with boundary conditions

$$f=\frac{\partial f}{\partial \eta}=0,\ \theta'=-1\ \ \text{at}\ \ \eta=0$$
$$\frac{\partial f}{\partial \eta}\to 0,\ \theta\to 0\ \ \text{as}\ \ \eta\to\infty \quad (16)$$

It can be seen that near the lower stagnation point of the sphere i.e. $\xi\approx 0$, equations (14) and (15) reduce to the following ordinary differential equations:

$$f'''+2ff''-f'^2+\theta=0 \quad (17)$$

$$\frac{1}{Pr}\left[\left\{1+\frac{4}{3}Rd\left(1+\Delta\theta\right)^3\right\}\theta'\right]'+2f\theta'=0 \quad (18)$$

Subject to the boundary conditions

$$f(0)=f'(0)=0,\ \theta'(0)=-1$$
$$f'\to 0,\ \theta\to 0\ \ \text{as}\ \ \eta\to\infty \quad (19)$$

In the above equations primes denote the differentiation with respect to η.

Equations (17) and (18) with the boundary conditions (19) are used to get initial numerical solutions of the main problem.

In practical applications, the physical quantities of principal interest are the shearing stress, the heat transfer rate in terms of the skin-friction coefficients C_f and Nusselt number Nu_x respectively, which can be written as

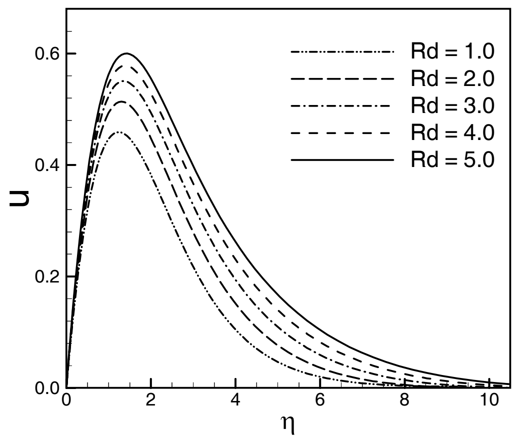

Figure 2: Velocity profiles for different values of Rd when Pr = 1.0 and Δ = 0.1.

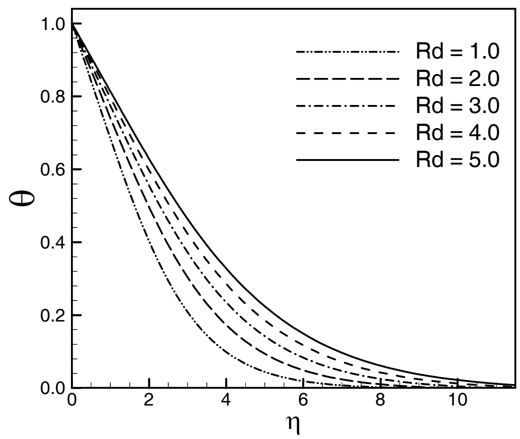

Figure 3: Temperature profiles for different values of Rd when Pr = 1.0 and Δ = 0.1.

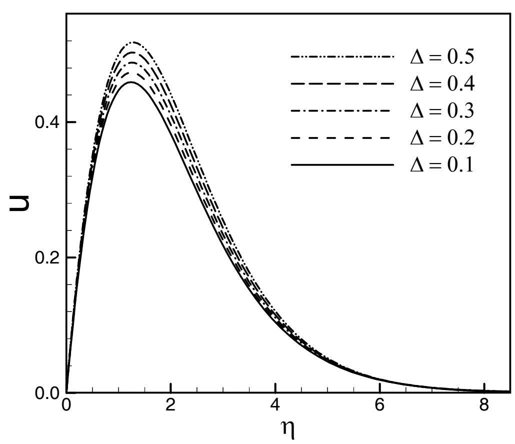

Figure 4: Velocity profiles for different values of Δ while Rd=1.0 and Pr = 1.0.

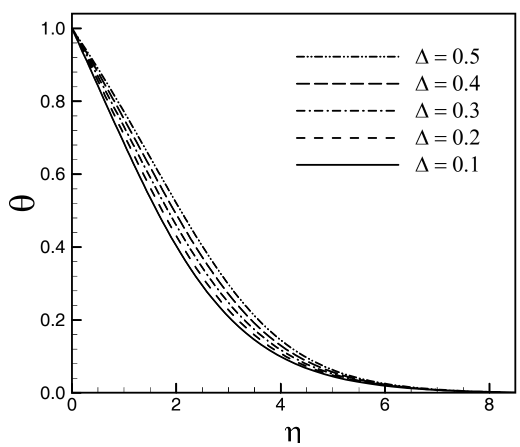

Figure 5: Temperature profiles for different values of Δ while Rd=1.0 and Pr = 1.0.

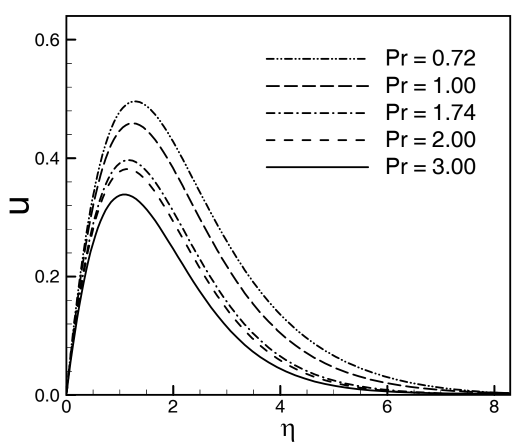

Figure 6: Velocity profiles for different values of Pr while Δ=1.0 and Pr = 1.0.

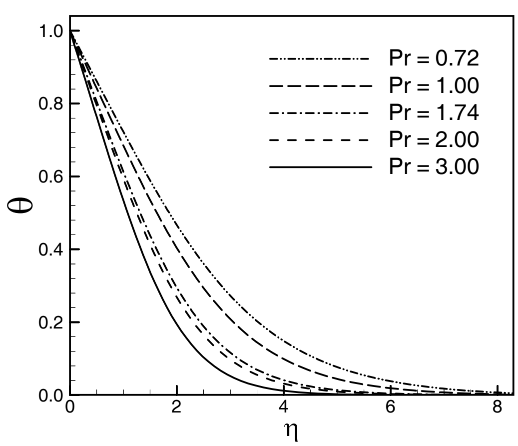

Figure 7: Temperature profiles for different values of Pr while Δ=1.0 and Pr = 1.0.

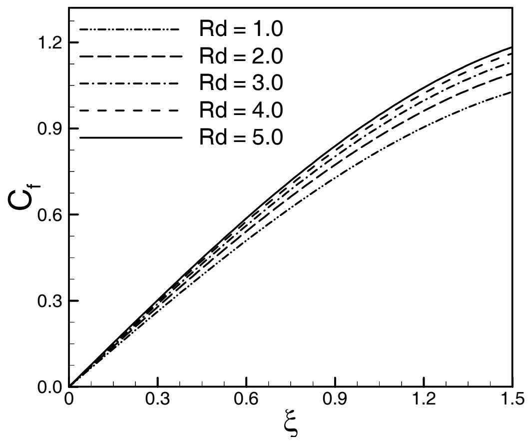

Figure 8: Skin friction coefficients for different values of Rd= while Pr =1.0 and Δ = 0.1.

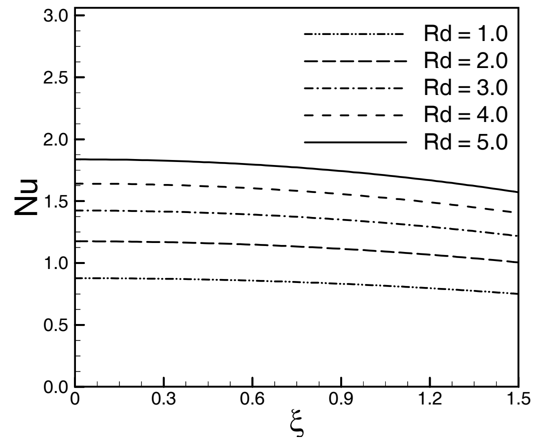

Figure 9: Heat transfer coefficients for different values of Rd while Pr=1.0 and Δ = 0.1.

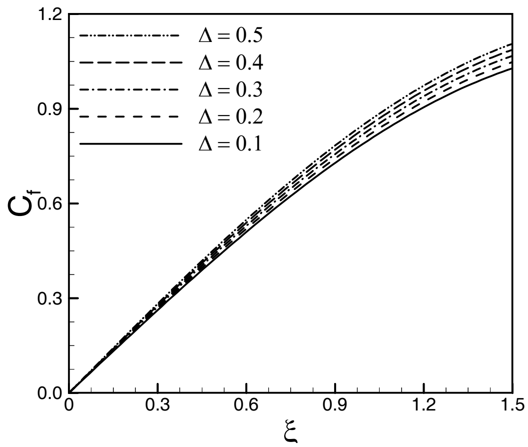

Figure 10: Skin friction coefficients for different Δ, while Rd = 1.0 and Pr =1.0

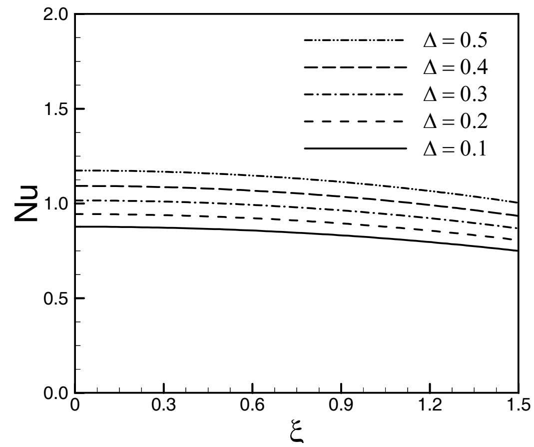

Figure 11: Heat transfer coefficients for different values of Δ, while Rd = 1.0 and Pr =1.0

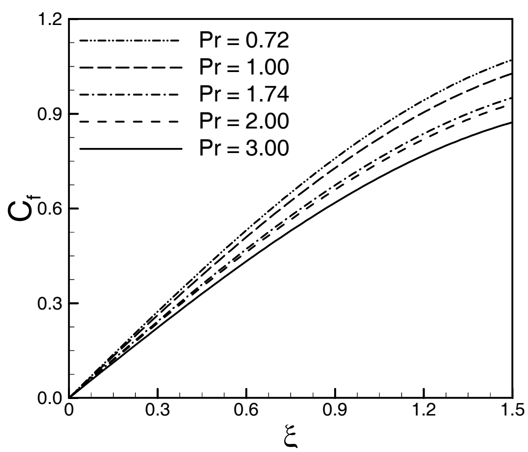

Figure 12: Skin friction coefficients for different values of Pr, while Rd = 1.0 and Δ = 0.1

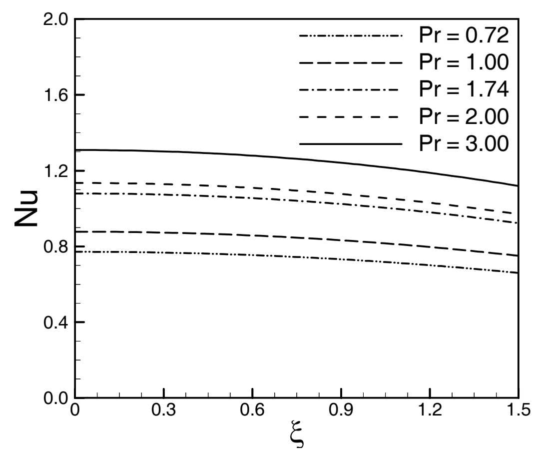

Figure 13: Heat transfer coefficients for different values of Pr, while Rd= 1.0 and Δ = 0.1

$$C_f = \frac{(\tau_w)_{y=0}}{\rho U^2} \quad \text{and} \quad Nu = \frac{a q_w / k}{(T_w - T_\infty)} \tag{20}$$

$$\text{where } \tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \text{ and } q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0} \tag{21}$$

Here we have used a reference velocity $U = \frac{\nu Gr^{1/5}}{a}$

Using the variables (6) and (13) and the boundary condition (16) into (20)-(21), we get

$$C_f Gr^{-1/5} = \xi f''(\xi, 0) \tag{22}$$

$$Nu Gr^{-1/5} = 1/\theta(\xi, 0) \tag{23}$$

The values of the velocity and temperature distribution are calculated respectively from the following relations:

$$u = \frac{\partial f}{\partial \eta}, \quad \theta = \theta(\xi, \eta) \tag{24}$$

RESULTS AND DISCUSION

To show the effects of radiation on natural convection flow with uniform heat flux on a sphere for different values of relevant physical parameters, we have obtained the solutions for fluids having Prandtl number Pr = 0.72 (air) and for some test values of Pr = 1.0, 1.74, 2.0, 3.0 against η for a wide range of radiation parameter Rd. The values of radiation parameter Rd = 1.0, 2.0, 3.0, 4.0 and 5.0 have been taken while Pr = 1.0 and Δ = 0.1. Different values of surface temperature parameter Δ = 0.1, 0.2, 0.3, 0.4 and 0.5 are considered while Pr = 1.0 and Rd = 1.0. We calculated the numerical values of local rate of heat transfer in terms of local Nusselt number (Nu) for the surface of the sphere from lower stagnation point to upper stagnation point. The effect for different values of radiation parameter (Rd) on local skin friction coefficient (C_f) and the local Nusselt number (Nu), as well as velocity and temperature profiles with the Prandtl number, Pr = 1.0 and surface temperature parameter Δ = 0.1 are discussed.

We have got the results for the velocity and temperature profiles, from the Figs 2-3 for different values of radiation parameter (Rd) while Prandtl number, Pr = 1.0 and surface temperature parameter Δ = 0.1. From Figs 2 and 3 we have seen that as the radiation parameter (Rd) increase, both the velocity and the temperature increase. Which means that higher radiation occur for higher values of temperature, which cause the increase of velocity as well. The change of velocity profiles in the η direction reveal the typical velocity profile for natural convection boundary layer flow i.e. the velocity is zero at the boundary wall and then the velocity increases to the peak value as η increases and finally the velocity approaches to zero (the asymptotic value).

Again the variation of the surface temperature parameter (Δ), the velocity and temperature profiles while Prandtl number Pr = 1.0 and radiation parameter Rd =1.0 are shown in the Figs 4 and 5. Here we have seen that surface temperature parameter (Δ) increase, the velocity profile and the temperature increase but velocity increase near the surface of the sphere and then temperature decrease slowly and finally approaches to zero. Moreover, in the Figs 6 and 7, when the Prandtl number (Pr) increase the velocity and the temperature both decrease while Δ = 0.1 and Rd = 1.0.

Figs 8-9 show that skin friction coefficient (C_f) and local rate of heat transfer (Nu) increase for increasing values of radiation parameter Rd, while Prandtl number Pr = 1.0 and surface temperature parameter Δ = 0.1. It is observed from the Fig 8 that the skin friction increase gradually from zero value at lower stagnation point along the ξ direction and from figure 9, it reveals that the rate of heat transfer increase slightly along the ξ direction from lower stagnation point to the downstream.

It can also be seen from Figs 10 - 11 that an increase in the surface temperature parameter (Δ) leads to increase the local skin friction coefficient (C_f) and the local rate of heat transfer *(*Nu) slightly while Prandtl number Pr = 1.0 and radiation parameter Rd = 1.0. Also it is observed that at any position of ξ, the skin friction coefficient (C_f) and the local Nusselt number (Nu) increase as Δ increases from 0.1 to 0.5. This phenomenon can easily be understood from the fact that when the surface temperature parameter Δ increase, the temperature of the fluid rises and the thickness of the velocity of the boundary layer grows i.e. the thermal boundary layer becomes thinner than the velocity at the boundary layer. Therefore the skin friction coefficient (C_f) and the local Nusselt number (Nu) increase.

Variations of the local skin friction coefficient (C_f) and local Nusselt number (Nu) for different values of Prandtl number Pr while Δ = 0.1 and Rd = 1.0 are shown in the figures 12 and 13. We have observed from the Figs that as the Prandtl number Pr increase, the skin friction coefficient decrease and heat transfer coefficient increase but the rate of increase in the heat transfer coefficient is higher than that of the skin friction coefficient. So the effect of Prandtl number Pr on heat transfer coefficient is more than of the effect of Pr on skin friction coefficient.

The following table depicts the comparisons of the present numerical results of the Nusselt number (Nu) with those obtained by Nazar *et al.* [1] and Huang and Chen [2]. Here, Prandtl numbers Pr = 0.7 and 7.0 have been chosen. Nazar *et al.* [1] compared their results in absence of micropolar parameter with Huang and Chen [2]. The present results have been compared with those in absence of Radiation and imposing the same conditions in the programming code of the present paper. It is observed from the table that the present result agreed well with the results of [1] and [2].

CONCLUSION

The Radiation effect on natural convection flow around a sphere with uniform heat flux has been studied for different values of relevant physical parameters including Prandtl number (Pr) and surface temperature parameter (Δ). From the present investigation following conclusions may be drawn:

- Significant effects of radiation on velocity and temperature profiles as well as on skin friction and the rate of heat transfer have been found in this study. An increase in the values of radiation leads to both the velocity and the temperature profiles increase, the local skin friction coefficient (C_f) and the local rate of heat transfer (Nu) also increase for increase of radiation at different position of ξ for Pr = 1.0. and Δ = 0.1.

Table 1: Comparisons of the present numerical results of (Nu) for the Prandtl numbers Pr = 0.7, 7.0 with those obtained by Nazar *et al.* [1] and Huang and Chen [2]

	Pr = 0.7			**Pr = 7.0**		
x in degree	Nazar et el. [1]	Huang and chen [2]	Present results	Nazar et el. [1]	Huang and chen [2]	Present results
0	0.4576	0.4574	0.4416	0.9595	0.9581	0.9355
10	0.4565	0.4563	0.4405	0.9572	0.9559	0.9333
20	0.4533	0.4532	0.4374	0.9506	0.9496	0.9268
30	0.4480	0.4480	0.4323	0.9397	0.9389	0.9162
40	0.4405	0.4407	0.4251	0.9239	0.9239	0.9008
50	0.4308	0.4312	0.4157	0.9045	0.9045	0.8819
60	0.4189	0.4194	0.4042	0.8801	0.8805	0.8581
70	0.4046	0.4053	0.3904	0.8510	0.8518	0.8297
80	0.3879	0.3886	0.3743	0.8168	0.8182	0.7964
90	0.3684	0.3694	0.3555	0.7774	0.7792	0.7580

- If the Prandtl number (Pr) is one and the values of surface temperature increase, then all the velocity profiles, temperature profiles, the local skin friction coefficients (C_f) and the local rate of heat transfer (Nu) increase significantly.
- Again for increasing values of Prandtl number Pr leads to decrease on the velocity and temperature profiles and the local skin friction coefficient (C_f), but the local rate Nusselt number (Nu) increase as the increase of the Prandtl number (Pr) while Rd = 1.0 and $\Delta = 0.1$.

NOMENCLATURE

a	Radius of the sphere [m]
C_f	Skin-friction coefficient
C_p	Specific heat at constant pressure [$kJkg^{-1}k^{-1}$]
f	Dimensionless stream function
g	Acceleration due to gravity [ms^{-2}]
Gr	Grashof number
k	Thermal conductivity [$wm^{-1}k^{-1}$]
Nu	Nusselt number
Pr	Prandtl number
q_c	Conductive heat flux [w/m^2]
q_r	Radiative heat flux [w/m^2]
q_w	Heat flux at the wall [w/m^2]
R_d	Radiation parameter
r	Distance from the symmetric axis to the surface [m]
T	Temperature of the fluid in the boundary layer [K]
T_∞	Temperature of the ambient fluid [K]
T_w	Temperature at the surface [K]
$\hat{u}$	Velocity component along the surface [ms^{-1}]
$\hat{v}$	Velocity component normal to the surface [ms^{-1}]
u	Dimensionless velocity along the surface
v	Dimensionless velocity normal to the surface
x	Coordinate along the surface [m]
y	Coordinate normal to the surface [m]

Greek symbols

α_r	Rosseland mean absorption co-efficient [cm^3/s]
β	Volumetric coeff. of thermal expansion [K^{-1}]
η	Dimensionless Coordinates Normal to the surface
θ	Dimensionless temperature
θ_w	Surface temperature parameter
μ	Viscosity of the fluid [m^2/s]
ν	Kinematic viscosity [$kg^{-1}m^5s^{-1}$]
ξ	Dimensionless Coordinates along the surface
ρ	Density of the fluid [kgm^{-3}]
σ	Stefan Boltzman constant [$Js^{-1}m^{-1}K^{-1}$]
σ_s	Scattering co-efficient [m^{-1}]
τ_w	Shearing stress [$dynes/cm^2$]
ψ	Stream function [m^2s^{-1}]

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