MULTI-ITEM CAPACITY CONSTRAINED DYNAMIC LOT-SIZING AND SEQUENCING WITH SETUP TIME

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Abstract: *Production lot-sizing has a special significance in supply chain taking into account the fact that majority of the lot-sizing problems are associated with NP-hard scheduling and sequencing problems. The complexity increases exponentially when multi-item capacitated dynamic lot-sizing is considered. The basic economic production quantity (EPQ) model minimizes the sum of setup and holding cost under certain favorable assumptions. However, when assumptions are removed by introducing more complex constraints, the solution procedure becomes extremely difficult to solve. As a result NP-hardness arises which necessitates the use of heuristics. The objective of this paper is to minimize the sum of setup and inventory holding costs over a time horizon subject to constraints of capacity limitations and elimination of backlogging. As reports reveal, algorithm for an optimal solution exists in case of a single item production. But for multi-item problems, no algorithm exists which can provide global optimality. This paper develops a model for multi-item problem with setup time. Based on the model a program has been executed and feasible solutions have been obtained for the bench-mark data.*

Keywords: *Capacitated scheduling, heuristics, multi-item, production lot-sizing.*

INTRODUCTION

Due to their importance in industry, dynamic demand lot-sizing problems are frequently studied. This paper considers the determination of lot-sizes for multiple products with capacity restriction that can be replenished. A fixed setup cost is incurred whenever any product is produced, independently of the number or type of products. The demand for each item is discrete and varies in time, but is known over a given time horizon. Linear holding costs are charged on the end-of-period inventories and backlogging is not permitted. The variable unit purchase cost for each product is constant throughout the horizon, so that the purchase cost of any item for total demand in the horizon is invariant of the replenishment policy. The problem is to determine a replenishment schedule for all items that minimizes the total setup plus inventory holding cost over the horizon. A variety of taxonomies are proposed for classifying lot-sizing problems [1]. An important problem characteristic is the nature of demand. Static demand problems assume a stationary or constant demand pattern, while dynamic demand problems permit demand to vary. If all demand values are known for the duration of the planning horizon, the demand stream is defined as deterministic. Otherwise, the demand is considered to be stochastic. There are two principal types of production system in terms of number of products. In single-item production planning there is only one end item (final product) for which the planning activity has to be organized, while in multi-item production planning there are several end items. The complexity of multi-item problems is much higher than that of single-item problems. When there is no restriction on capacity, the problem is said to be uncapacitated, and when capacity constraints are explicitly stated, the problem is named capacitated. Capacity restriction is important and more realistic and directly affects problem complexity. Problem solving will be more difficult when capacity constraints exist.

Many dynamic programming solutions exist for lot-sizing problems, but they are computationally complex. For example, when specialized to the multi-product dynamic lot-size problem Zangwill's method [2] has a computational complexity that is exponential in the number of products, while Veinott's [3] solutions are computationally exponential in the number of time periods. Other solutions that are computationally exponential in the number of products have also been proposed [1]. However, these solutions are of no use for practical problems, which usually involve many items and many time periods. So efforts have shifted to the development of heuristic solutions [4].

OVERVIEW OF LOT-SIZING PROBLEMS

After an introduction to lot-sizing problems, this paper will focus on the single level multi-item dynamic capacitated lot-sizing problem (CLSP) which is an NP-hard problem [1]. There are five other problem variants. These are: the economic lot scheduling problem (ELSP), the discrete lot-sizing and scheduling problem (DLSP), the continuous setup lot-sizing problem (CSLP), the proportional lot-sizing and scheduling problem (PLSP), and the general lot-sizing and scheduling problem (GLSP). The ELSP [5] is a single-level, multi-item problem with stationary demand. The time is continuous and planning horizon is infinite. Solving the ELSP where capacity restrictions are involved is NP-hard. The NP-hard problem DLSP [6] subdivides the (macro) periods of the CLSP into several (micro) periods. The fundamental assumption of the DLSP is the so-called all-or-nothing production, which means only one item may be produced per period, and, if so, the production amount would be as much as using full capacity. From this viewpoint, DLSP is called a small bucket problem. The CSLP [7] is a step towards a more realistic situation compared to DLSP. In CSLP the all-or-nothing assumption, that seems to be strict and makes efficient implementation of mathematical programming approaches possible, does not exist any more, but still only one item may be produced per period. The basic idea behind the PLSP [8] is to use the remaining capacity for scheduling a second item in the particular period, if the capacity of a period is not used in full. This is in fact the shortcoming of the CSLP. The underlying assumption of the PLSP is that the setup state of the machine can be changed at most once per period. Production in a period could take place only if the machine is properly setup either at the beginning or at the end of the period. Hence, at most two products may be produced per period. GLSP [9] integrates lot-sizing and scheduling of several products on a single capacitated machine. Continuous lot sizes are determined and scheduled, thus generalizing models using restricted time structures. The single-item CLSP has been shown by Florian et al. [10] to be NP-hard. In consequence, Chen and Thizy [11] have shown that the multi-item CLSP problem is strongly NP-hard. Due to the vastness of the lot-sizing literature, this paper only focuses on multi-item single-level capacitated lot-sizing decisions with deterministic demand.

The Problem Definition

In the context of single-level production planning, with finite planning horizon and known dynamic demand without incurring backlogs, the capacitated lot-sizing problem (CLSP), consists of determining the amount and the timing of production of the products in the planning horizon. Capacity restrictions constrain the production quantity in each period. A fixed setup cost is specified and there is also an inventory holding cost proportional to the inventory amount and time carried. In the CLSP, although the setup costs may vary for each product, they are sequence independent. A multi-item, multi-echelon inventory problem with dynamic variables is extremely difficult to solve in a realistic time period, which leads to NP-hardness, quite similar to scheduling problem [12]. Several other mathematical models have been developed to solve these types of NP-hard inventory problem; those are computationally harder and thus require more time in information processing. Often, they become near NP-hard problem, with global search options [13-15]. Hence, it appears highly unlikely that an efficient optimal algorithm will ever be developed. So the search for a good heuristic method is definitely warranted. The literature review indicates the existence of several efficient and effective problem formulations, heuristics for the CLSP, but the CLSP still poses many challenges for researchers. As a consequence, many heuristics were developed for this problem. Eisenhut's procedure [16] could be called period-by-period heuristic. His procedure was later extended by many, including Dixon and Silver [17]. Basic assumptions of the Dixon-Silver model are: (i) the requirements for each product are known period by period, out to the end of some common time horizon. (ii) for each product there is a fixed setup cost incurred each time production takes place, (iii) unit production and holding costs are linear, (iv) the time required to setup the machine is negligible, (v) all costs and production rates can vary from product to product but not with respect to time, and (vi) in each period there is a finite amount of machine time available that can vary from period to period. The objective is to determine lot-sizes so that (i) costs are minimized, (ii) no backlogging occurs, and (iii) capacity is not exceeded. It would be more realistic for multi-item problem to assume a setup time since production changeover from one item to another item incurs setup time. This setup time is usually independent of the item sequence but different for each item. In Dixon-Silver heuristic, setup time has been neglected. But for a multi-item problem, consideration of the setup time would be more realistic. The current research work has thus been directed toward an extension of the Dixon-Silver model considering the above mentioned situation. Based on the extended model, a program has been executed with the data of a hypothetical problem and feasible solutions have been obtained. Mathematically the model may be presented as follows.

Mathematical Model

Minimize $Z(X) = \sum_{i=1}^{N}\sum_{j=1}^{H}(S_i\delta(x_{ij}) + h_i I_{ij})$

Subject to

$I_{ij} = I_{i,j-1} + x_{ij} - D_{ij}$ $\quad i = 1, \ldots, N$ and $j = 1, \ldots, H$

$I_{i0} = I_{iH} = 0$ $\quad i = 1, 2, \ldots, N$

$\sum_{i=1}^{N}\left[k_i x_{ij} + St_i.\delta(x_{ij})\right] \le C_j$ $\quad j = 1, 2, \ldots, H$

$x_{ij}, I_{ij} \ge 0$ $\quad i = 1, \ldots, N$ and $j = 1, \ldots, H$

where N = the number of items, H = the time horizon, D_{ij} = the given demand for item i in period j, I_{ij} = the inventory of item i at the end of period j (after period j production and demand satisfied), x_{ij} = the lot-size of item i in period j, S_i = the setup cost for item i, h_i = the unit holding cost for item i, k_i = the capacity absorption rate for item i, C_j = the capacity in period j, St_i = setup time for item i, and

$$\delta(x_{ij}) = \begin{cases} 1 & if\ x_{ij} > 0 \\ 0 & if\ x_{ij} = 0 \end{cases}$$

$\delta(x_{ij})$ is a binary setup variable indicating whether a setup cost must be incurred for item i in period j or not.

The unit production cost is assumed to be constant for each item. Therefore, the total production cost (excluding setup costs) will be a constant and hence is not included in the model. If initial inventory exists, or if positive ending inventory is desired, then the net requirements should be determined. That is, use the initial inventory to satisfy as much demand as possible in the first few periods. The net requirements, will be that demand not satisfied by the initial inventory. Hence, an equivalent problem is created with zero starting inventories. Now increase the demand in the last period, H, by the desired ending inventory. Now the equivalent problem satisfies the starting and ending inventory constraints.

STEPS OF THE HEURISTIC

This paper extends the basic Dixon-Silver heuristic to accommodate setup time. The purpose of this section is to outline the steps of the proposed heuristic. The original multi-item problem with constant capacity is NP-hard. In the present work a new constraint, setup time is considered. With this new constraint the problem is also NP-hard. Therefore, a heuristic has been developed which guarantees a feasible solution. The heuristic method of solution is presented below in steps.

Step 1 Creation of an equivalent demand matrix:

- Convert the initial demand matrix into equivalent demand matrix with the use of initial inventory, ending inventory and safety stock.
- Use the initial inventory to satisfy as much demand as possible in the first few periods. The net requirements will be that demand not satisfied by the initial inventory. During the calculation of the net demands, the amount of the safety stock should be maintained. Let

  Iin_i = initial inventory for item i,

  $Iend_i$ = ending inventory for item i,

  $Irem_i$= remaining initial inventory for item i, and

  SS_i = safety stock for item i.

  d_{ij} = equivalent demand for product i in period j.

  Initially set $Irem_i = Iin_i - SS_i$ and period $j = 1$.

  Then set $d_{ij} = \begin{cases} 0 & if\ Irem_i > D_{ij} \\ D_{ij} - Irem_i & if\ Irem_i \le D_{ij} \end{cases}$.

  Compute $Irem_i = Irem_i - D_{ij}$.

  Set $j = j + 1$ and recycle till $Irem_i > 0$.
- Since the amount of the safety stock is always maintained, the demand in the last period H would be partially satisfied by the safety stock of the period H-1. If ending inventory is desired, then the requirements in period H should be increased by the desired ending inventory. Then

  $d_{iH} = D_{iH} + Iend_i - SS_i$.
- Compute the net demands for all $i = 1, 2,\dots, N$.

Step 2 Check the feasibility of the problem:

$$\sum_{j=1}^{H} CR_j \le \sum_{j=1}^{H} C_j ,$$

where $CR_j = \sum_{i=1}^{N} k_i d_{ij}$,

CR_j = demand in terms of capacity unit for period j, and k_i = capacity absorption rate for product i.

If the feasibility condition is not satisfied, the problem is infeasible i.e. all demands cannot be met with the available capacity.

Step 3 Use the Dixon-Silver heuristic with inclusion of setup time [through steps 3.1 to 3.12]:

Step 3.1

- Start at period 1, i.e. set R=1 [R = 1, 2, …, H]. When lot-sizing of period 1 is complete, then lot-sizing is started for period 2 up to period H.

Step 3.2

- Initialize lot-size x_{ij} by equalizing to demand d_{ij} , i.e.,

$$x_{ij} = d_{ij} \quad i = 1, 2, \dots, N \text{ and } j = 1, 2, \dots, H.$$

Step 3.3

- Initially set the value of the time supply to one i.e., $T_i = 1$, where $i = 1, 2,\dots, N$.

  Time supply (T_i) denotes the integer number of period requirements that this lot will exactly satisfy.

Step 3.4

- Produce $d_{iR} > 0$, in the lot-sizing period R, where $i = 1, 2, \dots, N$.
- After producing d_{iR} calculate remaining capacity in period R, denoted by RC_R, by

$$RC_R = C_R - \sum_{i=1}^{N} k_i\, d_{iR} .$$

- Let I'_{ij} be the amount of inventory at the end of period j for item i, resulting from only the currently scheduled production in period R. Initialize I'_{ij} with zero, i.e.,

$$I'_{ij} = 0, \quad i = 1, 2, \dots, N \text{ and } j = 1, 2, \dots, H.$$

Step 3.5

- Let AP_j be the amount of inventory (in capacity units) resulted from the production of period R that will be used in period j. Then

$$AP_j = \sum_{i=1}^{N} k_i \left(I'_{i,j-1} - I'_{i,j} \right).$$

- Let CR_j be the total demand (in capacity units) in period j. Then

$$CR_j = \sum_{i=1}^{N} k_i \, d_{ij} .$$

- The production plan for period R is feasible if and only if the following condition is satisfied for t = 2,…, H.

$$\sum_{j=R+1}^{R+t-1} AP_j \geq \sum_{j=R+1}^{R+t-1} (CR_j - C_j + St_j) .$$

- Determine the earliest period t_c at which the above feasibility constraint is not satisfied, i.e.,

$$t_c = \min\left\{ t \,\middle|\, \sum_{j=R+1}^{R+t-1} AP_j < \sum_{j=R+1}^{R+t-1} (CR_j - C_j + St_j) \right\}.$$

To remove infeasibility upto t_c, extra amount is to be produced with the use of remaining capacity RC_R of period R.

If there is no infeasibility, set $t_c = H + 1$.

Step 3.6

- Consider only items i' which have
  (1) $T_{i'} < t_c$,
  (2) RC_R is sufficient to produce $d_{i',R+T_{i'}}$, and
  (3) $d_{i',R+T_{i'}} > 0$.

To decide the best item (from a cost standpoint) to be produced in period R, calculate the priority index $U_{i'}$ for all of these items, where

$$U_{i'} = \frac{AC(T_{i'}) - AC(T_{i'} + 1)}{k_{i'} d_{i',T_{i'}+1}} \text{ , and} \qquad (1)$$

$$AC(T_{i'}) = \left\{ S_{i'} + h_{i'} \sum_{j=R}^{R+T_{i'}-1} (j - R) d_{i'j} \right\} / T_{i'} .$$

Among these find the one, denoted by i, that has the largest U_i.

- U_i is the marginal decrease in average costs per unit of capacity absorbed.
- $AC(T_i)$ is average cost per unit time of a lot of item i which will satisfy T_i periods' requirements. This is from the Silver-Meal model in which future setup cost may be saved at the expense of added inventory holding cost.

Step 3.7

- Check the value of U_i.

(a) If Ui > 0, then it is economic to produce $d_{i,R+T_i}$ in period R.

Increase the value of lot-size x_{iR} and inventory I'_{ij} by $d_{i,R+T_i}$, i.e.,

$$x_{iR} = x_{iR} + d_{i,R+T_i}$$

$$I'_{ij} = I'_{ij} + d_{i,R+T_i} \quad j = R+1, \ldots, R+T_i.$$

Decrease the value of lot-size $x_{i,R+T_i}$, demand $d_{i,R+T_i}$ and remaining capacity RC_R by $d_{i,R+T_i}$, i.e., set

$$x_{i,R+T_i} = x_{i,R+T_i} - d_{i,R+T_i}$$

$$d_{i,R+T_i} = d_{i,R+T_i} - d_{i,R+T_i} = 0$$

$$RC_R = RC_R - d_{i,R+T_i} .$$

- Set $T_i = T_i + 1$ and continue from Step 3.5.

(b) If $U_i \leq 0$, then it is not economic to increase T_i of any item, because of the increase of the total cost.

- Check the value of t_c.
  (i) If $t_c > H$, then no infeasibilities left and lot-sizing of the current period is complete. Go to Step 3.12.
  (ii) If $t_c < H$, there are infeasibilities and production of one or more item is to be increased and it is done through Steps 3.8 to 3.11.

Step 3.8

- Calculate the value of Q, where

$$Q = \max_{R+t_c-1 \leq t \leq H} \left[\sum_{j=R+1}^{t} \left(CR_j - (C_j - St_j) - AP_j \right) \right].$$

- Q is the amount of production still needed in the current period to eliminate infeasibilities in the later period because the available capacity is not sufficient to meet the demands of those periods.

Step 3.9

- Consider only items i' for which
  i. $T_{i'} < t_c$, and
  ii. $d_{i',R+T_{i'}} > 0$.

To decide the best item (from a cost standpoint) to be produced in period R, calculate the priority index $\Delta_{i'}$ for all of these items, where

$$\Delta_{i'} = \frac{AC(T_{i'} + 1) - AC(T_{i'})}{k_{i'} d_{i',T_{i'}+1}} .$$

- Find the one, denoted by i, that has the smallest Δ_i .

Step 3.10

- Let $W = k_i d_{i,R+T_i}$.
- Compare the value of Q with W.

(a) If $Q > W$,

Increase the value of lot-size x_{iR} , and inventory I'_{ij} by $d_{i,R+T_i}$, i.e.,

$$x_{iR} = x_{iR} + d_{i,R+T_i}$$

$$I'_{ij} = I'_{ij} + d_{i,R+T_i} \quad j = R+1, \ldots, R+T_i.$$

Decrease the value of lot-size $x_{i,R+T_i}$, demand $d_{i,R+T_i}$ and remaining capacity RC_R by $d_{i,R+T_i}$, i.e.,

$$x_{i,R+T_i} = x_{i,R+T_i} - d_{i,R+T_i}$$

$$d_{i,R+T_i} = d_{i,R+T_i} - d_{i,R+T_i} = 0$$

$RC_R = RC_R - d_{i,R+T_i}$.

Set $Q = Q - W$ and $T_i = T_i + 1$.

Continue from Step 3.9.

(b) If $Q \leq W$, set $IQ = \left\lceil \frac{Q}{k_i} \right\rceil$.

Increase the value of lot-size x_{iR} and inventory I'_{ij} by IQ, i.e.,

$$x_{iR} = x_{iR} + IQ$$

$$I'_{ij} = I'_{ij} + IQ .$$

Decrease the value of lot-size $x_{i,R+T_i}$ and demand $d_{i,R+T_i}$ by IQ, i.e.,

$$x_{i,R+T_i} = x_{i,R+T_i} - IQ$$

$$d_{i,R+T_i} = d_{i,R+T_i} - IQ .$$

Step 3.11

- Set $R = R + 1$.
- Check the value of R.
  (a) If $R < H$, then continue from Step 3.3.
  (b) If $R > H$, lot-sizing is complete up to period H.

Step 3.12

- Calculate the values of
  i. Forecasted machine time required/period.
  ii. Total expected setup cost.
  iii. Total expected inventory holding cost.
  iv. Total expected safety stock cost.
- Stop.

COMPUTATIONAL RESULTS

The proposed heuristic has been implemented with bench-mark data as obtained from [17]. It is assumed that entire production to meet demands is done in the plant and no subcontracting is permissible. A further assumption is made that plant capacity could not be increased.

Product Data

In this problem, the machine setup time to produce each product item is included. Relevant product data (e.g., holding cost, setup cost, production rate, safety stock, initial inventory and ending inventory) including setup time for each item has been presented in Table 1. The problem size has been restricted to 12 products and 12 time periods; each time period corresponds to a month.

Table 1. Relevant product data for the proposed heuristic with setup time.

Item No (i)	Holding Cost (h_i)	Setup Cost (S_i)	Setup Time (St_i)	Production Rate ($1/k_i$)	Safety Stock (SS_i)	Initial Inventory (Iin_i)	Ending Inventory ($Iend_i$)
01	0.0167	322.0	1.40	524	0	19320	18893
02	0.0167	81.0	2.00	349	10602	200180	124225
03	0.0167	124.0	1.00	245	4577	24460	43294
04	0.0167	124.0	1.50	172	1974	23260	21757
05	0.0167	81.0	0.25	349	7581	55489	92168
06	0.0167	124.0	0.70	245	4861	-2727	44394
07	0.0167	124.0	0.50	172	2026	9659	8466
08	0.0167	105.0	1.20	847	11117	29705	40273
09	0.0167	105.0	0.40	464	9533	11362	84717
10	0.0167	106.0	0.60	575	20417	242944	227344
11	0.0167	105.0	1.00	1261	16634	324215	271627
12	0.0167	105.0	1.30	663	9794	45439	69068

Table 2. Forecasted demand and capacity of the machine.

Item No	**Period**											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
01	11456	11456	10501	13365	13365	11456	8592	1909	1909	1909	4773	4773
02	53124	53124	48697	61977	61977	53124	39842	8854	8854	8854	22135	22135
03	18099	18099	16591	21116	21116	18099	13574	3016	3016	3016	7541	7541
04	9250	9250	8480	10792	10792	9250	6938	1542	1542	1542	3854	3854
05	39546	39546	36250	46137	46137	39546	29659	6591	6591	6591	16478	16478
06	18363	18363	16833	21423	21423	18363	13772	3060	3060	3060	7651	7651
07	4976	4976	4562	5806	5806	4976	3732	829	829	829	2074	2074
08	41690	41690	38216	48638	48638	41690	31267	6948	6948	6948	17371	17371
09	32816	32816	30081	38285	38285	32816	24612	5469	5469	5469	13673	13673
10	96745	96745	88683	112868	112868	96745	72559	16124	16124	16124	40310	40310
11	119220	119220	109285	139088	139088	119220	89415	19870	19870	19870	49675	49675
12	27715	27715	25405	32333	32333	27715	20786	4619	4619	4619	11548	11548
	Available Machine Hours											
	706	729	729	706	729	706	729	729	660	729	706	729

Product Demand Plant Capacity

Product demands are quite seasonal and the same seasonal indices are used for all the products. Forecasted demand and the capacity of the machine are shown in Table 2. It has been assumed that the capacity per month is the total number of hours available per month. It is assumed that 2% of the capacity is reserved as a buffer to guard against uncertainty in the actual production rate. In this problem, Period 1 corresponds to the month of June, Period 2 corresponds to the month of July. Thus the machine capacity in Period 1 is 98% of the total hours in June, i.e., $30 \times 24 \times 0.98 = 706$ hours. To be in the safe side, it has been assumed that the number of days in February is 28. Then the machine capacity in Period 9 is $28 \times 24 \times 0.98 = 660$ hours. Similarly the machine capacity for the other periods has been calculated.

Equivalent Demand Schedule

An equivalent demand schedule is generated such that starting and ending inventory are accommodated. In addition, demands are adjusted such that in the heuristic solution, the inventory at the end of any period never drops below the safety stock level. Table 3 depicts the equivalent demand after considering initial inventory, ending inventory and safety stock.

Table 3. Equivalent demand with the use of initial inventory, ending inventory and safety stock.

Item No	Period											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
01	0	3592	10501	13365	13365	11456	8592	1909	1909	1909	4773	23666
02	0	0	0	27344	61977	53124	39842	8854	8854	8854	2135	135758
03	0	16315	16591	21116	21116	18099	13574	3016	3016	3016	7541	46258
04	0	0	5694	10792	10792	9250	6938	1542	1542	1542	3854	23637
05	0	31184	36250	46137	46137	39546	29659	6591	6591	6591	16478	101065
06	25951	18363	16833	21423	21423	18363	13772	3060	3060	3060	7651	47184
07	0	2319	4562	5806	5806	4976	3732	829	829	829	2074	8514
08	23102	41690	38216	48638	48638	41690	31267	6948	6948	6948	7371	46527
09	30987	32816	30081	38285	38285	32816	24612	5469	5469	5469	13673	88857
10	0	0	59646	112868	112868	96745	72559	16124	16124	6124	0310	247237
11	0	0	40144	139088	139088	119220	89415	19870	19870	19870	9675	304668
12	0	19785	25405	32333	32333	27715	20786	4619	4619	4619	1548	70822

Table 4. Final lot-sizes and forecasted machine time requirements.

Item No	Period											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
01	3592	23866	0	13365	11456	0	10501	3818	0	28439	0	0
02	0	0	27344	61977	0	53124	39842	17708	68079	0	98668	0
03	37814	0	16208	21116	18099	0	13574	9048	0	53799	0	0
04	0	27278	0	0	9250	0	6938	8480	0	0	23637	0
05	31184	82387	0	46137	0	39546	29659	13182	124134	0	0	0
06	61147	0	21423	21423	16899	1464	13772	9180	0	54835	0	0
07	18493	0	0	0	4976	0	4561	11912	0	0	334	0
08	23102	41690	41862	44992	48638	41690	31267	13896	6948	17371	0	46527
09	30987	62897	38285	11166	27119	32816	24612	10938	5469	45743	56787	0
10	0	14322	158192	0	112868	96745	72559	16124	32248	40310	64595	182642
11	0	0	40144	139088	139088	119220	89415	19870	19870	19870	49675	304668
12	0	45190	32333	0	32333	27715	20786	9238	4619	11548	0	70822
	Forecasted Machine Requirements (hours)											
	707.2	724.7	727.8	704.4	728.3	703.5	727.9	398.3	656.3	727.6	702.2	725.1

Table 5. Inventories at the end of each period for all items.

Item No	Period											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
01	11456	23866	13365	13365	11456	0	1909	3818	1909	28439	23666	18893
02	147056	93932	72579	72579	10602	10602	10602	19456	78681	69827	146360	124225
03	44175	26076	25693	25693	22676	4577	4577	10609	7593	58376	50835	43294
04	14010	32038	23558	12766	11224	1974	1974	8912	7370	5828	25611	21757
05	47127	89968	53718	53718	7581	7581	7581	14172	131715	125124	108646	92168
06	40057	21694	26284	26284	21760	4861	4861	10981	7921	59696	52045	44394
07	23176	18200	13638	7832	7002	2026	2855	13938	13109	12280	10540	8466
08	11117	11117	14763	11117	11117	11117	11117	18065	18065	28488	11117	40273
09	9533	39614	47818	20699	9533	9533	9533	15002	15002	55276	98390	84717
10	146199	63776	133285	20417	20417	20417	20417	20417	36541	60727	85012	227344
11	204995	85775	16634	16634	16634	16634	16634	16634	16634	16634	16634	271627
12	17724	35199	42127	9794	9794	9794	9794	14413	14413	21342	9794	69068

Results

Table 4 shows the final lot-sizes and forecasted machine hour requirements for each period, and Table 5 shows the inventories at the end of each period for all items as obtained by applying the proposed heuristic. Tables 6 and 7 show the results that have been found after applying the heuristics without setup time and with setup time, respectively.

Table 6. Costs obtained by applying Dixon-Silver heuristic.

Total available machine time ($\sum_{t=1}^{H} C_t$)	: 8587.0 hour
Total forecasted machine time	: 8139.8 hour
Total inventory-holding cost, $C_{inv} = \sum_{i=1}^{N}\sum_{t=1}^{H}(I_{it} - SS_i)$	: Tk. 64674.05
Total expected safety-stock cost, $C_{ss} = \sum_{i=1}^{N} SS_i$	: Tk. 19862.85
Total expected setup cost, $C_{set} = \sum_{i=1}^{N} n_i S_i$	: Tk. 11959.00
where n_i is the number of setups for item *i*.	
Total expected cost ($C_{inv} + C_{ss} + C_{set}$)	: Tk. 96495.90

Table 7. Costs obtained by applying the proposed heuristic.

Total available machine time ($\sum_{t=1}^{H} C_t$)	: 8587.0 hour
Total setup time ($\sum_{i=1}^{N} n_i St_i$)	: 93.5 hour
where n_i is the number of setup for item *i*.	
Total forecasted machine time	: 8233.2 hour
Total inventory-holding cost, $C_{inv} = \sum_{i=1}^{N}\sum_{t=1}^{H}(I_{it} - SS_i)$	: Tk. 65896.46
Total expected safety-stock cost, $C_{ss} = \sum_{i=1}^{N} SS_i$	: Tk. 19862.85
Total expected setup cost, $C_{set} = \sum_{i=1}^{N} n_i S_i$	: Tk. 11853.00
Total expected cost ($C_{inv} + C_{ss} + C_{set}$)	: Tk. 97612.31

Effect of Setup Time on Different Parameters

In Table 1, the setup time for each item of the hypothetical problem has been chosen so that any increase in this time will make the problem infeasible. To see the effect of setup time on different parameters, the value of setup time of each item of Table 1 has been varied step by step at a 5% interval. With these variations the changes of the machine utilization time, total inventory cost, and total cost have been determined and shown in Figure 1, Figure 2 and Figure 3. The setup time 0% indicates that there is no setup time for each item. This is same as the Dixon-Silver's original algorithm.

Figure 1 shows the machine utilization time for various percentage of the setup time. This time increases linearly with the setup time. The increase in setup time increases the time to produce an item. This increase in production time results an increase in the machine utilization time.

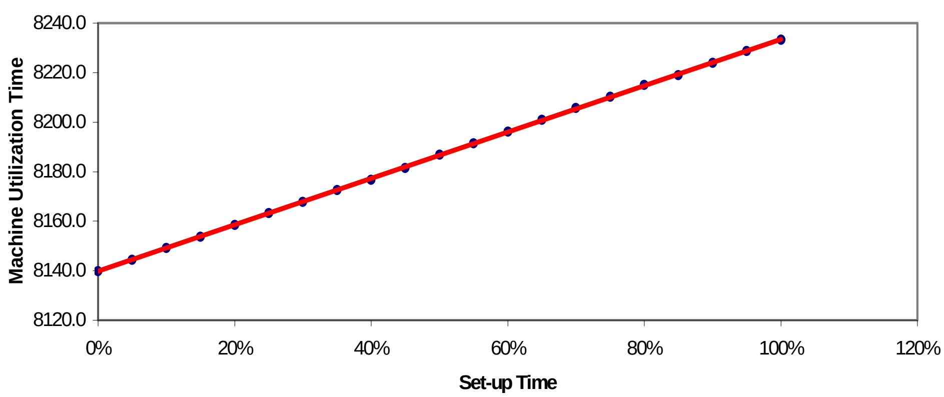

Figure 1. Variation of machine utilization time with setup time.

Figure 2 shows the variation of inventory holding cost with setup time. With the increase of setup time, inventory holding cost increases gradually. Since the increase of setup time decreases the available capacity in a period, there could be periods in which total demand exceeds total capacity. To overcome this unbalance situation some inventory will have to be built up in earlier periods with available slack capacity. When setup time increases, number of capacity violating period would increase. Thus the inventory will be more. As a result inventory holding cost increases with the increase of setup time.

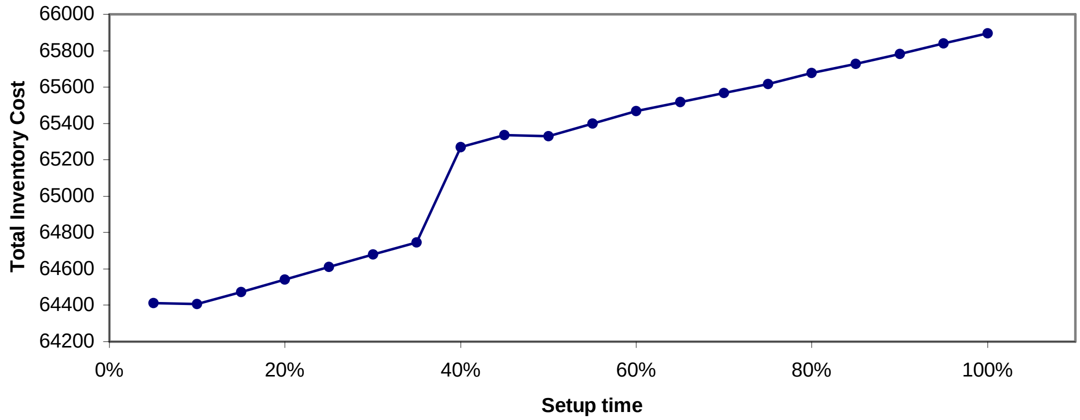

Figure 2. Variation of inventory holding cost with setup time.

Figure 3 shows the variation of total cost with setup time. With the increase of setup time, total cost increases, since the inventory holding cost increases, and the setup cost and safety stock cost remain almost unchanged.

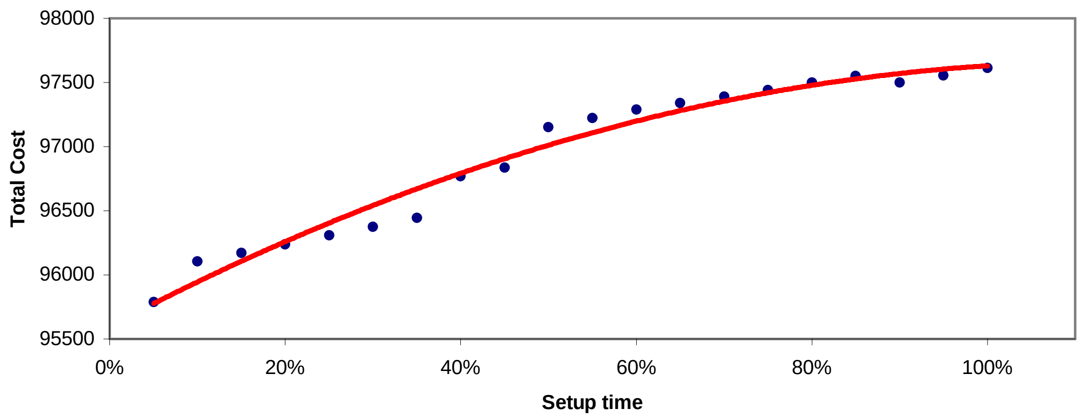

Figure 3. Variation of total cost with setup time.

CONCLUSION

Due to their importance in industry, deterministic and dynamic demand lot-sizing problems are frequently studied. Researchers typically develop specialized formulations and solution procedures for each particular lot-sizing problem class. However, The CLSP provides a comprehensive modeling framework for single and multiple items, coordinated and uncoordinated scheduling, and capacitated and uncapacitated problem variants. This paper extends Dixon-Silver heuristic with setup time which is very important from practical point of view, especially when the problem is multi-item and capacitated. The proposed heuristic has given feasible solutions for bench-mark data.

Other promising research areas are available. While genetic algorithms, tabu search and capacitated network flow models are successfully applied to solve other lot-size problems, their potential to solve CLSP is unknown. Research examining sensitivity analysis of dynamic lot-sizing heuristics with other parameters of CLSP is also worthwhile. The applicability of these problems arises commonly in operations in industries which consist of a single production process, or where all production process can be considered as a single operation, such as some medical or chemical industries.

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