Heat Transfer in a Porous Medium over a Stretching Surface with Internal Heat Generation and Suction or Injection in the Presence of Radiation

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Abstract: Heat transfer in a porous medium over a stretching surface with internal heat generation and suction or injection has been analyzed numerically in the presence of radiation. In this analysis, the governing equations are transformed into a system of ordinary differential equations and solved them numerically using Nachtsheim-Swigert shooting iteration technique. The local similarity solutions for the flow and the heat transfer characteristics are presented graphically for various material parameters entering into the problem. The effects of the pertinent parameters on the local skin friction coefficient (viscous drag) and the Nusselt number (rate of heat transfer) are also displayed graphically.

Keywords: Internal heat generation, suction, injection, radiation, Nusselt number.

INTRODUCTION

Boundary layer flow on continuous moving surface is a significant type of flow occurring in several engineering applications. Aerodynamic extrusion of plastic sheets, cooling of an infinite metallic plate in a cooling bath, the boundary layer along a liquid film in condensation processes and a polymer sheet or filament extruded continuously from a dye, or a long thread traveling between a feed roll and a wind-up roll, are examples for practical applications of continuous flat surface. As examples on stretched sheets, many metallurgical processes involve the cooling of continuous strips or filament by drawing them through a quiescent fluid and that in the process of drawing, when these strips are stretched.

The flow field of a stretching wall with a power-law velocity variation was discussed by Banks1. Ali2 and Elbashbeshy3 extended the work of Banks1 for a porous stretched surface with different values of the injection parameter. Gupta and Gupta4 analyzed the stretching problem with constant surface temperature. Sriramula et al.5 studied steady flow and heat transfer of a viscous incompressible fluid flow through porous medium over a stretching sheet. Pop and Na6 studied free convection heat transfer of non-Newtonian fluids along a vertical wavy surface in a porous medium. Ali et al.2 studied radiation effect on natural convection flow over a vertical surface in a gray gas. Following Mansour7 studied the interaction of mixed convection with thermal radiation in laminar boundary layer flow over a horizontal, continuous moving sheet with constant suction/injection. Bakier and Gorla8 considered the effect of thermal radiation on the mixed convection from horizontal surfaces in saturated porous media.

Elbashbeshy and Bazid9-14 re-analyzed the stretching problem discussed earlier by Elbashbeshy3 including variable viscosity, internal heat generation, suction/injection and porous medium. Subhas and Veena15 analyzed visco-elastic fluid and heat transfer in a porous medium over a stretching surface. In the present study, the thermal radiation interaction of the boundary layer flow over a stretching surface embedded in a porous medium with internal heat generation and suction or injection has been investigated. The similarity solutions have been

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>( C_f )</td>
<td>Skin friction coefficient</td>
</tr>
<tr>
<td>( e_p )</td>
<td>Specific heat at constant pressure</td>
</tr>
<tr>
<td>( h(x) )</td>
<td>Local heat transfer coefficient</td>
</tr>
<tr>
<td>( K )</td>
<td>Permeability parameter</td>
</tr>
<tr>
<td>( N )</td>
<td>Radiation parameter</td>
</tr>
<tr>
<td>( Nu )</td>
<td>Nusselt number</td>
</tr>
<tr>
<td>( Pr )</td>
<td>Radiative Prandtl number</td>
</tr>
<tr>
<td>( Pr )</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>( Q )</td>
<td>Volumetric rate of heat generation</td>
</tr>
<tr>
<td>( q_r )</td>
<td>Radiative heat flux</td>
</tr>
<tr>
<td>( q_o )</td>
<td>Heat flux</td>
</tr>
<tr>
<td>( Re )</td>
<td>Local Reynolds number</td>
</tr>
<tr>
<td>( T )</td>
<td>Temperature within the boundary layer</td>
</tr>
<tr>
<td>( T_w )</td>
<td>Temperature at the plate</td>
</tr>
<tr>
<td>( T_\infty )</td>
<td>Temperature outside the boundary layer</td>
</tr>
<tr>
<td>( u_o )</td>
<td>Velocity of the plate</td>
</tr>
<tr>
<td>( u_s )</td>
<td>Suction/Injection parameter</td>
</tr>
<tr>
<td>( u_x, u_y )</td>
<td>x and y velocity components</td>
</tr>
<tr>
<td>( x, y )</td>
<td>Cartesian co-ordinates</td>
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</table>

Greek Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
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</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>Dimensionless temperature</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Thermal conductivity</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Density of the fluid</td>
</tr>
<tr>
<td>( \kappa_1 )</td>
<td>Mean absorption coefficient</td>
</tr>
<tr>
<td>( \Psi )</td>
<td>Stream function</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Coefficient of the dynamic viscosity</td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>Stefan-Boltzmann constant</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Similarity parameter</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Heat generation or absorption parameter</td>
</tr>
<tr>
<td>( \mu_e )</td>
<td>Effective viscosity</td>
</tr>
<tr>
<td>( \alpha_p )</td>
<td>Thermal diffusivity of the porous medium</td>
</tr>
<tr>
<td>( \tau_\nu )</td>
<td>Shear stress</td>
</tr>
<tr>
<td>( \nu_a )</td>
<td>Apparent kinematic viscosity</td>
</tr>
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numerically for various parameters entering into the problem using shooting method and interpreted the obtained data from the physical point of view.

MATHEMATICAL FORMULATION

Let us consider a steady, two-dimensional laminar flow of a viscous, incompressible fluid through a porous medium of permeability $K$ over a stretching surface with a uniform temperature $T_w$ and velocity $u, v$, moving axially through a stationary fluid. Two equal and opposite forces are introduced along $x$- axis so that the wall is stretched keeping the origin fixed and $y$- axis is perpendicular to it. The fluid is considered to be gray; absorbing-emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The radiative heat flux in the $x$- direction is considered negligible in comparison to the $y$- direction. The conservation equations of the laminar boundary layer are as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\rho \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\mu_x}{K} u
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} - \frac{1}{\rho c_p} \frac{\partial q}{\partial y} + Q(T - T_w) - \frac{1}{\rho c_p} \frac{\partial q}{\partial y}
\]

where, $u, v$ are the velocity components along $x, y$ coordinates respectively, $\rho$ is the density of the fluid, $K$ is the permeability of the porous medium, $\mu$ is the dynamic viscosity, $\mu_x$ is the coefficient of the dynamic viscosity, $\mu_y$ is the effective viscosity, $T$ is the temperature of the fluid in the boundary layer, $T_w$ is the temperature of the fluid outside the boundary layer, $\alpha$ is the effective thermal diffusivity of the saturated porous medium and $Q$ is the volumetric rate of heat generation.

By using Rosseland approximation, $q_r$ takes the form

\[
q_r = \frac{4\sigma_1 T_4^4}{3\kappa_1} \frac{\partial T}{\partial y}
\]

where, $\sigma_1$ is the Stefan-Boltzmann constant and $\kappa_1$ is the mean absorption coefficient.

We assume that the temperature differences within the flow are sufficiently small such that $T^4$ may be expressed as a linear function of temperature. This is accomplished by expanding $T^4$ in a Taylor’s series about $T_w$ and neglecting higher-order terms, thus

\[
T^4 \approx T_w^4 + (T - T_w) \times 4T_w^3 = 4T_w^3 T - 3T_w^4
\]

By using Eqs. (4) and (5), (3) gives

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + Q(T - T_w) - \frac{1}{\rho c_p} \frac{\partial q}{\partial y} + \frac{16\sigma_1 T_w^3}{3\rho c_p \kappa_1} \frac{\partial^2 T}{\partial y^2}
\]

The corresponding boundary conditions for the above problem are given by

\[
\begin{align*}
 u &= u_o x, \quad v = \pm v_w, \quad T = T_w \text{ at } y = 0 \\
 u &= 0, \quad T = T_w \text{ as } y \to \infty
\end{align*}
\]

Positive and negative values for $v_w$ indicate blowing and suction respectively, while $v_w = 0$ corresponds to an impermeable plate.

The following non-dimensional variables are introduced in order to obtain the non-dimensional governing equations:

\[
\eta = \sqrt{\frac{\mu u_o}{\rho}}, \quad \psi(x, y) = \sqrt{\frac{\mu u_o}{\rho}} f(\eta), \quad \theta = \frac{T - T_w}{T_w - T_{\infty}}
\]

where, $\psi$ is the stream function.

Since $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, we have from Eq. (8) that

\[
u = u_o x f'(\eta) \quad \text{and} \quad v = -\sqrt{\frac{\mu u_o}{\rho}} f(\eta)
\]

Here $f$ is non-dimensional stream function and prime denotes differentiation with respect to $\eta$. Now substituting Eqs. (8)-(9) into Eqs. (1)-(2) and (6) we obtain,

\[
\frac{f'''}{f'} + \frac{f''}{f'} - K_1 f' = 0
\]

\[
\theta' + Pr(P_s + f \theta') = 0
\]

where,

\[
K_1 = \frac{\rho K u_o}{\kappa}
\]

is the permeability parameter

\[
\lambda = \frac{Q}{u_o}
\]

is the heat generation or absorption parameter

\[
Pr = \frac{\rho u_o}{3N_1 + 4}
\]

is the radiative Prandtl number

\[
n = \frac{KN_1}{4\sigma_1 T_w^3}
\]

is the radiation parameter

\[
Pr = \frac{\rho u_o}{3N_1 + 4}
\]

is the Prandtl number

The corresponding boundary conditions (7) becomes,

\[
\begin{align*}
 f &= f_w, \quad f' = 1, \quad \theta = 0 \text{ at } \eta = 0 \\
 f' &= 0, \quad \theta = 0 \text{ as } \eta \to \infty
\end{align*}
\]

where, $f_w = \mp v_w \sqrt{\frac{\rho}{\mu u_o}}$ is the suction and injection velocity at the plate for $f_w > 0$ and $f_w < 0$ respectively.

SKIN FRICTION COEFFICIENT

The quantities of main interest in such problem are the skin friction coefficient and the Nusselt number (rate of heat transfer). The shearing stress on the surface is defined by

\[
\tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}
\]

The local skin friction coefficient is defined as

\[
C_f = \left( \frac{\tau_w}{\frac{1}{2} \rho u_o^2} \right) f'(0)
\]

where $Re_x = \frac{u_{w}}{v_{w}}$ is the local Reynolds number.

Thus from Eq. (14) we see that the local values of the skin friction coefficient $C_f$ is proportional to $f'(0)$.

NUSSELT NUMBER

The local heat flux may be written by Fourier’s law as

\[
q_w(x) = \kappa \left( \frac{\partial T}{\partial y} \right)_{y=0}
\]

The local heat transfer coefficient is given by

\[
h(x) = \frac{q_w(x)}{T_w - T_{\infty}}
\]

The local Nusselt number may be written as

\[
Nu = \frac{h(x)}{K} = \left( Re_x \right)^{1/2} \theta'(0)
\]

Thus from Eq. (17) we see that the local Nusselt number $Nu$ is proportional to $\theta'(0)$. 

NUMERICAL SOLUTION

The set of nonlinear ordinary differential equations (10)-(11) with boundary conditions (12) have been solved by using sixth order Runge-Kutta method along with Nachtsheim-Swigert shooting iteration technique with $f_m$, $K$, $\lambda$, $Pr$, $N$ as prescribed parameters. A step size of $\Delta \eta = 0.001$ is selected to be satisfactory for a convergence criterion of $10^{-6}$ in all cases. The value of $\eta_\infty$ is found for each iteration loop by the statement $\eta_\infty = \eta_\infty + \Delta \eta$. The maximum value of $\eta_\infty$ to each group of parameters $f_m$, $K$, $\lambda$, $Pr$, $N$ is determined when the value of the unknown boundary conditions at $\eta = 0$ does not change to successful loop with error less than $10^{-6}$.

CODE VERIFICATION

To assess the accuracy of our code, we calculated the velocity for the case of injection and suction. Setting $N = 0$, we have compared these with that of Elbashbeshy and Bazid. Figure 1 shows the comparison of the data produced by the present code and those of Elbashbeshy and Bazid. In fact, the results show a close agreement, hence an encouragement for the use of the present code.

RESULTS AND DISCUSSION

For discussing the results, the numerical calculations are presented in the form of non-dimensional velocity and temperature profiles. In the calculations, the values of permeability parameter $K$, suction parameter $f_m$, Prandtl number $Pr$, radiation parameter $N$, Heat generation or absorption parameter $\lambda$ are chosen arbitrarily.

Figure 2(a) shows the dimensionless velocity profiles for different values of injection and suction parameter $f_m$. It can be seen that the velocity profiles decrease monotonically with the increase of injection parameter as well as suction parameter. Figure 2(b) also shows the similar behavior (like as the velocity profiles) of the effect of suction and injection parameter on the temperature profiles. Figures 3(a)-(b) show the velocity profiles for different values of permeability parameter $K$ in case of suction and injection. From these figures, we see that the velocity decreases with the increase of the permeability parameter $K$ in both case of suction and injection.
Figures 4(a)-(b) show that dimensionless temperature profiles increase with increasing $K$ in case of both suction and injection. From these figures, we see that the temperature profile increases very slowly for the case of suction; whereas for the case of injection its increasing behavior is significant. The effect of the heat generation and absorption parameter $\lambda$ on the temperature field is shown in the Figures 5(a)-(b). From these figures, we observe that temperature profiles increase with the increasing $\lambda$ in case of both suction and injection. We also observe that after a certain distance from the plate the profiles overlap and decrease monotonically for both the cases of fluid suction and injection.
Figures 6(a)-(b) show the temperature profiles for different values of Prandtl number $Pr$. From these figures we see that the dimensionless temperature profile decreases with increasing $Pr$ in case of suction and injection. All the above calculations have been carried out for a fixed radiation parameter $N$. Therefore, the effects of radiation parameter $N$ on velocity and temperature profiles are not clear from the earlier discussions. Figures 7(a)-(b) show the effect of radiation parameter $N$ on the temperature profiles in the case of suction and injection. We observe from these figures that temperature profile decreases with the increase of radiation parameter for both cases. For large $N$, it is clear that temperature decreases more rapidly with the increase of $N$. Therefore using radiation, we can control the temperature distributions.

Figures 8(a)-(b), respectively, show that skin friction coefficient for different values of permeability parameter $K$ and the heat generation and absorption parameter $\lambda$. Here we found skin friction coefficient decreases with increasing permeability in the case of suction and injection and increases with increasing heat source parameter in the case of suction and injection. Figures 9(a)-(b) show that dimensionless rate of heat transfer coefficient decreases with increasing heat source parameter and permeability parameter in the case of suction and injection respectively. Figures 10(a)-(b), respectively, show the effects of Prandtl number $Pr$ and radiation parameter $N$ on the Nusselt number. The Nusselt number $Nu$ increases with the increase of $Pr$ as well as with $N$ in case of suction and injection.
Figure 8. Variation of Skin Friction Coefficient as a Function of $\lambda$ for Various Values of $K$ at: (a) $f_w=0.5$ and (b) $f_w=-0.5$

Figure 9. Variation of Nusselt Number as a Function of $\lambda$ for Various Values of $K$ at: (a) $f_w=0.5$ and (b) $f_w=-0.5$

Figure 10. Variation of Nusselt Number as a Function of $N$ for Various Values of $Pr$ at: (a) $f_w=0.5$ and (b) $f_w=-0.5$
CONCLUSIONS

In the present work, we have analyzed heat transfer in a porous medium over a stretching surface with internal heat generation and suction or injection. Using usual similarity transformations, the governing equations have been transformed into a system of non-linear ordinary differential equations and are solved for similar solutions by using Nachtsheim-Swigert shooting iteration technique. A numerical study has been performed to investigate the influence of permeability parameter, heat source (sink) parameter, Prandtl number and radiation parameter on the skin friction coefficient, the rate of heat transfer coefficient, dimensionless velocity and temperature profiles. From the above investigation, we may draw the following conclusions:

(i) Radiation has significant effect on temperature profiles.
(ii) Velocity profiles decrease with increasing injection as well as suction.
(iii) The temperature profiles increase with increasing permeability and heat source parameter and decreases with increasing Prandtl number and radiation parameter.
(iv) The skin friction coefficient increases with heat source and decreases with permeability.
(v) The rate of heat transfer coefficient decreases with heat source parameter as well as permeability and increases with Prandtl number as well as radiation parameter.

REFERENCES