INFLUENCE OF CURVE FITTING TECHNIQUES ON HYDROSTATIC CHARACTERISTICS OF SHIP

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Abstract: *This paper presents influence of curve fitting techniques on hydrostatic characteristics of marine vessels. Three vessels having lengths of 32.16m, 70.0m and 25.6m are selected for this study. The hull geometries of these vessels are represented by three curve fitting techniques such as least square, cubic spline and B-spline methods. At first, hydrostatic characteristics of the vessels are calculated using data only available in offset table and then using more data generated by curve fitting techniques. Finally, results obtained by offset table data are compared to those by fitted data. It is seen that there are some influences of curve fitting technique on the hydrostatic characteristics of ship especially at lightly loaded condition.*

Keywords*: Curve fitting technique, ship hull form, offset table, hydrostatic characteristics, ship stability, cross curve***.**

INTRODUCTION

The surface of a ship is traditionally described by a mesh of intersecting curves called ship lines. Ship hull form is very much complicated due to its 3D non-linear curvature since it may consist of flats, curves, arcs, knuckles, etc. depending on the complexity of the form. For this reason, various mathematical approaches are developed to deal with this problem. Fairing of ship hull surface is a principal design requirement of ship design process as it affects the ship's hydrodynamic performance, stability characteristics, sea-keeping performance etc. The offset table generated from lines plan[1] is usually used for the calculation of ship hydrostatics and stability. However, due to few data available in the offset table at lightship condition, error may arise in the computational results. Many past accidents occurred at lightship condition forces designer to find better way to compute ship hydrostatics and stability more precisely especially at light draught condition. In this paper, different curve fitting techniques are used for fairing of ship lines and their effects on computation of hydrostatic characteristics are studied.

In the field of surface modeling, geometric complexity of ship hull form[1-4] gives many difficulties in surface modeling technique that can describe the irregular topological characteristics precisely. A number of methods has been developed to generate curves and surfaces from a given set of data points, e.g., least square method, cubic spline, Coons patch etc. In general, the principal shortcomings of these methods are their global behaviors that any local change affects the complete shape. The B-spline curves and surfaces[5-7] introduced the concept of control polygons and meshes which provide more intuitive geometric control over the shape.

At preliminary stage of ship design, it is essential to have the data of a fair hull surface[8] before proceed to perform the production tasks and detailed design calculations. Clearly, if the hull surface is not fair it is necessary to carry out a fairing process to obtain the surface with required fairness. The naval architect requires the detailed hydrostatic particulars to check whether the hull form generated at the preliminary design phase has satisfied selected design criteria.

The aims of the present study is to investigate the effect of different curve fitting techniques such as least square, B-Spline and cubic spline on fairing of ship lines and computation of ship hydrostatics.

THEORETICAL BACKGROUND

There is a number of curve fitting techniques among which three methods, i.e., least square, cubic spline and B-spline techniques are chosen to represent hull geometry of ships.

Least Square Curve Fitting Technique

Least square is a mathematical optimization technique which, when given a series of measured data, attempts to find a function which closely approximates the data (a 'best fit'). It attempts to minimize the sum of the squares of the ordinate differences (called *residuals*) between points generated by the function and corresponding points in the data. Specifically, it is called *least mean squares* (LMS) when the number of measured data is 1 and the gradient descent method is used to minimize the squared residual. LMS is known to minimize the expectation of the squared residual, with the smallest operations (per iteration). But it requires a large number of iterations to converge. Suppose that the data set consists of the points (x_i,y_i) with i=1,2,……,n. We want to find a function *f* such that $f(x_i) \approx y_i$

To attain this goal, we suppose that the function *f* is of a particular form containing some parameters which need to be determined. For instance, suppose that it is quadratic, meaning that $f(x) = ax^2 + bx + c$, where *a*, *b* and *c* are not yet known. We now seek the values of *a*, *b* and *c* that minimize the sum of the squares of the residuals given by[9]:

$$S = \sum_{i=1}^{n} (y_i - f(x_i))^2$$

Cubic Spline Fitting Technique

Real world numerical data is usually difficult to analyze. Any function which would effectively correlate the data would be difficult to obtain. To this end, the idea of the cubic spline[6,11] was developed. Using this process, a series of unique cubic polynomials are fitted between each of the data points, with the stipulation that the curve obtained be continuous and appear smooth. These cubic splines can then be used to determine rates of change and cumulative change over an interval.

Table1: Principal Particulars of Vessels				
	Length, L_{BP} (m)	**Breadth, B (m)**	**Depth, D (m)**	**Draft, d (m)**
Vessel 1	32.158	7.50	2.10	1.400
Vessel 2	70.000	12.00	3.00	2.368
Vessel 3	25.588	5.80	2.80	2.262

Table 2: Offset Table for Vessel 1 (All dimensions in mm)

ST	Wl-1	Wl-2	Wl-3	Wl-4	Deck at side	Keel	BTK-1	BTK-2	BTK-3	Deck
0	0	0	0	0	3750	1300	1401	1437	1583	2286
0.5	0	0	0	3160	3750	1062	1156	1250	1343	2253
1	0	0	2348	3620	3750	823	912	1001	1090	2230
1.5	0	1207	3516	3646	3750	585	674	763	847	2189
2	0	3320	3719	3750	3750	350	415	508	600	2131
3	3426	3691	3750	3750	3750	0	50	126	194	2100
4	3577	3739	3750	3750	3750	0	50	100	150	2100
5	3635	3750	3750	3750	3750	0	0	0	0	2100
6	3501	3693	3744	3750	3750	0	50	100	151	2100
7	2800	3357	3557	3666	3718	0	75	181	354	2100
8	1397	2472	2878	3127	3628	0	217	473	975	2163
8.5	790	1523	2175	2706	3505	0	408	885	1492	2205
9	407	819	1243	1775	3201	0	806	1462	2048	2275
9.5	104	293	539	880	2694	0	1453	2060	0	2373
10	0	0	0	0	1678	1300	2187	0	0	2524

Table 3: Offset Table for Vessel 2 (All dimensions in mm)

ST	Wl-1	Wl-2	Wl-3	Wl-4	Deck at side	Keel	BTK-1	BTK-2	BTK-3	Deck
0	0	0	0	1013	5009	2237	2467	2711	0	3592
0.5	0	0	1829	4964	5600	1513	1776	2020	2447	3487
1	0	2296	5167	5583	5741	822	1092	1355	1711	3388
1.5	2083	5066	5651	5854	5854	263	526	822	1184	3276
2	3490	5516	5882	5921	5921	0	316	559	855	3158
3	4953	5854	6000	6000	6000	0	197	428	592	3000
4	5516	5854	6000	6000	6000	0	197	329	461	3000
5	5516	5854	6000	6000	6000	0	197	329	461	3000
6	5066	5741	5966	6000	6000	0	197	329	592	3000
7	3901	4908	5448	5769	5966	0	197	414	1316	3079
8	2026	3377	4092	4728	5426	0	461	1184	2697	3375
8.5	1295	2319	3152	3827	4953	0	855	1993	0	3553
9	675	1396	2071	2730	4221	0	1447	2908	0	3717
9.5	225	563	991	1463	3107	0	2566	0	0	3862
10	0	0	0	0	1351	2368	0	0	0	4079

Let (x_1, f_1), (x_2, f_2), (x_3, f_3), …………,(x_n, f_n) be given data points. On each interval $[x_i, x_{i+1}]$, a cubic spline S(x) has the following form

$$s_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$$

$(i = 1, 2, \ldots, n-1)$

Where the coefficients a_i, b_i, c_i, d_i are to be determined from the definition of "cubic splines" and the interpolatory requirement.

B-Spline

The B-Spline function has been studied for almost five decades. Schoenberg[10] first proposed the theory of the B-Spline function. Cox[12] and de Boor[5] proposed recursive schemes for computing the coefficients. Riesenfeld[12] utilized blending functions for curves of arbitrary shape. Thereafter, representation of curves using B-Spline functions became popular. Usually, three important

Table3 (cont.): Offset Table for Vessel 3 (All dimensions in mm)

ST	Wl-1	Wl-2	Wl-3	Wl-4	Deck at side	Keel	BTK-1	BTK-2	Deck
0	0	746	1668	2228	2700	898	1260	2001	3036
0.5	0	1081	1878	2335	2742	608	1087	1824	2927
1	596	1505	2109	2463	2786	191	804	1567	2891
1.5	1103	1874	2318	2585	2823	27	508	1262	2860
2	1501	2198	2511	2680	2853	0	331	918	2835
2.5	1853	2440	2668	2764	2877	0	243	655	2815
3	2134	2630	2787	2840	2893	0	180	493	2801
4	2563	2829	2875	2900	2900	0	99	249	2797
5	2788	2892	2900	2900	2900	0	54	125	2797
6	2536	2797	2890	2900	2900	0	98	245	2828
7	1833	2302	2583	2764	2900	0	181	726	2881
7.5	1462	1957	2281	2578	2897	0	274	1198	2915
8	1109	1547	1874	2263	2880	0	473	1896	2955
8.5	714	1097	1402	1843	2803	0	961	2411	3000
9	312	602	899	1313	2602	0	1861	2764	3051
9.5	0	114	325	648	2178	774	2590	3049	3107
10	0	0	0	0	1139	2262	3105	0	3195

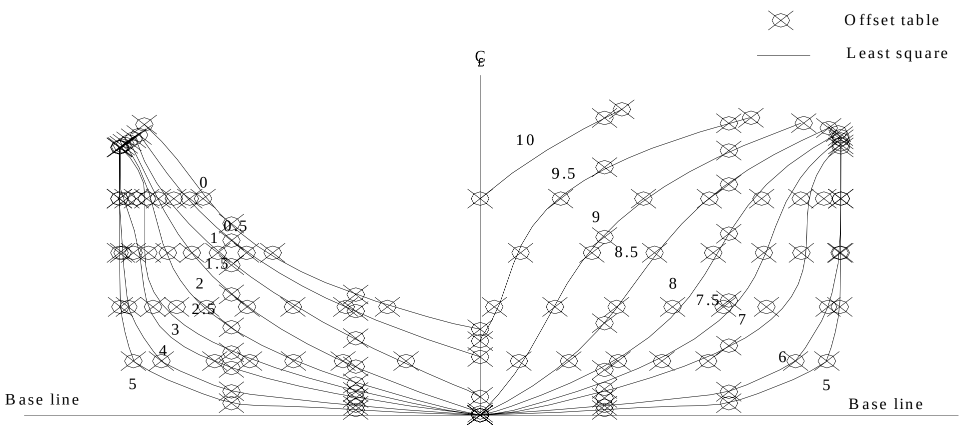

Figure 1: Body plan of Vessel 1 using offset table and least square fitted data

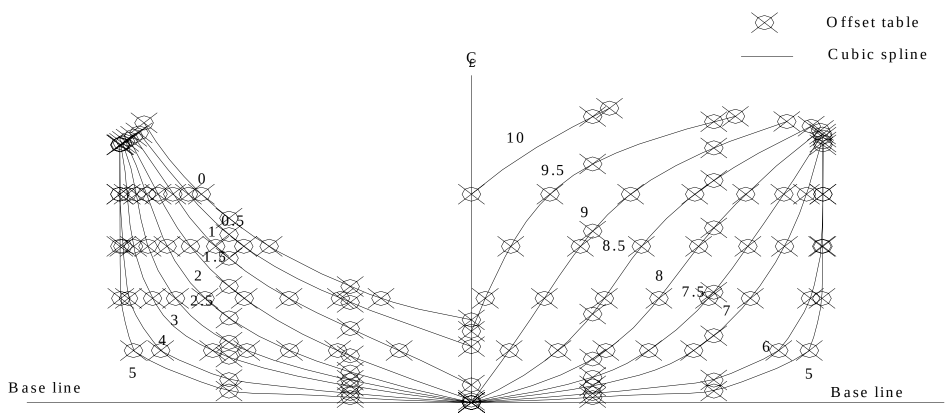

Figure 2: Body plan of Vessel 1 using offset table and cubic-spline fitted data

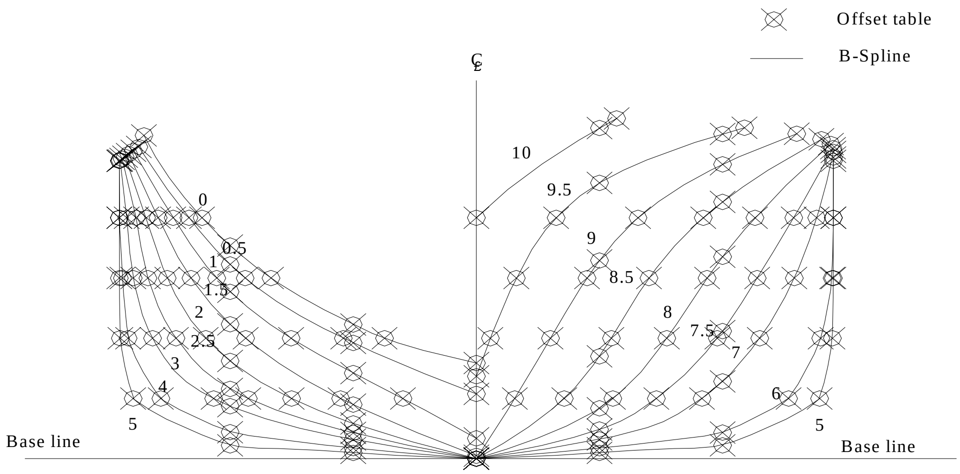

Figure 3: Body plan of Vessel 1 using offset table and B-spline fitted data

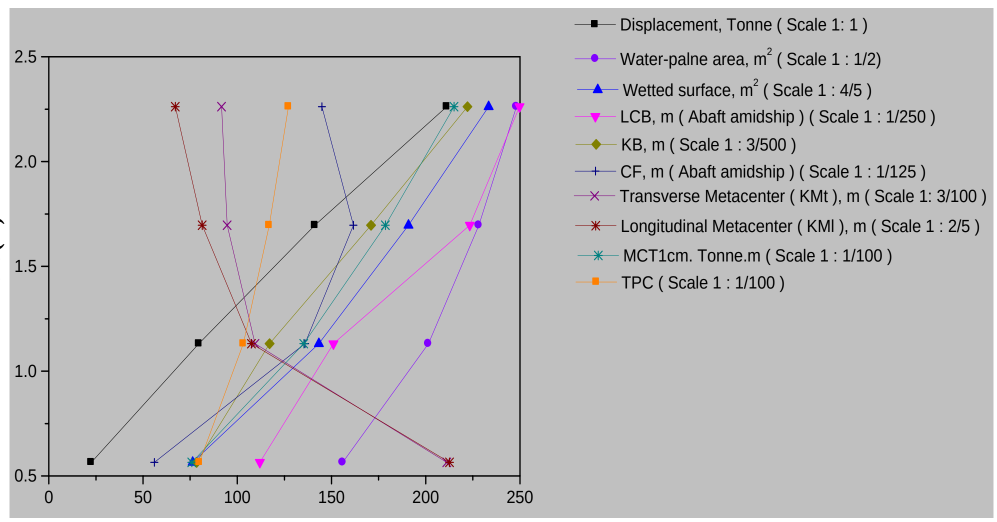

Figure 4: Hydrostatic curves of Vessel 1 using offset table

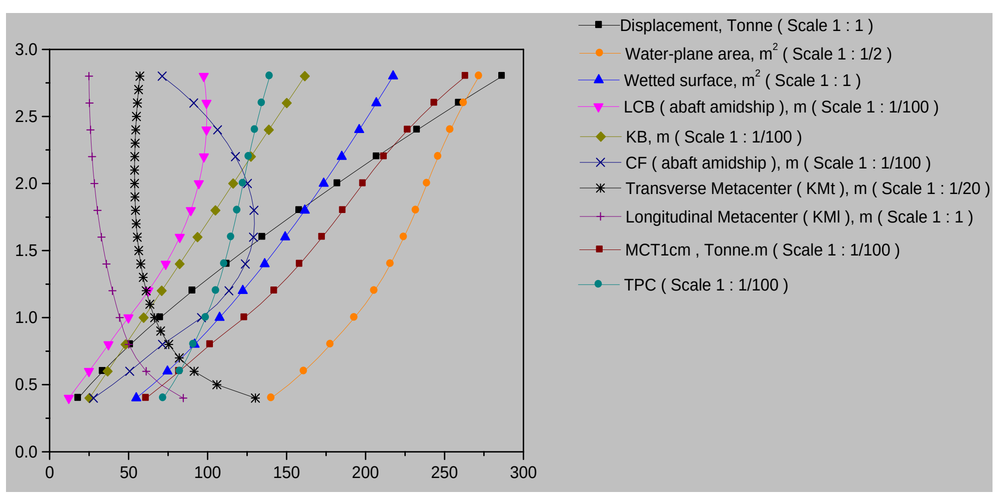

Figure 5: Hydrostatic curves of Vessel 1 using B-Spline fitted data.

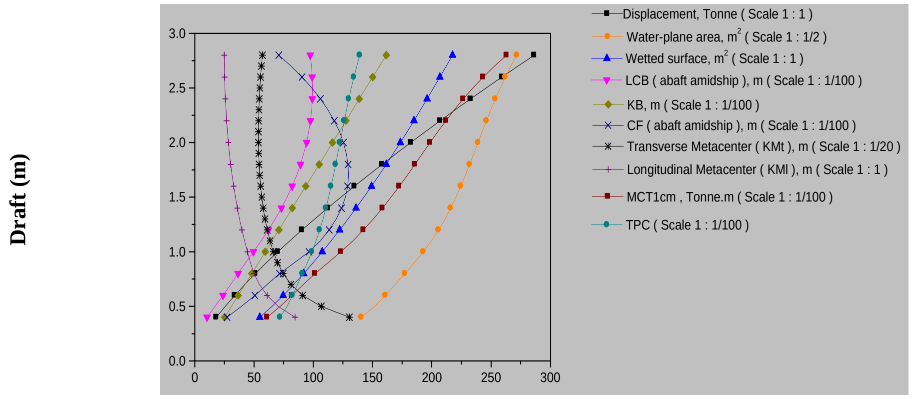

Figure 6: Hydrostatic curves of Vessel 1 using cubic spline fitted data

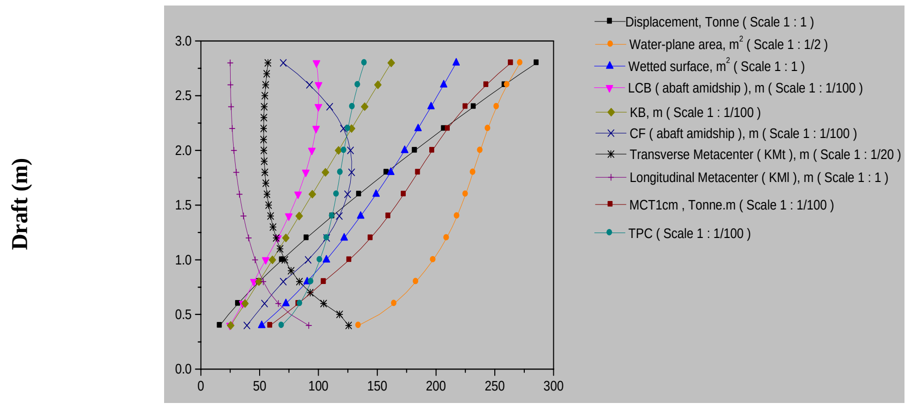

Figure 7: Hydrostatic curves of Vessel 1 using least square fitted data

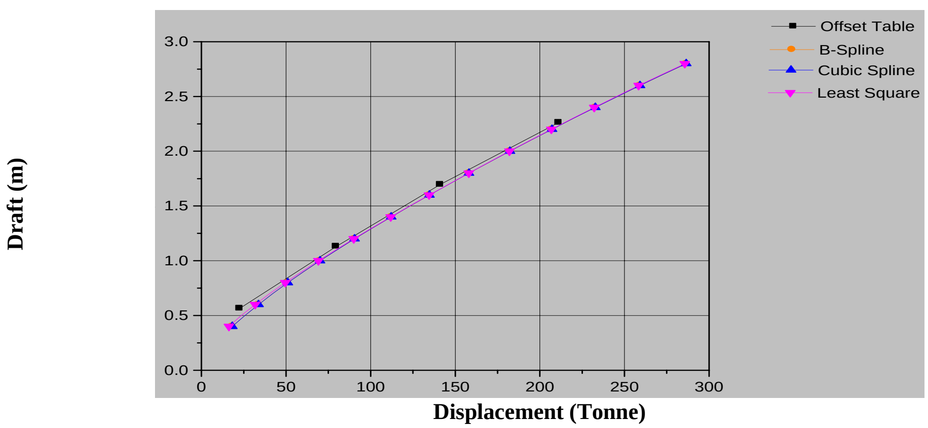

Figure 8: Curve of displacement of Vessel 1 using offset table and fitted data.

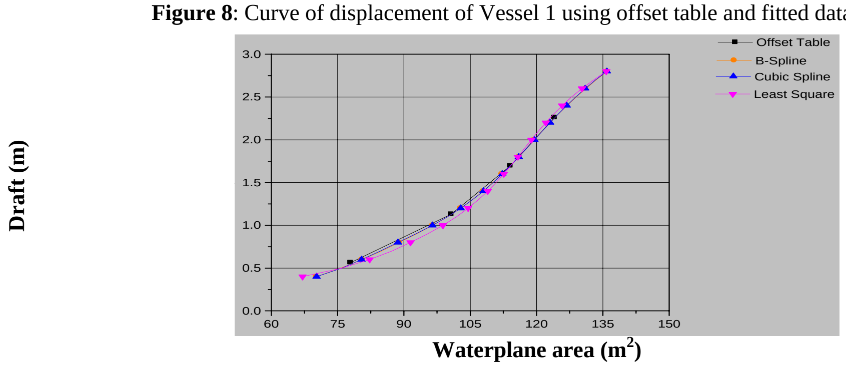

Figure 9: Curve of water-plane area of Vessel 1 using offset table and fitted data.

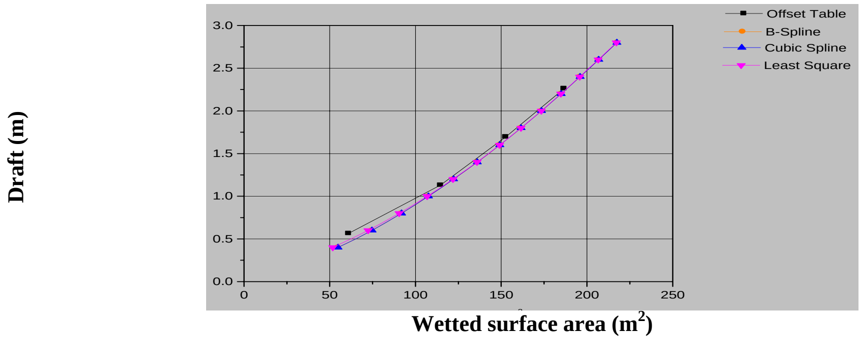

Figure 10: Curve of wetted surface area of Vessel 1 using offset table and fitted data.

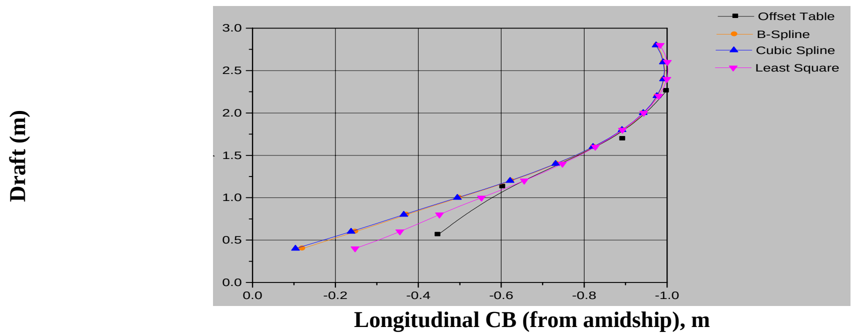

Figure 11: Curve of longitudinal center of buoyancy of Vessel 1 using offset table and fitted data.

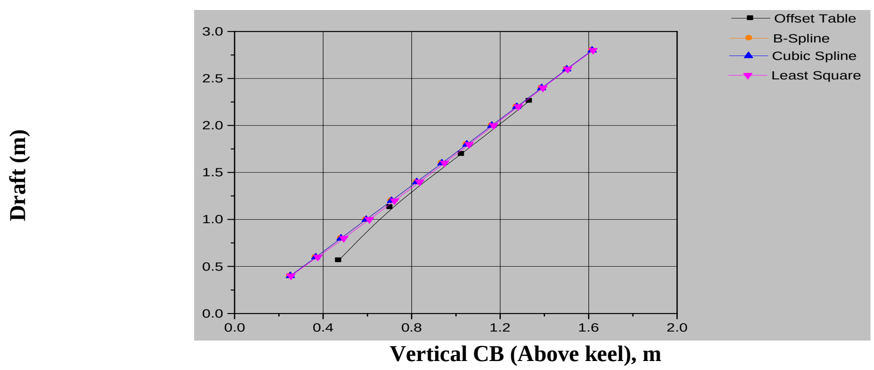

Figure 12: Curve of vertical center of buoyancy of Vessel 1 using offset table and fitted data

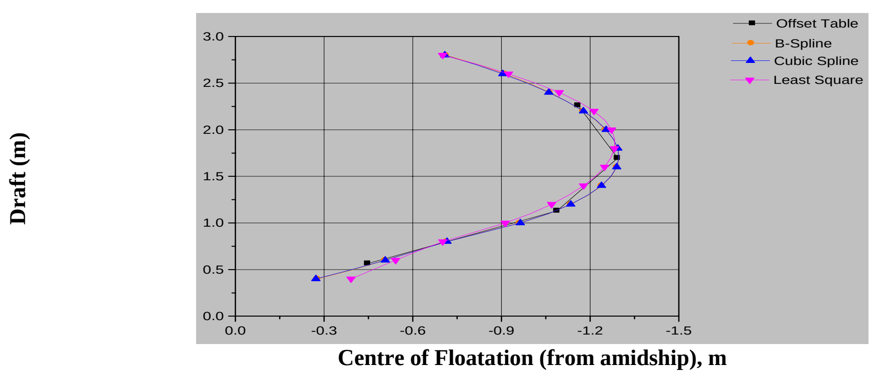

Figure 13: Curve of longitudinal center of flotation of Vessel 1 using offset table and fitted data

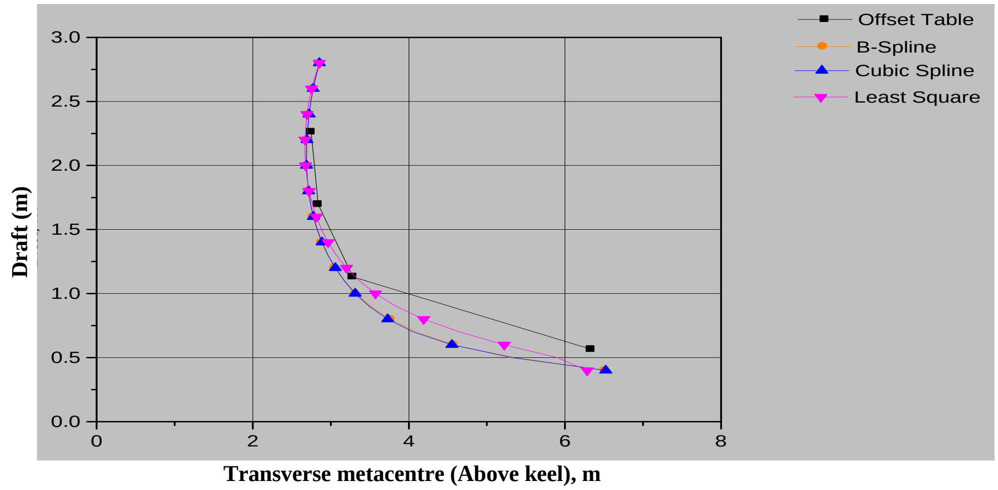

Figure 14: Curve of transverse metacenter of Vessel 1 using offset table and fitted data

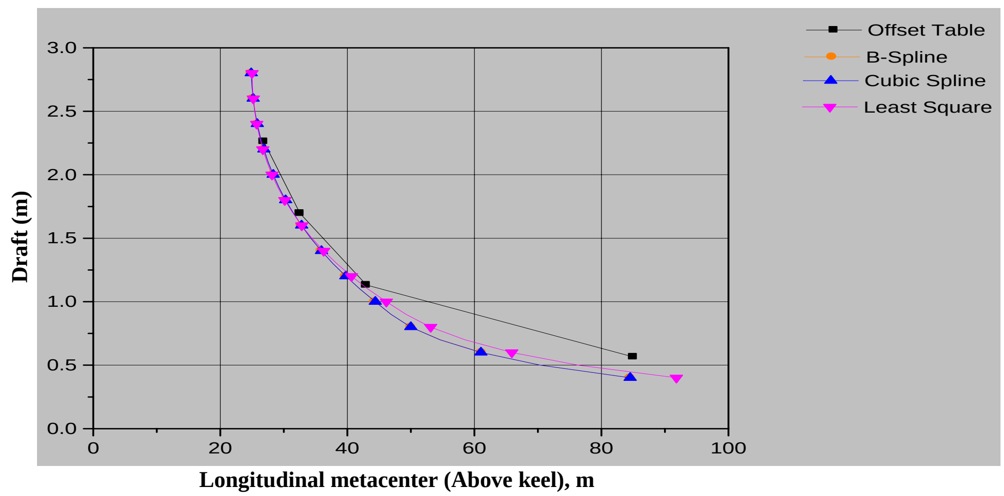

Figure 15: Curve of longitudinal metacenter of Vessel 1 using offset table and fitted data

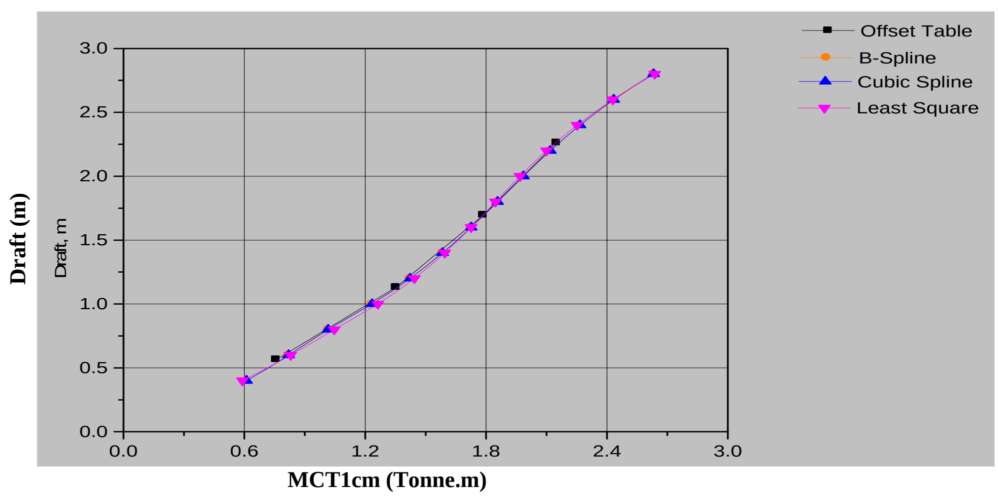

Figure 16: Curve of MCT1 cm of Vessel 1 using offset table and fitted data

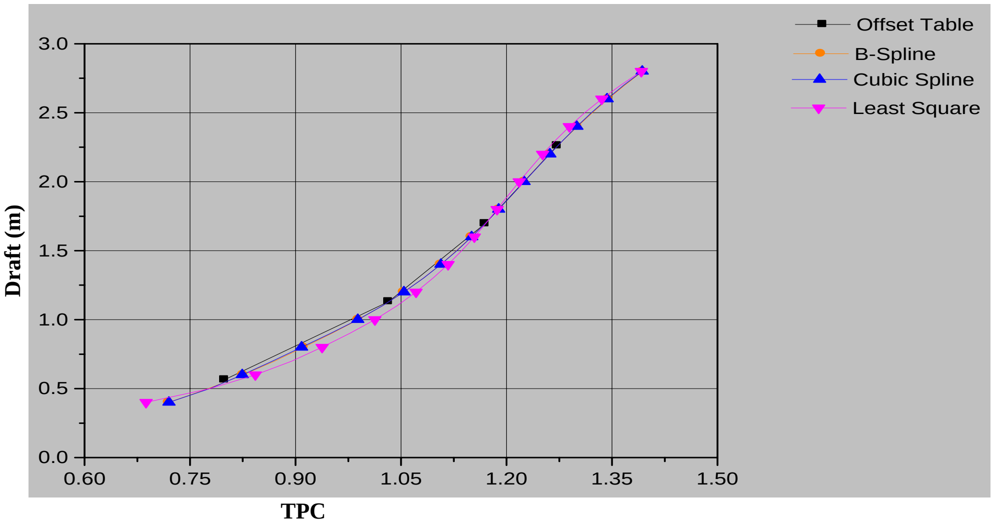

Figure 17: Curve of TPC of Vessel 1 using offset table and fitted data

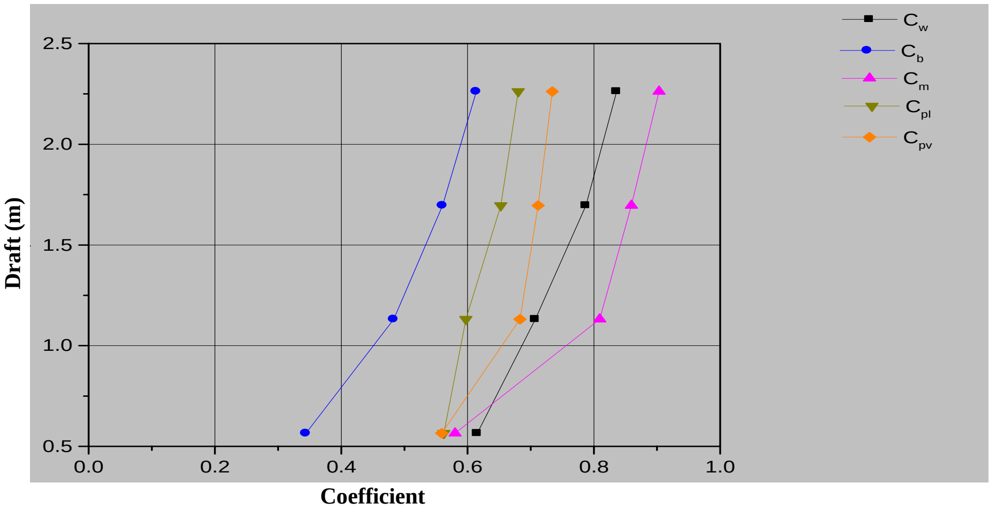

Figure 18: Curves of form coefficients of Vessel 1 using offset table

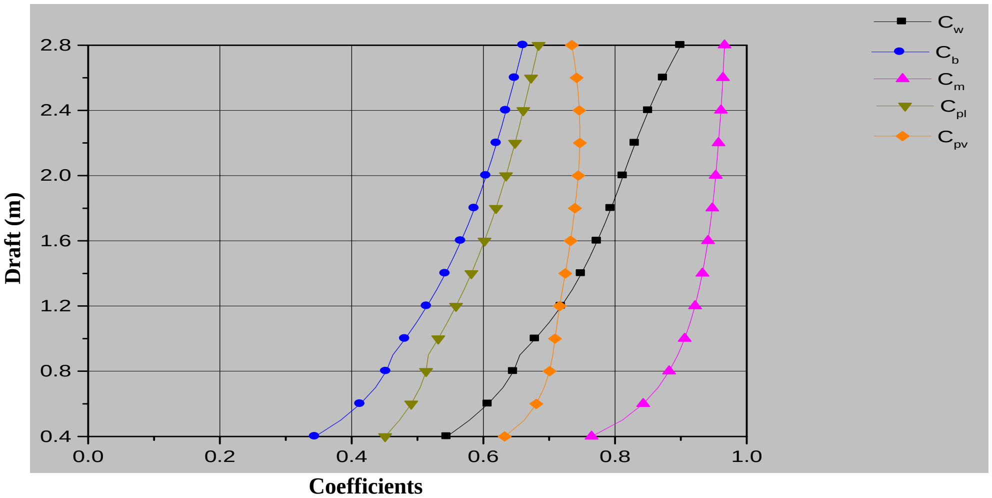

Figure 19: Curves of form coefficients of Vessel 1 using B-Spline fitted data

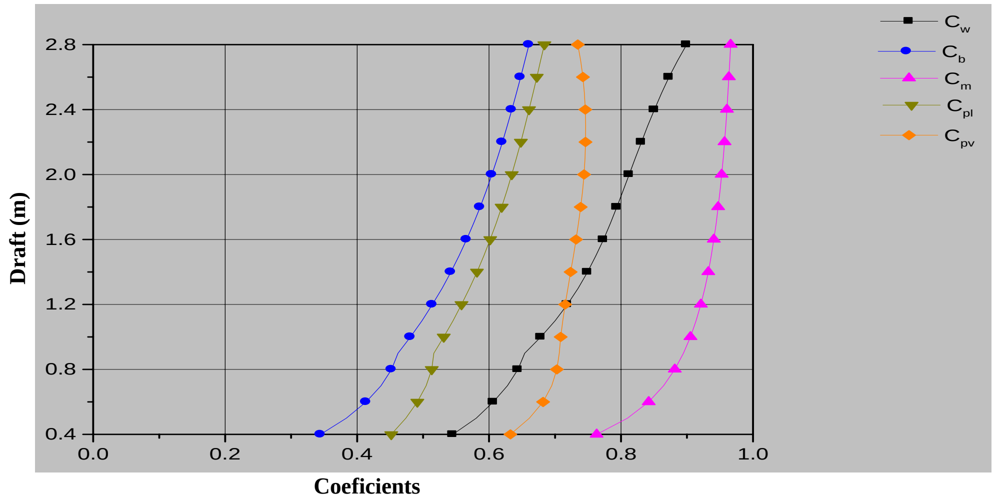

Figure 20: Curves of form coefficients of Vessel 1 using cubic spline fitted data

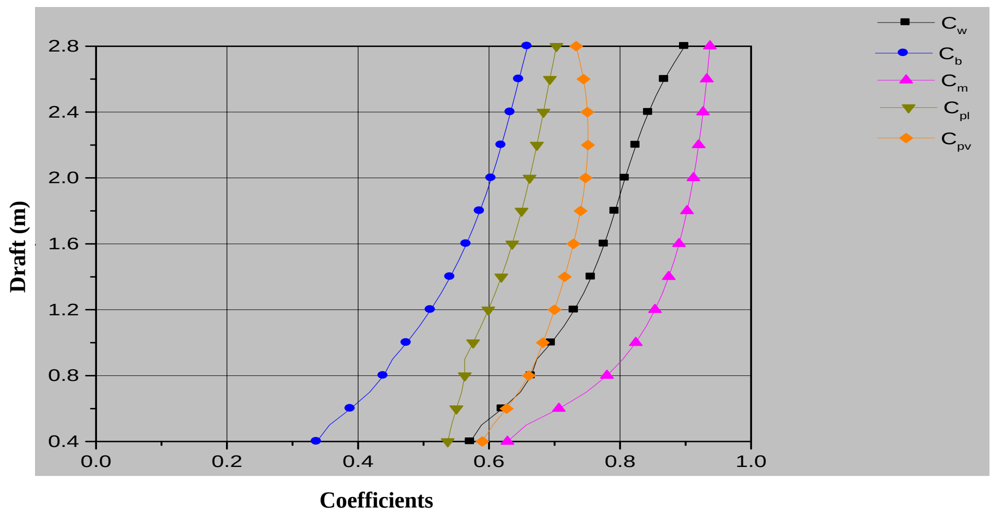

Figure 21: Curves of form coefficients of Vessel 1 using least square fitted data

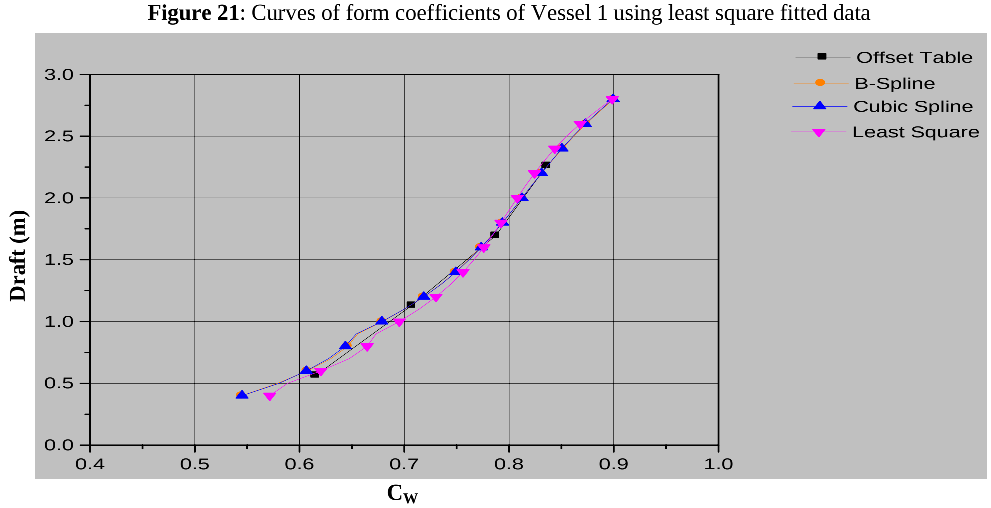

Figure 22: Curves of water-plane area coefficient (C_w) of Vessel 1 using offset table and fitted data

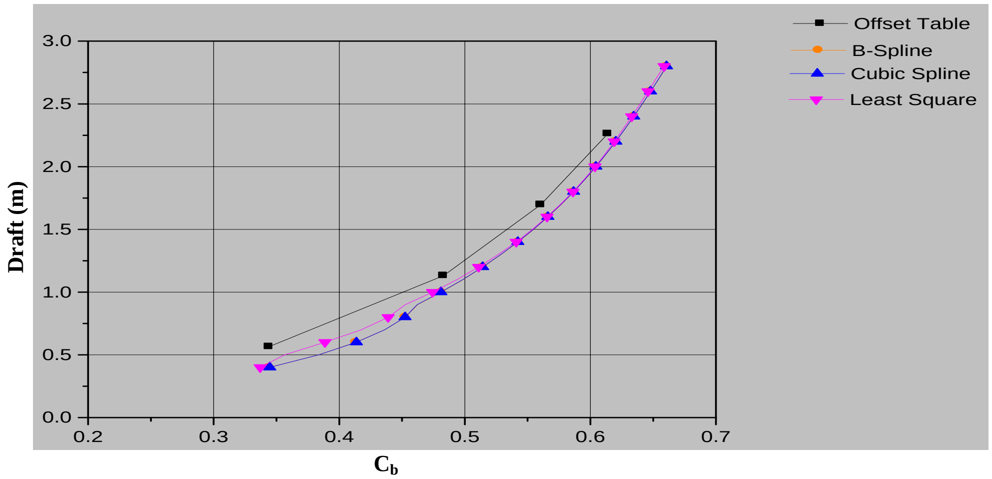

Figure 23: Curves of block coefficient (C_b) of Vessel 1 using offset table and fitted data

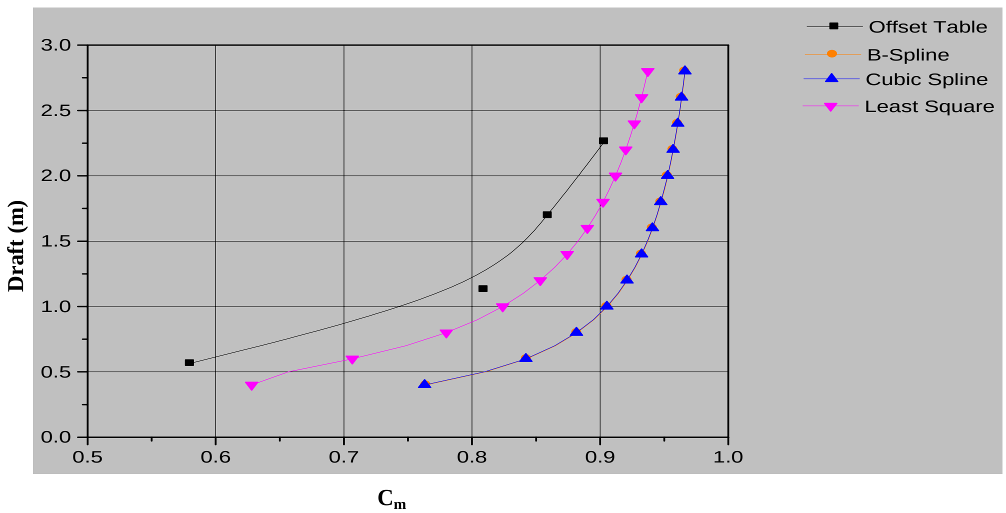

Figure 24: Curves of midship area coefficient (C_m) of Vessel 1 using offset table and fitted data

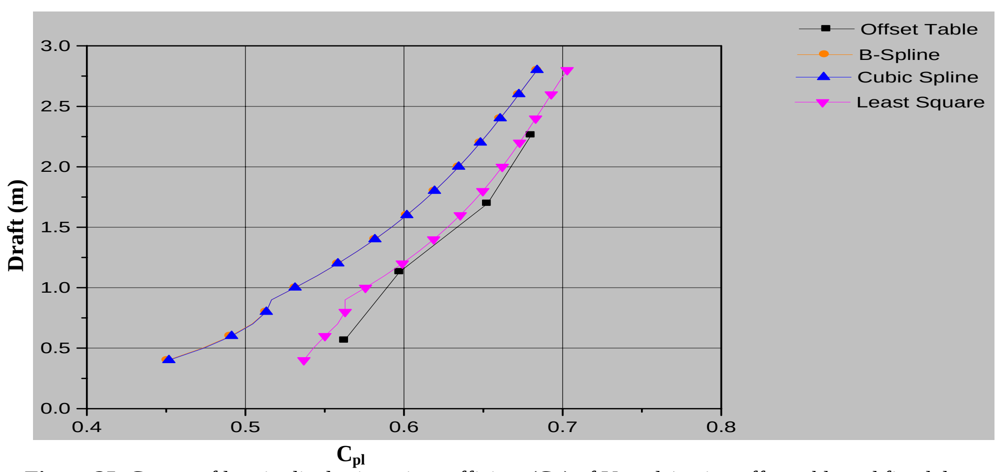

Figure 25: Curves of longitudinal prismatic coefficient (C_{pl}) of Vessel 1 using offset table and fitted data

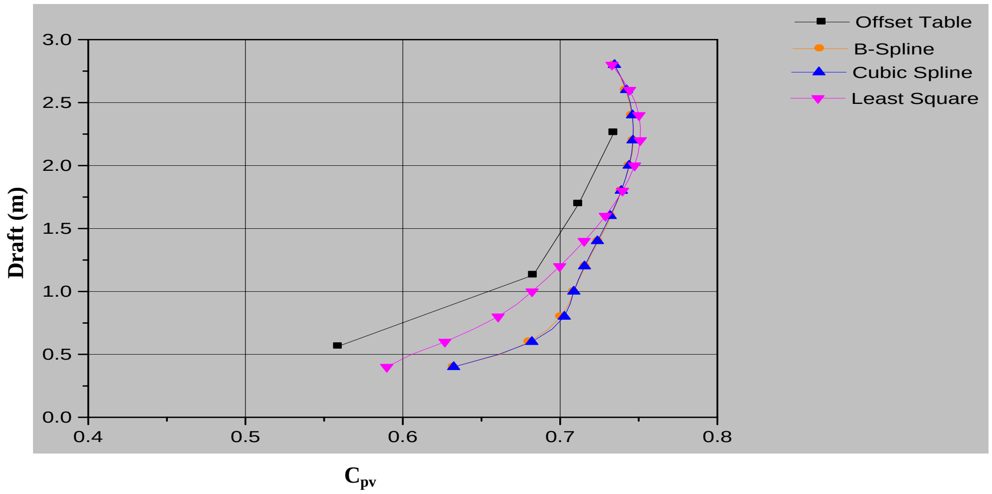

Figure 26: Curves of vertical prismatic coefficient (C_{pv}) of Vessel 1 using offset table and fitted data

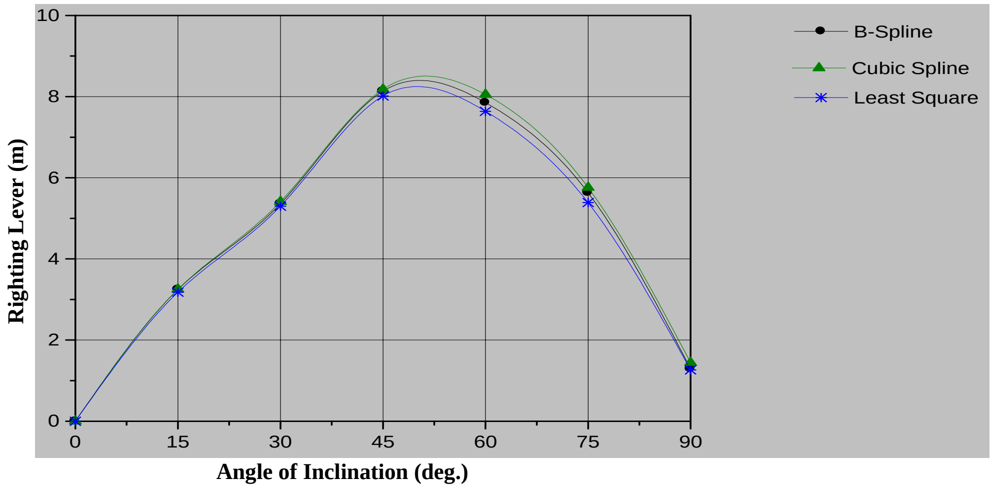

Figure 27: Comparison of curves of stability between curve fitting techniques for Vessel 2 at 0.60 m draft

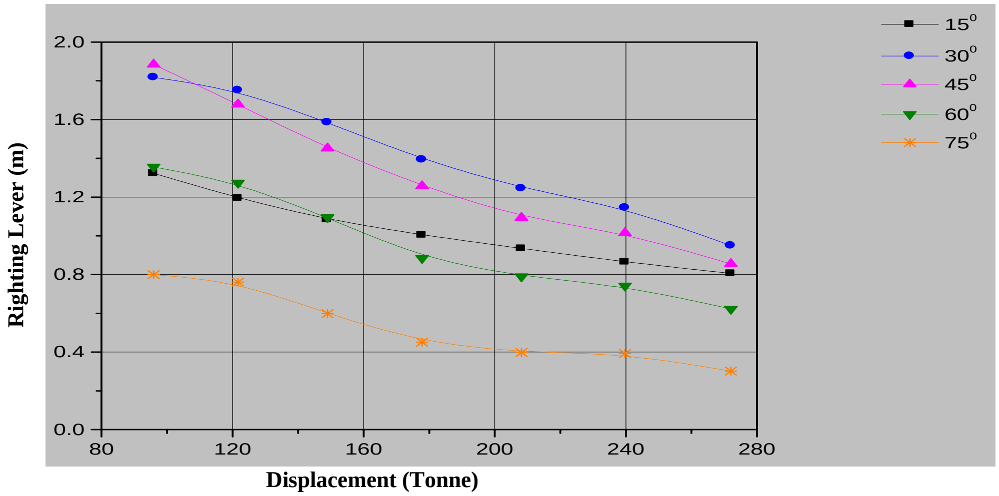

Figure 28: Cross Curves of Stability of passenger Vessel 3 using B-Spline

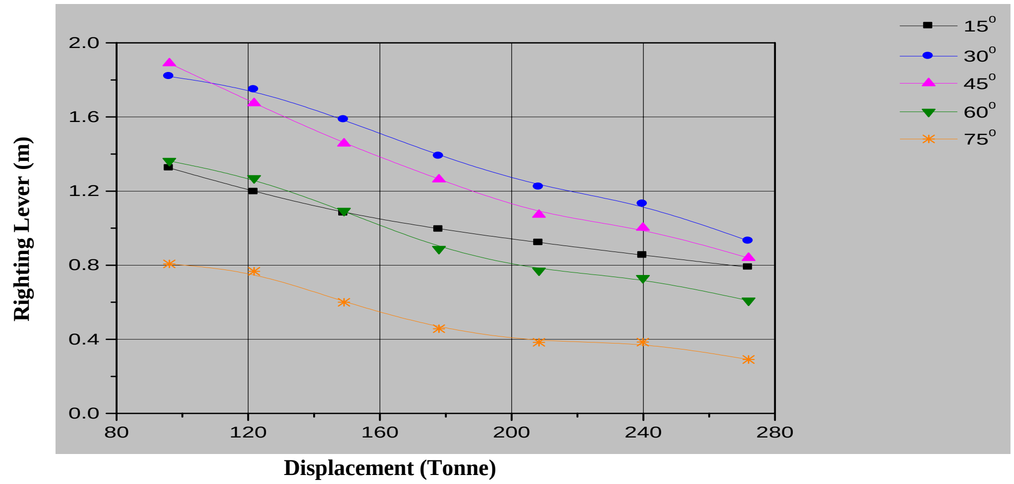

Figure 29: Cross Curves of Stability of passenger Vessel 3 using Cubic Spline

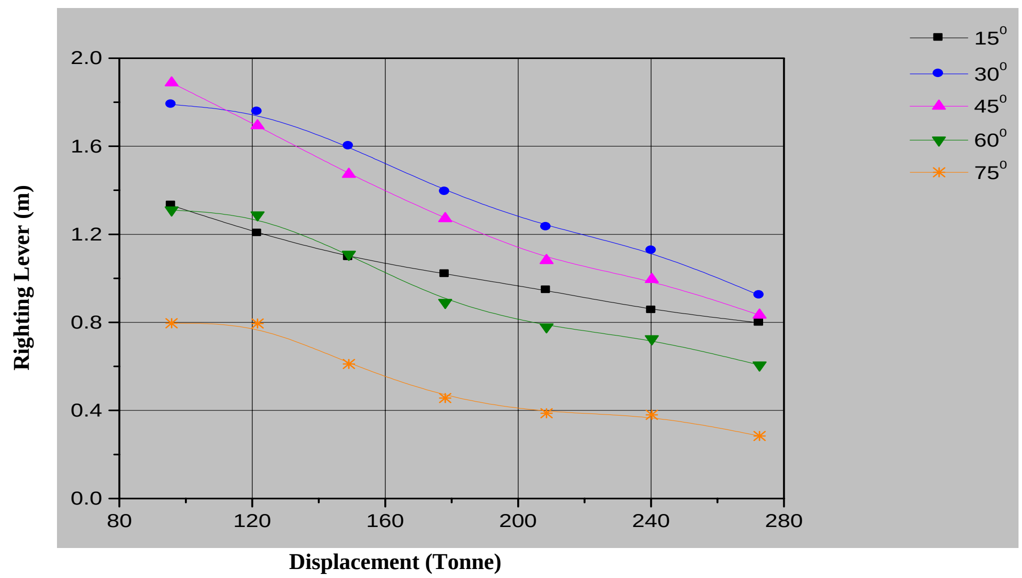

Figure 30: Cross Curves of Stability of passenger Vessel 3 using Least Square

properties are inherent in a B-Spline curve. First, the curve is completely controlled by control points. Second, curve can have different degrees without affecting the number of control points. Thirdly, if a control point is moved, only the segments around this control point are affected. Most applications employ the cubic B-Spline function[13-16] because of the inherent C^2continuity. Wu et al.[17] derived a formula to obtain the control points from the given knot points in order to fit a curve using a cubic B-Spline function.

The equation for k-order B-Spline with $n+1$ control points ($\boldsymbol{P}_0$, $\boldsymbol{P}_1$, ... , $\boldsymbol{P}_n$) is

$$P(t) = \sum_{i=1}^{n+1} B_i N_{i,k}(t) \qquad t_{\min} \le t \le t_{\max}$$

where the B_i are the position vectors of the n+1 defining polygon vertices and the $N_{i,k}$ are the normalized B-Spline basis functions.

In a B-Spline each control point is associated with a basis function $N_{i,k}$ which is given by the recurrence relations.

$N_{i,k}(t) = N_{i,k-1}(t)\ (t-t_i)/(t_{i+k-1}-t_i)+N_{i+1,k-1}(t)\ (t_{i+k}-t)/(t_{i+k} - t_{i+1})$,

$N_{i,1} = \{1 \;\; \text{if} \;\; t_i \le t < t_{i+1} , \quad 0 \;\; \text{otherwise} \}$

$N_{i,k}$ is a polynomial of order k (degree $k-1$) on each interval $t_i < t < t_{i+1}$. k must be at least 2 (linear) and can be not more, than $n+1$ (the number of control points). A knot vector $(t_0 , t_1 , ... , t_{n+k})$ must be specified. Across the knots basis functions are C^{k-2} continuous.

RESULTS AND DISCUSSION

Three vessels are chosen for this study. The principal particulars of the vessels are given in Table1. Table 2-4 shows offset tables of those vessels. Offset table shown in Table 1 consists of ten stations with spacing 3215.8mm, four water lines with spacing 350mm and three buttock lines with spacing 937.5 mm. Offset table shown in Table 2 consists of ten stations with spacing 7000mm, four water lines with spacing 592mm and three buttock lines with spacing 1689mm. Offset table shown in Table 3 consists of ten stations with spacing 2558.8, four water lines with spacing 565mm and two buttock lines with spacing 1000 mm. Figures 1-3 show the body plans of Vessel 1 which are obtained by using both offset table and B-spline, cubic spline and least square curve fitted data respectively. From Figures 1 & 2, it is clear that all interpolated values agree well with the original offset table data. The body plan using least square curve fitted data is compared with that using offset table data in Figure 3. From this figure, it is seen that sections at station 2.5, 3, 7, 7.5 & 8 generated using fitted data show some discrepancies with those with original offset table data.

Hydrostatic curves[1, 18] of Vessel 1 using offset table and fitted data are shown in Figures 4-30. Figures 4, 5, 6 and 7 show hydrostatic curves of Vessel 1 using offset table data, B-spline fitted data, cubic spline fitted data and least square fitted data respectively. In Figures 8, 9, 16 and 17, curve of displacement, curve of water-plane area, curve of MCT1cm and curve of TPC of vessel 1 using offset data coincide with those obtained by using interpolated data. In Figure 10, curve of wetted surface area using offset data coincide with those obtained by using interpolated data except at low draught. Similarly, in Figures 11, 12, 14, 15, curve of longitudinal center of buoyancy, curve of vertical center of buoyancy, curve of transverse metacenter, curve of longitudinal metacenter using offset table data do not match with those obtained by using interpolated data at low draught.

Figure 27 shows the curve of righting lever with respect to angle of inclination for Vessel 2 at draught of 0.6 m using different curve fitting techniques. The righting levers computed using three techniques coincide except slight discrepancies at 60 and 75 degrees of angle of inclination. Figures 28, 29 and 30 show the cross curve of stability using B-spline, cubic spline and least square methods. All curves computed by different techniques are of mostly same nature and same values.

CONCLUSION

The ship lines are faired using B-spline, cubic spline and least square techniques and comparative studies of their hydrostatic characteristics are performed in this research. From above mentioned study, following conclusion can be drawn:

i) B-spline and cubic spline curve fitting techniques can be useful in defining and fairing lines plan of ships.
ii) To compute hydrostatic characteristics of ship, the results obtained by offset table data are more or less accurate at loaded condition but in most of the cases accuracy is lost at lightly loaded condition. So offset table data is not enough. More interpolated values are required.
iii) The interpolation method based on B-spline has been found more effective than those based on cubic spline and least square method for faring of ship lines and computation of hydrostatics and stability of ship.
iv) In fairing ship lines, the least square method is not suitable. It shows many deviations at each section from the original values. However, the computed results using least square fitted data shows better consistence with B-spline and cubic spline.
v) Improvement in curve fitting can be obtained by providing more data in a curve region where the slope of a curve changes sharply.

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