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Minority Carrier Profile and Storage Time of a Schottky Barrier Diode for All levels of injection

M M Shahidul Hassan[1], Orchi Hassan[2], and Md. Azharul Haque[3]

[1]Department of Electrical and Electronic Engineering, BUET, Dhaka 1000, Bangladesh.
[2] Senior Undergraduate student, Department of Electrical and Electronic Engineering, BUET, Bangladesh.
[3]Department of Electrical and Electronic Engineering, PAU, Star Building, Kamal Atatuk Avenue, Banai, Dhaka, Bangladesh.
Email: shassan@eee.buet.ac.bd[1] orchi_hassan@gmail.com[2] azhar_iiuc@hotmail.com[3]

Abstract —**The minority carrier injection and minority carrier stored charge of an n-Si Schottky barrier diode (SBD) considering carrier recombination and blocking properties of the low-high (n^-n^+) are analyzed. Based on the assumption of slow variation of electric field within the quasi-neutral Si, solution of minority carrier profile is obtained. For the first time, a closed form expression for minority carrier profile p(x) for uniformly doped n-Si SBD is obtained, which is applicable for all levels of injection. Present analysis shows that minority carrier current, charge storage time and current injection ratio depend not only on the length of the n region but also on doping density, recombination within n-Si and effective surface recombination velocity at the low-high (n^-n^+) interface. Results obtained from the present model are also compared with experimental data available in the literature and are found to be in good agreement.**

Index Terms— **Schottky barrier diode; recombination, minority carrier profile; minority carrier current; storage time and injection ratio.**

I. INTRODUCTION

Schottky barrier diodes are still a subject of considerable attention [1]-[3] because of their two important properties, (i) fast switching speed and (ii) low forward voltage. Solutions for minority carrier profile and minority carrier current in the silicon region under different levels of injection are two of the most important subjects of interest for SBDs. A Schottky barrier diode with a high barrier injects minority carrier at forward bias [4]. At low level of injection, SDB is considered as a majority carrier device [5]-[6]. At large forward bias, the minority carrier current can not be neglected [7]-[8]. Therefore, minority carrier injection must be considered in studying the characteristics of SDB. The work [9] was done considering both minority and majority carrier currents as constant and neglecting recombination within n-Si. On the other hand, the work [10] considered both minority carrier current and recombination but the analysis was applicable only for high level of injection. No analysis considering recombination was carried out for intermediate level of injection where minority carrier current can not be neglected. Using the equations for high level injection and traditional low level currents, the authors [10] obtained an empirical expression for minority carrier to model the complete range of injection for SBD. In the present work, p(x) and hole current density J_p are obtained considering drift and diffusion currents, recombination and also the finite surface recombination velocity S_{eff} at the low-high (n^- n+) interface. The storage time τ_s and injection ratio γ are obtained from p(x) and J_P. The mathematical expression developed for p(x) is applicable for all levels of injection.

II. DERVATIONS

The structure of a metal - n^-n^+ Schottky barrier diode is shown in Fig. 1. Minority carrier holes will be injected from metal into n-Si when Schottky barrier diode is forward biased. The minority carrier hole profile p(x) within the drift region of length l_d can be obtained from drift and diffusion current equations. The electron and hole current densities in the quasi-neutral drift region are given by

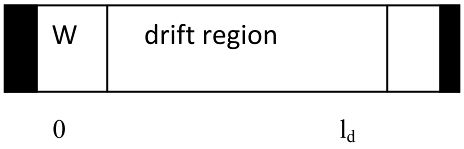


Fig.1. A planer n-Si Schottky barrier diode with drift region of length l_d and depletion region of width W.

$$J_n = qn(x)\mu_n(x)E(x) + qD_n(x)\frac{dn(x)}{dx} \tag{1}$$

$$J_p = qp(x)\mu_p(x)E(x) - qD_p(x)\frac{dp(x)}{dx} \tag{2}$$

where, $D_n(x)(D_p(x))$ is the electron(hole) diffusivity and $\mu_n(x)(\mu_p(x))$ is the electron (hole) mobility [11]. For uniformly doped silicon, μ_p *and* μ_n are constants.

The current continuity equations are

$$\frac{dJ_n(x)}{dx} = qrn(x) \tag{3}$$

$$\frac{dJ_p(x)}{dx} = -qrp(x) \tag{4}$$

where r is the net recombination rate [12].

The quasi-neutral condition is

$$n(x) = p(x) + N_d \tag{5}$$

where N_d is the doping density of n-Si region.

Using eq.(1)- (5), a second order differential equation for p(x) can be obtained which is not analytically tractable. On the other hand, for low level of injection $p(x) << N_d$ and eq.(5) will be reduced to $n(x)=N_d$, which leads to a analytically tractable differential quation for p(x). Incase of high level of injection $p(x)>>N_d$ and eq.(5) will be reduced to $n(x)=p(x)$ which also leads to an analytically tractable differential equation for p(x). But for intermediate level of injection, p(x) is comparable to N_d and no simplifications to eq.(5) cannot be done. Therefore, in order to obtain an expression for p(x) applicable for all levels of injection, an appropriate assumption needs to be made.

It is known that the electric field in the quasi-neutral n-Si varies very slowly. So, in order to obtain an analytically tractable differential equation applicable for all levels of injection, derivative of the slow varying electric field is neglected. The differential equation then reduces to the form given below

$$\frac{dp^2(x)}{dx^2} - \frac{r(1+m)}{2D_p} p(x) = \frac{r}{2D_n} N_d \tag{6}$$

where, $m = \frac{D_p}{D_n}$.

The solution of (6) can be written as

$$p(x) = A\exp\left(\sqrt{a}x\right) + B\exp\left(-\sqrt{a}x\right) - \frac{m}{1+m} N_d \tag{7}$$

where A and B are arbitray constants and

$$a = \frac{r(1+m)}{2D_p}.$$

At x = 0, p(0) = p_o, and from (7)

$$p_o + \frac{m}{2(1+m)} N_d = A + B \tag{8}$$

where p_o is given by [9]

$$p_o = \frac{N_d}{2}\left(\sqrt{1 + \frac{4n_{ie}^2\left(J_{no}/J_{ns}+1\right)}{N_d}} - 1\right). \tag{9}$$

The effective intrinsic carrier concentration, n_{ie} depends upon N_d as [13]

$$n_{ie}^2(x) = n^2_{io}\left(\frac{N_d(x)}{N_{ref}}\right)^{\alpha} \tag{10}$$

Where n_{io} is the intrinsic carrier concentration for uniformly doped Silicon and N_{ref} is the reference doping density.

For $N_d < N_{ref}$, $n_{ie} = n_{io}$.

The thermionic emission diode current at x = 0 is given by [14]

$$J_{no} = J_{ns}\left(\exp\left(qV_f/kT\right) - 1\right) \tag{11}$$

and

$$J_{ns} = A^{**}T^2 \exp\left(-\frac{q\phi_B}{kT}\right)$$

where, V_f is the forward biased voltage across the Schottky contact, A** is the effective Richardson constant, k is the Boltzmann constant and T is the diode temperature in Kelvin. The width of the space charge region W depends on applied voltage and is given by [14]

$$W = \frac{2\varepsilon_s\left[\phi_B - \frac{kT}{q}\ell n\frac{N_C}{N_d} - \frac{kT}{q}\ell n\left(1+\frac{J_{no}}{J_{ns}}\right) - \frac{kT}{q}\right]}{qN_d} \tag{12}$$

where N_C is the effective density of states in the conduction band.

From eq. (1), (2) and (7), $J_n(x)$ can be obtained

$$J_n(x) = \frac{p(x)+N_d}{mp(x)} J_p(x) + qD_n\left(\frac{2p(x)+N_d}{p(x)}\right) \times\left[\sqrt{a}\left(A\exp\left(\sqrt{a}x\right) - B\exp\left(-\sqrt{a}x\right)\right)\right] \tag{13}$$

Substituting x = 0, and $J_n(0) = J_{no}$ in (13), it can be shown

$$\frac{J_{no}p_o}{q\sqrt{a}D_n\left(2p_o+N_d\right)} - \frac{\left(p_o+N_d\right)}{q\sqrt{a}mD_n\left(2p_o+N_d\right)} J_{po} = A + B \tag{14}$$

The constant A and B can be obtained from (8) and (14) . Finally, p(x) can be written as

$$p(x)=\frac{1}{2}\left(p_o+\frac{m}{(1+m)}N_d\right)\left(\exp\left(\sqrt{a}x\right)+\exp-\left(\sqrt{a}x\right)\right)+\frac{1}{2q\sqrt{a}D_n(2p_o+N_d)}\times$$
$$\left(J_{no}p_o-\frac{p_o+N_d}{m}J_{po}\right)\left(\exp\left(\sqrt{a}x\right)-\exp-\left(\sqrt{a}x\right)\right)-\frac{m}{1+m}N_d \qquad (15)$$

The hole current density J_{po} can be obtained by integrating (4) from x = 0 to x =l_d. Integration gives

$$J_{po}=\frac{1}{2}\left(p_o+\frac{m}{(1+m)}N_d\right)\left(\frac{C}{D}\right)+\frac{J_{no}p_o}{2\sqrt{a}D_n(2p_o+N_d)}\times \qquad (16)$$
$$\left(\frac{F}{D}\right)+\frac{F}{D}$$

where,

$$C=\frac{qS_{eff}}{2}\left(\exp\left(\sqrt{a}x\right)+\exp\left(-\sqrt{a}x\right)\right)+$$
$$\frac{qr}{2\sqrt{a}}\left(\exp\left(\sqrt{a}x\right)-\exp\left(-\sqrt{a}x\right)\right)$$

$$D=S_{eff}\left(\exp\left(\sqrt{a}x\right)-\exp\left(-\sqrt{a}x\right)\right)+\frac{r}{\sqrt{a}}\left(\exp\left(\sqrt{a}x\right)+\exp\left(-\sqrt{a}x\right)-2\right)$$

and

$$F=-q\frac{mN_d}{1+m}\left(S_{eff}+rl_d\right)$$

In obtaining (16), the hole current density at n^-n^+ interface, $J_{pl} = qS_{eff}p_l$ is used. Hole density p_l is obtained by putting x=l_d in (15). In this work, the effective surface recombination velocity S_{eff} of the low-high (n^-n^+) junction given in [15] is used.

There are two important parameters that characterize SBD. One is the storage time τ_s and the other is the injection ratio γ. The storage time can be derived by dividing the stored charge Q_s by the reverse sweeping current density J_r [7].

$$\tau_s=\frac{Q_s}{J_r} \qquad (17)$$

where Q_s is the total excess minority charges stored in the quasi-neutral drift region. Q_s can be found by integrating (15)

$$Q_s=q\int_0^{l_d}\left(p(x)-\frac{n_{ie}{}^2}{N_d}\right)dx=\left(\exp\left(\sqrt{a}l_d\right)-\exp\left(-\sqrt{a}l_d\right)\right)G$$
$$+\left(\exp\left(\sqrt{a}l_d\right)+\exp\left(-\sqrt{a}l_d\right)-2\right)H-ql_d\left(\frac{mN_d}{1+m}+\frac{n^2{}_{ie}}{N_d}\right) \qquad (18)$$

where

$$G=\frac{q\left(p_o+\dfrac{mN_d}{1+m}\right)}{2\sqrt{a}}$$ and

$$H=\frac{1}{2aD_n}\left(\frac{J_{no}p_o}{2p_o+N_d}-\frac{J_{po}}{m}\left(\frac{p_o+N_d}{2p_o+N_d}\right)\right)$$

The minority current injection ratio γ is obtained through

$$\gamma=\frac{J_{po}}{J_{no}+J_{po}}=\frac{J_{po}}{J} \qquad (19)$$

where J is the total current density.

The electric field E(x) can be obtained from (1), (2) and (15). Integration of electric field from x = 0 to x = l_d gives

For K^2 -4AB > 0

$$V_{inj}=\frac{J}{q\sqrt{a}\mu_n(1+m)\sqrt{K^2-4AB}}\times$$
$$\left[\begin{array}{l}\ell n\left(\dfrac{2A\exp\left(\sqrt{a}l_d\right)+K-\sqrt{K^2-4AB}}{2A\exp\left(\sqrt{a}l_d\right)+K+\sqrt{K^2-4AB}}\right)-\\ \ell n\left(\dfrac{2A+K-\sqrt{K^2-4AB}}{2A+K+\sqrt{K^2-4AB}}\right)\end{array}\right] \qquad (20)$$
$$+\frac{(1-m)V_t}{1+m}\ell n\frac{(1+m)p_o+N_d}{(1+m)p_l+N_d}$$

For K^2 -4AB = 0

$$V_{inj}=2\left(\frac{1}{K+2A}-\frac{1}{L+2A\exp\left(\sqrt{a}l_d\right)}\right)$$ (20a) For K^2 - 4AB < 0

$$V_{inj}=\frac{2J}{q\sqrt{a}\mu_n(1+m)}\left[\begin{array}{l}\tan^{-1}\left(\dfrac{2A\exp\left(\sqrt{a}l_d\right)+K}{\sqrt{K^2-4AB}}\right)-\\ \tan^{-1}\left(\dfrac{2A+K}{\sqrt{K^2-4AB}}\right)\end{array}\right]+$$
$$\frac{(1-m)V_t}{1+m}\ell n\frac{(1+m)p_o+N_d}{(1+m)p_l+N_d} \qquad (20b)$$

where,

$$K = \frac{1-m}{1+m} N_d$$

The applied voltage V_a is given by

$$V_a = V_f + V_{inj} + JR_C \qquad (21)$$

where R_C is the specific series contact resistance [10] expressed in Ω/cm^2.

III. RESULTS AND DISCUSSIONS

The equations derived in section II are used to study the characteristics of a Schottky barrier diode. Fig. 2 shows hole density p(x) within the drift region for three different values of N_d. Hole density p(x) increases with x. But for a given x, p(x) is higher for smaller N_d. The minority carrier hole density (n_{ie}^2/N_d) in n-Si at thermal equilibrium decreases with increase of N_d.

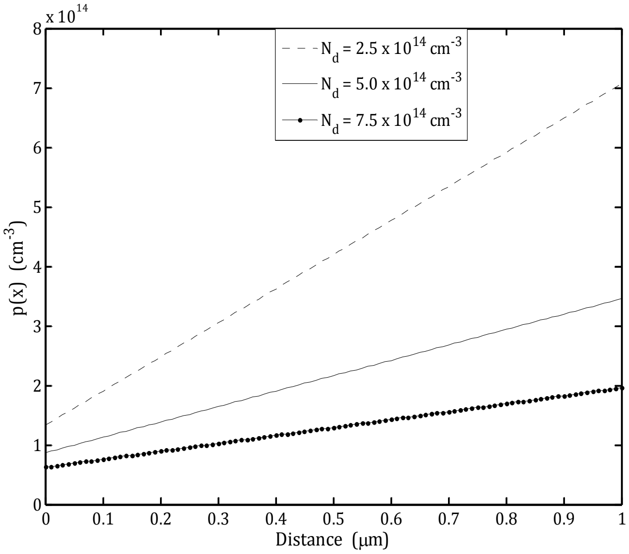


Fig. 2. Hole density p(x) within n-Si region of a Schottky barrier diode for three different values of N_d. Device parameters: φ_B = 0.85V, l_d = 1 μm and $S_{eff} = 10^4$ cm.s^{-1} and V_a = 0.5V.

Therefore, p(x) decreases with increase of N_d at a given voltage across junction. Fig. 3 shows the minority carrier current density J_{po} as a function of applied voltage V_a for three different doping densities.

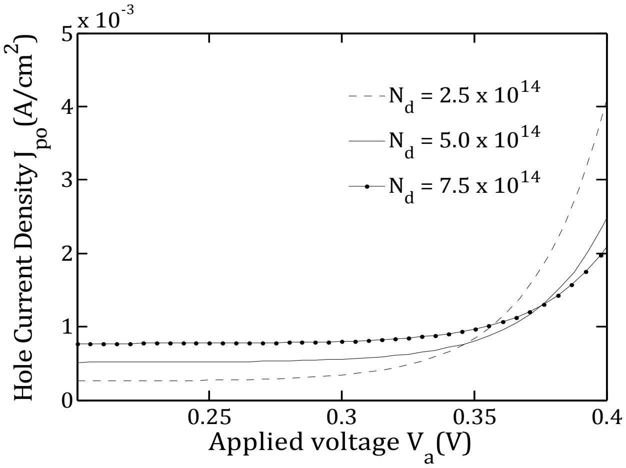


Fig.3. Minority carrier current density J_{po} as a function of applied voltage V_a for three different doping densities. Device parameters: φ_B =0.85V, l_d = 1 μm and $S_{eff} = 10^4$ cm.s^{-1}.

For a given N_d, J_{po} increases with V_a. But at high voltages, J_{po} increases faster with small N_d. At low applied voltage, the voltage across drift region decreases with increase of N_d and voltage across the junction increases. Therefore, J_{po} increases with N_d. At high applied voltage V_a, voltage across drift region increases with N_d and consequently the voltage across junction will decrease resulting in decrease of J_{po} with increase of N_d. The results for J_{po} as a function of V_a for three different barrier heights are plotted in Fig. 4. J_{po} increases with φ_B. For low φ_B, contribution of J_{po} to τ_s and γ may be neglected.

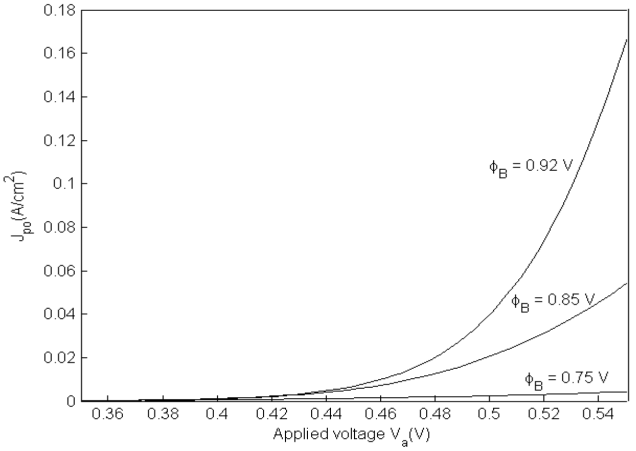


Fig. 4. Forward biased Schottky barrier diode currents as a function of applied voltage V_a for three different values of φ_B. Device parameters: $N_d = 1x10^{15}$ cm^{-3}, l_d = 1μm and $S_{eff} = 10^4$ cm.s^{-1}.

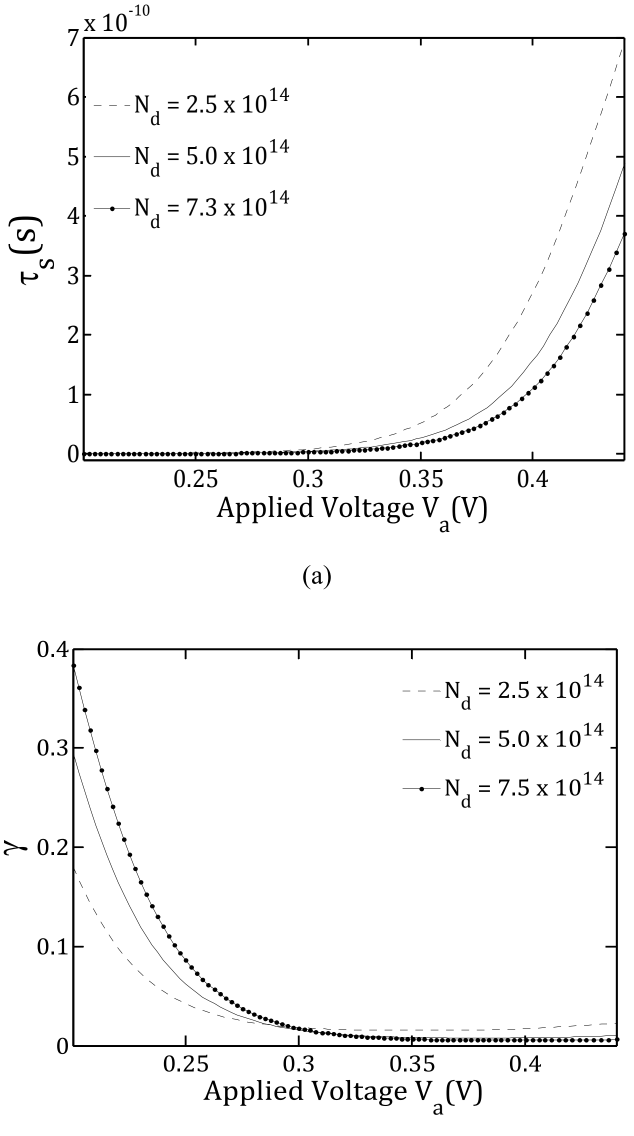


Fig. 5. (a) Storage time τ_s as a function of V_a for three different doping densities. (b) Injection ratio γ as a function of V_a for three different doping densities. Device parameters: $\varphi_B = 0.85V$, $l_d = 1\mu m$ and $S_{eff} = 10^4\,cm.s^{-1}$.

Fig. 5(a) shows τ_s while Fig. 5(b) shows variation of γ as a function of V_a for three different values of N_d. For a given V_a, the storage charge per unit area Q_s decreases with increase of N_d. The profile p(x) for higher N_d falls below that for lower N_d (Fig. 2) and consequently Q_s, the total charge under the profile p(x), decreases with increase of N_d.

Dependence of τ_s and γ upon drift length l_d is shown in Fig. 6(a) and 6(b) respectively. Both τ_s and γ depend on l_d and increase with l_d.

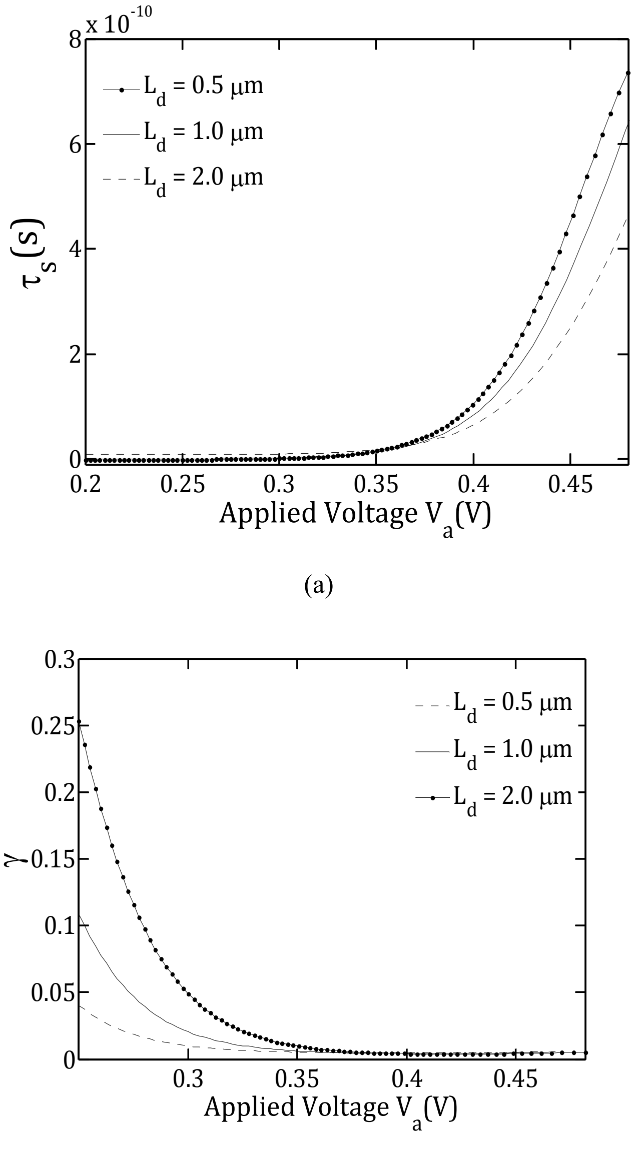
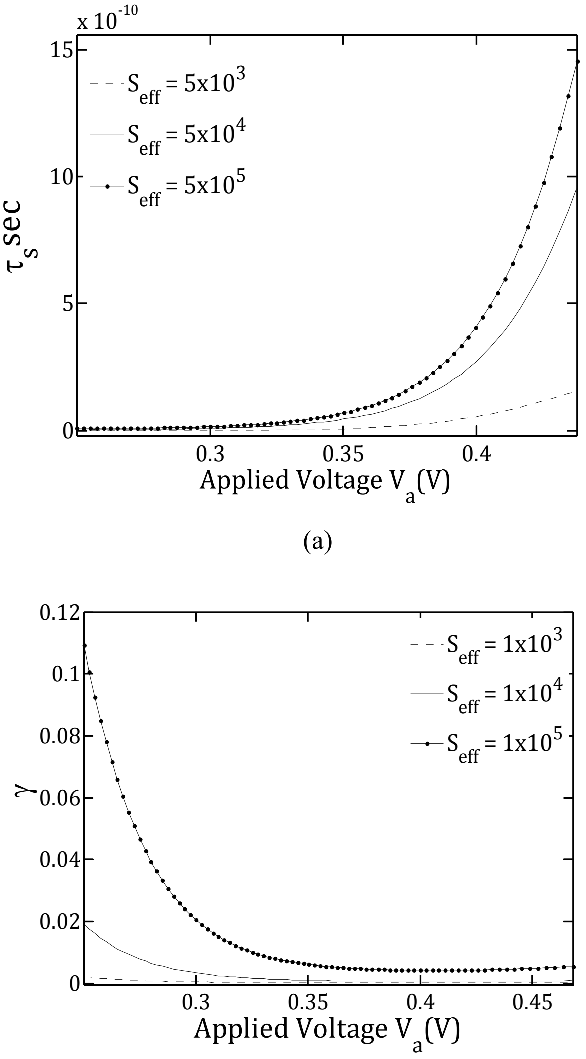


Fig. 6. Variation of (a) τ_s and (b) γ as a function of V_a for three different values of drift length l_d. Device parameters: $\varphi_B = 0.85V$, $N_d = 1x10^{15}\,cm^{-3}$ and $S_{eff} = 10^4\,cm.s^{-1}$.

Fig. 7(a) and 7(b) show dependence of τ_s and γ on S_{eff} respectively. The storage time τ_s increases with S_{eff}. As the recombination increases with S_{eff}, more holes will be required to sustain that increased current within n-Si and Q_s will increase.



(a)

(b)

Fig. 7. (a) Storage time τ_s as a function of V_a for three different values of S_{eff}. (b) Injection ratio γ as a function of V_a for three different values of S_{eff}. Device parameters: $N_d = 1x10^{15}$ $cm^{-3,}$ $l_d = 1\mu m$ and $\varphi_B = 0.65$ V.

The total forward biased current density can be obtained from equations (11) and (16). The variation of current-voltage characteristics are compared with the experimental data [16]. Fig.8 shows I-V characteristics obtained from the present model and also from experimental data for two different barrier heights. For higher barrier height, the current is smaller than that for lower barrier height at a given applied voltage V_a. The two results are found to be in good agreement.

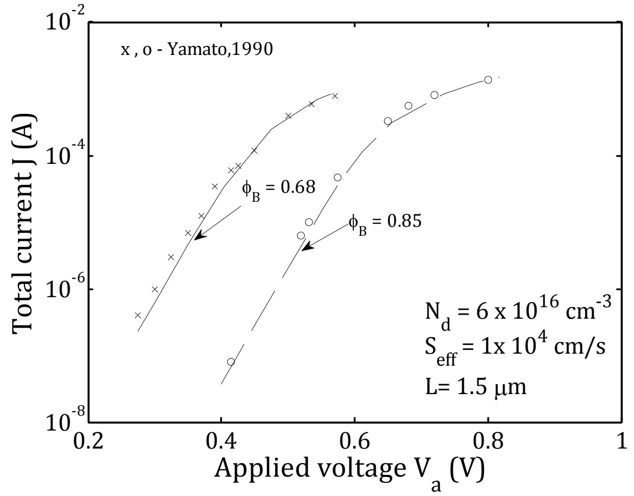


Fig.8. Comparison of calculated current-voltage characteristics with experimental results.

IV. CONCLUSION

Mathematical expression for p(x) is obtained considering drift, diffusion and recombination components of both minority and majority carrier currents. In this analysis the blocking property of low-high (n-n+) interface is also considered. The equation for p(x) is applicable for all levels of injection. Study shows that the storage charge Q_s and storage time depend upon minority carrier current and effective surface recombination velocity. It is concluded that minority current must not be neglected in obtaining J and Q_s for higher barrier height φ_B. The study also shows that SDBs with thin drift region, higher doping density and small effective recombination velocity give smaller τ_s andγ.

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