Development of the wave spectrum of the northern region of the Bay of Bengal based on Indian National Centre for Ocean Information Services (INCOIS) moored buoy data

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Abstract: The wave spectrum of the northern region of the Bay of Bengal is developed and analyzed. A meticulous analysis shows variations of the wave spectrum of this region in the four meteorological seasons namely spring (March, April, May), summer (June, July, August), autumn (September, October, November) and winter (December, January, February). Real time wave data have been collected from a moored buoy of Indian National Centre for Ocean Information Service (INCOIS) located at 89.675° E longitude and 18.191° N latitude. Varying two-parameter (mean wave period and significant wave height) wave spectra for the four seasons are obtained and formulae for all the four spectra are derived. The adequacy of the different spectra is indicated by comparing with different wave spectra of known formulations.

Keywords: Wave spectrum, Bay of Bengal, meteorological seasons, significant wave height, mean wave period.

INTRODUCTION

Waves in the sea or ocean, generated by wind, generally consist of locally generated waves and the swells propagating from the far-flung locations1. These waves are not just simple sinusoids. The surface appears to be composed of random waves of various height, lengths, periods and shape. This randomness results in limited predictability. They can be described as a stochastic process2. As such we cannot describe this surface very easily. However, with some simplifications, we can come close to describing the surface. The simplifications lead to the concept of the spectrum of waves. The spectrum gives the frequency wise distribution of wave energy on the sea surface1. Thus we can say that the wave spectrum is an expression for the distribution of energy in the waves. The knowledge of waves and wave spectrum has widespread scientific and practical applications. These range from engineering applications such as the design and operational safety of harbors, ships, and offshore structures to coastal and marine management including, e.g. coastal stability, marine and coastal pollution and marine operations planning3-6.

However, the Earth’s climate is dynamic and varies naturally over a range of time-scales7. These variations and trends tend to instigate a number of physical processes culminating a ‘herculean’ impact on the marine environment. The varying atmospheric and climatic conditions observed in these time-scales or in other words, in different seasons stimulate variations in the spectral characteristics of the sea or ocean waves8-26. The monsoons of the Indian Ocean are the beau ideals of particularly strong ocean–atmosphere interaction. The large-scale wind field in the northern Indian Ocean reverses between summer and winter, leading to changes in the waves11. Hence, it is important to know how the wave spectrum varies with the change in seasons over a period of one year.

Many studies have been conducted in this region regarding this wave spectral characteristics and its variability. Kumar and Nair11 studied the inter-annual variations in wave spectral characteristics at a location off the central west coast of India. Patra and Bhaskaran17 tried to find the trends in wind-wave climate over the head Bay of Bengal region and Shanais and Kumar27, on the other hand, focused on the long-term wind and wave in the central Bay of Bengal and studied its seasonal variability. Jena et al.23 conducted a study on the seasonal variation in near shore wave characteristics off Cuddalore, southeast coast of Tamil Nadu, India. Potemra et al.5 researched on the seasonal circulation of the upper ocean in the Bay of Bengal. Cheng et al.23 studied the intraseasonal variability of sea surface height in the Bay of Bengal. Seasonal and annual variations in the wave climate over the near shore waters off Puduchery, south western Bay of Bengal induced by monsoon and cyclone were studied by Glejin et al.26. Chaudhary et al.27 derived significant wave height in the Bay of Bengal basin of the Indian Ocean near the Indian city of Visakhapatnam for the years 2010–2012. Kumar et al.28 studied the wave spectral characteristics off Gaggavaram, Bay of Bengal. Islam et al.29 calculated wave energy and analyzed energy fluctuation at various months of different consecutive years in the Bay of Bengal. However, no study has been conducted to develop and analyze the wave spectrum in the northern region of the Bay of Bengal for the different seasons. So, a research study is needed in this regard.
Meteorologists and climatologists divide the seasons into four with each having a grouping of three months. Meteorological spring includes March, April, and May; meteorological summer includes June, July, and August; meteorological autumn or fall includes September, October, and November; and meteorological winter includes December, January, and February. During these four seasons, various climatic factors vary which ultimately leads to the variations in wave spectra.

A study is carried out from March, 2015 to February, 2016. A location of study is selected where the measured wave data is available. The geographical co-ordinates of the location are 89.675° E longitude and 18.191° N latitude.

The objective of the study is to investigate the characteristics of wave spectrum of northern region of the Bay of Bengal and its seasonal variation and derive formulae for the wave spectra and thus, to have a better understanding of the nature of the waves occurring in the region.

DATA AND METHODOLOGY

Data

![Map of the northern Bay of Bengal](image1)

Figure 1: Map of the northern Bay of Bengal.

![Map showing 'Moored Buoy BD-08'](image2)

Figure 2: Map showing 'Moored Buoy BD-08'.

The data used in the study were taken from the website of Indian National Centre for Ocean Information Service (INCOIS), covering the period from March, 2015 to February, 2016. The URL link of the website is: [http://odis.incois.gov.in/](http://odis.incois.gov.in/). INCOIS receives voluminous oceanographic data in real time, from a variety of in-situ and remote sensing observing systems. To achieve this purpose INCOIS has installed many buoys. As a part of this, they installed a moored buoy at location 89.675° E longitude and 18.191° N latitude in the Bay of Bengal named ‘Moored Buoy BD-08’ (red circle in Figure 2). Data of this buoy is used in this study.

Methodology

The wave spectra for each of the seasons were obtained from the data using histogram. It is known that when umpteen sinusoidal waves of disparate wavelengths, heights and periods superpose on each other, irregular wave pattern is developed. This superposition results in an extremely irregular seaway that is never repeated anytime. The wave spectrum of this seaway can be obtained by measuring the total energy content of all the waves present. According to Bhattacharyya, the energy of a sinusoidal wave is given by,

$$ E_s = \frac{1}{2} \rho g \zeta^2 $$

where, $\zeta$ is the amplitude

The total energy is then given by

$$ E_T = \frac{1}{2} \rho g (\zeta_1^2 + \zeta_2^2 + \ldots + \zeta_n^2) $$

where, $\zeta_1, \zeta_2, \ldots, \zeta_n$ are the amplitudes of the waves

Hence, energy spectrum is the distribution of this total energy according to frequency of waves where the ordinates are obtained by dividing the individual energy content by the bandwidth, which is 0.2 in this study. The energy density or spectral density results when $\rho g$ is divided out from the energy, given by,

$$ S_w = \frac{1}{2} (\zeta_1^2 + \zeta_2^2 + \ldots + \zeta_n^2) $$

where, $S_w$ is the energy density

Wave spectrum is the distribution of energy density according to frequency of waves. Mathematically,

$$ S_w / \delta w = \frac{1}{2} (\zeta_1^2 + \zeta_2^2 + \ldots + \zeta_n^2) $$

where, $\delta w$ is the bandwidth.

The wave spectrum curve can be obtained from these histograms. By averaging, the final wave spectrum results. This method was exercised in the study. The data was processed and computations were done accordingly to obtain wave spectra for all the seasons. The significant wave height and mean time period for each season is also computed. Correction factor for the
Development of the wave spectrum of the northern region of the Bay of Bengal

Figure 3: Spring wave spectrum with histogram.

Figure 4: Summer wave spectrum with histogram.

Figure 5: Autumn wave spectrum with histogram.

Figure 6: Winter wave spectrum with histogram.

Wave spectra are derived showing the deviation from Rayleigh distribution from the equations given below:

\[
CF = (1 - \delta^2)^{1/2}
\]

where, \(CF\) is the correction factor and \(\delta^2\) is the broadness parameter which is given by the expression,

\[
\delta^2 = \frac{m_0m_4 - m_2^2}{m_0m_4}
\]

where the moment of the area under the spectrum is

\[
m_n = \int_0^\infty w^n S_w \, dw
\]

Here, \(n\) is any integer and \(m_n\) is the \(n\)th moment of the area under the spectrum.

Four equations using the parameters significant wave height and mean time period are set describing the spectra of the four seasons, henceforth, showing the seasonal variation. The spectra are compared with different standard wave spectra namely ITTC spectrum, JONSWAP spectrum, Scott spectrum, and Ochi-Hubble bi-modal spectrum measured for respective seasons at this location in order to assess the adequacy of the result obtained.

RESULTS AND DISCUSSIONS

Wave spectra

As per the method outlined in the previous section, histograms and wave spectra for the four seasons were obtained (Fig. 3, Fig. 4, Fig. 5 and Fig. 6).

The final wave spectra were obtained by averaging (Fig. 7, Fig. 8, Fig. 9 and Fig. 10).

Significant wave height and mean wave period

Significant wave height (SWH) and mean wave period for the four seasons were analyzed from the data (Table 1).

Table 1: Significant Wave Height (SWH) and Mean Wave Period.

<table>
<thead>
<tr>
<th>Season</th>
<th>Significant Wave Height(m)</th>
<th>Mean Wave Period(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring</td>
<td>1.77</td>
<td>6.40567861</td>
</tr>
<tr>
<td>Summer</td>
<td>3.32</td>
<td>6.328103411</td>
</tr>
<tr>
<td>Autumn</td>
<td>2.01</td>
<td>6.453918845</td>
</tr>
<tr>
<td>Winter</td>
<td>0.99</td>
<td>5.251915237</td>
</tr>
</tbody>
</table>
Correction factor
Correction factor of the wave spectra of the four seasons were calculated showing how much the spectra deviates from ‘narrow frequency spectrum’\(^3\) (Table 2). The deviation was different for different seasons leading to the seasonal variation of the correction factor as well.

Table 2: Correction Factor (CF).

<table>
<thead>
<tr>
<th>Season</th>
<th>CF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring</td>
<td>0.981772626</td>
</tr>
<tr>
<td>Summer</td>
<td>0.930123968</td>
</tr>
<tr>
<td>Autumn</td>
<td>0.981771781</td>
</tr>
<tr>
<td>Winter</td>
<td>0.891792832</td>
</tr>
</tbody>
</table>

Equations of the wave spectra
Equation was derived for the wave spectra of each season. Significant wave height and mean wave period are the parameters used in the equations.

The spring wave spectral equation of Northern Bay of Bengal is given by,

\[
S_n = \frac{a \cdot H_s^3 \cdot \exp\left(-\frac{(w-b)^2}{c \cdot (w+d+e \cdot T_m)} \right)}{g \cdot T_m^5} - f \cdot H_s^2 \cdot T_m \\
* \exp\left(\frac{H_s^2 \cdot w^4}{T_m} \right)
\]  

(8)

where, \(a = 7.099133\); \(b = 1.027581\); \(c = 0.0108036\); \(d = 0.8876376\); \(e = 0.1518847\); \(f = 5.914 \times 10^{-5}\);

\(w\) is the frequency in rad / sec;
\(g\) is the acceleration due to gravity in ms\(^2\);
\(H_s\) and \(T_m\) are significant wave height and mean wave period respectively whose values for spring are 1.77 m and 6.40567861 sec respectively (Table 1).

For summer, the spectral equation is given by,

\[
S_n = a \cdot g^2 \cdot T_m^5 \cdot \exp\left(-\frac{\left(\frac{T_m}{c} \right)^2}{\frac{T_m}{c}} \right) + d \cdot H_s^2 \cdot T_m \\
* \exp\left(-\frac{\left(\frac{T_m}{f} \right)^2}{\frac{T_m}{f}} \right)
\]  

(9)
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where, \( a = 0.0005708; \) \( b = 5.71172; \) \( c = 0.0417619; \)
\( d = 0.7029379; \) \( e = 7.191828; \) \( f = 0.0289901 \)
g is the acceleration due to gravity in \( \text{ms}^2; \)
\( H_s \) and \( T_m \) for summer are 3.32 m and 6.328103411 sec respectively (Table 1).

The wave spectral equation for the autumn season is,

\[
S_w = a \cdot g^2 \cdot T_m^5 \cdot \exp\left(-\left(\frac{w}{c}\right)^2\right) + d \cdot H_s^2 \cdot T_m^6
\]
\[
\cdot \exp\left(-\left(\frac{w}{T_m}\right)^2\right) - \frac{h \cdot H_s^3 \cdot \exp\left(\frac{H_s^2 \cdot w^4}{T_m}\right)}{g \cdot T_m}
\]

(10)

where, \( a = 5.548 \times 10^{-5}; \) \( b = 6.04062; \) \( c = 0.0283126; \)
\( d = 0.0133533; \) \( e = 6.496685; \) \( f = 0.3313183; \)
h is the acceleration due to gravity in \( \text{ms}^2; \)
\( H_s \) and \( T_m \) for autumn are 2.01 m and 6.453918845 sec respectively (Table 1).

The equation of the wave spectrum for the winter is,

\[
S_w = a \cdot g^2 \cdot T_m^5 \cdot \exp\left(-\left(\frac{w}{c}\right)^2\right) + d \cdot H_s^2 \cdot T_m^6
\]
\[
\cdot \exp\left(-\left(\frac{w}{T_m}\right)^2\right) - \frac{h \cdot g \cdot H_s^3 \cdot \exp\left(\frac{H_s^2 \cdot w^4}{T_m}\right)}{f}
\]

(11)

where, \( a = 6.702 \times 10^{-7}; \) \( b = 6.934648; \) \( c = 0.0901302; \)
\( d = 0.0098529; \) \( e = 5.654206; \) \( f = 0.5557179; \)
h is the acceleration due to gravity in \( \text{ms}^2; \)
\( H_s \) and \( T_m \) for winter are 0.99 m and 5.251915237 sec respectively (Table 1).

Comparison with other known wave spectra

The spectra of the four seasons are analogized to ITTC spectrum, JONSWAP spectrum, Scott spectrum and Ochi-Hubble bi-modal spectrum of respective seasons (Fig. 11, Fig. 12, Fig. 13 and Fig. 14).

It can be seen that the spectra of all the seasons except winter are comparable to the Scott spectrum of the corresponding seasons. The winter spectrum gives a unique representation.
Seasonal variations of the wave spectrum and deviation of the measured spectra from other known spectral formulations

The energy content at different frequency bands varied from season to season, thus, showing the state of the sea (according to ITTC recommended procedures and guidelines) at different seasons. Summer showed a rough state (SWH is 3.32 m) and winter showed a slight sea state (SWH is 0.999m). Sea state in both spring and autumn was moderate (SWH is 1.77m and 2.01 m for spring and autumn respectively). The range of frequency that is important for the contribution of energy to the seaway also showed seasonal variation. In spring the energy content is ranged between 0.4 to 1.8 rad/sec. In summer the range of frequency which contributes in the energy content is much lower (between 0.6 to 1.4 rad/sec). Autumn spectrum has its energy between a range of 0.2 to 1.8 rad/sec and winter spectrum between 0.4 to 2 rad/sec. The frequency at which maximum energy is supplied varied as well. Spring, summer and autumn all had their maximum energy supplied at 1 rad/sec but in winter the maximum energy was supplied at 1.4 rad/sec. Swell condition was different for different seasons. Winter showed swell condition ranged over comparatively more low-frequency bands. Swell effect is more prominent in lower sea states than the higher sea states.

The reason behind the variations of the wave spectra is the varying atmospheric and climatic conditions observed during the different seasons. The atmospheric circulation, and consequently the surface winds are impelled by these climatic variations35, 46. As surface wind waves (and storm surges) and its spectrum are generated and shaped by the action of wind on the sea surface32, so properties of wind-waves will vary in response to the climatological variations in the surface atmospheric circulation. Therefore, variations will occur in the spectrum of the waves as well. Variation in swell in different seasons41 is another cause that triggers variations in the wave spectral characteristics. Monsoon intensity changes with season42. This tends to bring inter seasonal changes in the wave spectral characteristics11. Summer season shows the highest values of energy density. This is probably due to higher monsoon intensity in summer season. Fetch length variation probably caused the spectral variations as well32. Longer fetch in summer presumably resulted in its rough sea state. Other spectra vary accordingly.

Measured spectra of the four seasons showed variation from ITTC spectrum, JONSWAP spectrum, Scott spectrum and Ochi-Hubble bi-modal spectrum of respective seasons. As mentioned earlier, the spring, the summer and the autumn spectra can be best compared to the Scott spectra of their respective seasons. Although the energy content is different, they show somewhat comparable shape. Winter spectrum, on the other hand, does not show similarity with any of these. It gives a unique representation. There might be some probable reasons for these deviations. The ITTC formulation has some limitations: fetch limitations, state of development or decay, sea-floor topography, local currents and swell limitations43. Hasselmann et al.36 found that the JONSWAP spectrum is a fetch-limited wave spectrum. The wave spectrum is never fully developed. It continues to develop through non-linear, wave-wave interactions even for very long times and distances. Therefore in the JONSWAP spectrum, waves continues to grow with distance (or time) and the peak in the spectrum is more pronounced. It has some basic assumptions: North Sea data, limited fetch, uni-directional seas and no swell37. These assumptions probably led to the deviation from the measured wave spectra of the northern region of the Bay of Bengal for all seasons. Scott spectral formula is independent of the wind speed, fetch or duration and represents a fully developed sea35, 37 and Ochi-Hubble spectrum incorporates the effect of swell35, 38. Real-time wave data of the northern region of the Bay of Bengal is used in this study. Thus new wave spectra for the four seasons are obtained, different from other known spectral formulations.

CONCLUSION

The principal focus of the study is to develop the wave spectrum of the northern region of the Bay of Bengal. How the wave spectrum varies with the change of seasons was analyzed as well. The mean wave period and significant wave height, varying with seasonal changes, were investigated. The correction factors of the wave spectra in the four seasons showing the deviation from Rayleigh distribution were calculated. The equations of the wave spectra using the parameters mean wave period and significant wave height were derived. The seasonal analysis shows that the energy content in summer is the highest and that in winter is the lowest. The comparison with other spectra for each season indicates that the result is encouraging. Thus, using the derived formulae, the wave spectrum of the northern part of Bay of Bengal in every season can be obtained, thereby, opening the door to new research opportunities in this region.

ACKNOWLEDGEMENT

Buoy data were obtained from the website of Indian National Centre For Ocean Information Services (INCOIS). We thank the topic editor and reviewers for critical comments and suggestions. This work forms the undergraduate thesis of the first and second author.

REFERENCES
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40. Letcher, T.M., 2015, Climate change: observed impacts on planet Earth, Elsevier.
43. Techet, A., 2005, 13.42 design principles for ocean vehicles.

APPENDIX

The data used in the study is obtained from the website of INCOIS. Data of 5 days of the month of March only is reproduced here in Table A1 for perusal.

Table A1: Wave data of 5 days of March’15

<table>
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