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Research Article

The Sandor-Smarandache function with a prime factor

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ABSTRACT

The Sandor-Smarandache function, denoted by SS(n), is a newly-introduced Smarandache-type arithmetic function. This paper focuses on the functions SS(30p), SS(60p), SS(210p), SS(420p) and SS(840p), where p (≥ 2) is a prime. At the end of the paper, four tables, giving the values of SS(30p), SS(60p), SS(210p) and SS(420p) for the first 200 primes, calculated on a computer, are given.

Introduction

The Sandor-Smarandache function, proposed by Sandor (2001), is denoted by $SS(n)$ and is defined as follows: For $n \geq 7$,

$$SS(n) = max\left\{k : 1 \leq k \leq n-2, n \text{ divides } \binom{n}{k}\right\}, \quad (1)$$

where by convention,

$$SS(1) = 1, SS(2) = 1, SS(6) = 1. \quad (2)$$

Let, for $0 \leq k \leq n$,

$$C(n, k) \equiv \binom{n}{k} = \frac{n(n-1)(n-2) \ldots (n-k+1)}{k!}. \quad (3)$$

Then, the problem of finding $SS(n)$ may be stated as follows: Given an integer n (≥ 7), find the minimum integer k such that $k!$ divides the number $(n-1)(n-2)\ldots(n-k+1)$, where $1 \leq k \leq n-2$. With this minimum k, $SS(n) = n - k$.

Thus, to find $SS(8)$, note that 2! does not divide 7, but $3! = 2\times 3$ divides 7×6. Hence, the minimum k such that $k!$ divides 7×6 is 3; consequently, $SS(8) = 8\text{-}3 = 5$.

An extensive study of the function was made by Majumdar (2018). Later, the problem was studied, to some extent, by Majumdar (2019), Islam et al. (2021), Majumdar and Ahmed (2021), Islam et al. (2021), and Islam et al. (2022). The following properties are known about $SS(n)$.

Lemma 1: $SS(n) = n - 2$ if and only if n (≥ 3) is an odd integer.

Lemma 2: $SS(n) = n - 3$ if and only if n (≥ 4) is an even integer, not divisible by 3.

From Lemma 1 and Lemma 2, it follows that $SS(n) \leq n - 4$ if n is of the form $n = 6m$, $m \geq 1$ being any integer.

This paper considers functions of the forms $SS(30p)$, $SS(60p)$, and $SS(210p)$, where p is a prime. This is done in the next section. In the analysis of the problem, the following results would be needed.

Lemma 3: Let a, b, and c be any three integers. The linear Diophantine equation $ax + by = c$ has a solution if and only if $d \equiv \gcd(a, b)$ divides c. Moreover, if (x_0, y_0) is a solution, then the general solution is given parametrically by $x = x_0 + (\frac{b}{d})t$, $y = y_0 + (\frac{-a}{d})t$ for any integer t.

Proof: See, for example, Gioia (2001,Theorem 12.2).

Lemma 4: For any integer m (≥ 1), the product of m consecutive integers is divisible by $m!$.

Proof: See Hardy and Wriht (2002, Theorem 74).

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Lemma 3 gives the complete solution of the linear Diophantine equation of the form $ax + by = c$. Recall that a Diophantine equation involves two or more variables for which positive integer solutions are required.

Main results

First, the following result is proved, which gives an explicit form of $SS(30p)$.

Lemma 5: Let $p \geq 2$ be a prime. Then,

$$SS(30p) = \begin{cases} 30p - 4, & if\ p = 4s + 3,\ s \geq 0 \\ 30p - 7, & otherwise \end{cases}$$

Proof: Consider the following expression:

$$C(30p, 4) \equiv 30p\left[\frac{(30p-1)(15p-1)(10p-1)}{4}\right].$$

Now, the problem is to find the condition such that the term inside the square bracket is an integer. In other words, the problem is to find the condition on p such that the term inside the square bracket is an integer. Now, p may be of one of the two forms, namely, $p = 4s + 3$ (for some integer $s \geq 0$), and $p = 4t + 1$ (for some integer $t \geq 1$). If $p = 4s + 3$, then

$$15p - 1 = 4(15s + 11),$$

This shows that 4 divides $15p - 1$, so the term inside the square bracket is an integer.

To complete the proof, consider the case when $p = 4t + 1$. Note that, in this case, 4 does not divide $15p - 1$. The expression

$$C(30p, 5) \equiv 30p\left[\frac{(30p-1)(15p-1)(10p-1)(15p-2)}{2\times 5}\right]$$

shows $SS(30p) \neq 30p - 5$ for any prime $p \geq 2$. Also, from the expression for $C(30p, 6)$

$$C(30p, 6) \equiv 30p\left[\frac{(30p-1)(15p-1)(10p-1)(15p-2)(6p-1)}{3\times 4}\right]$$

it follows that $SS(30p) \neq 30p - 6$ for any prime $p \geq 3$. Now, consider the expression

$$C(30p, 7) \equiv$$

$$30p\left[\frac{(30p-1)(15p-1)(10p-1)(15p-2)(6p-1)(5p-1)}{2\times 7}\right].$$

Here, one of the numbers, $15p - 2$ and $15p - 1$, is even, depending on whether $p = 2$ or p is odd. Also, $p \neq 7$ (since by part (1) of the lemma, $SS(210) = 206$). Thus, the term inside the square bracket is an integer by virtue of Lemma 4. All these establish the lemma.

The lemma below finds $SS(60p)$.

Lemma 6: Let $p \geq 2$ be a prime. Then,

$$SS(60p) = \begin{cases} 30p - 6, & if\ p = 6s + 5,\ s \geq 0 \\ 60p - 7, & if\ p = 6t + 1,\ t \geq 2 \end{cases}$$

Proof: Consider the following expression:

$$C(60p, 4) \equiv 60p\left[\frac{(60p-1)(30p-1)(20p-1)}{4}\right].$$

Clearly, the numerator of the term inside the square bracket is not divisible by 4. Also, the expression

$$C(60p, 5) \equiv 60p\left[\frac{(60p-1)(30p-1)(20p-1)(15p-1)}{5}\right]$$

shows that the term inside the square bracket cannot be an integer. Thus, for any prime p,

$SS(60p) \neq 60p - 4$, $SS(60p) \neq 60p - 5$, Now, consider the expression:

$$C(60p, 6) \equiv 60p\left[\frac{(60p-1)(30p-1)(20p-1)(15p-1)(12p-1)}{6}\right].$$

Note that, p is either of the form $p = 6s + 5$ (for some integer $s \geq 0$), or it is of the form $p = 6t + 1$ (for some integer $t \geq 0$). With $p = 6s + 5$,

$$20p - 1 = 3(40s + 33),$$

so that $20p - 1$ is divisible by 3; also, with this p, $15p - 1$ is even. Thus, the term inside the square bracket is an integer.

Next, consider the following expression:

$$C(60p, 7) \equiv$$

$$60p\left[\frac{(60p-1)(30p-1)(20p-1)(15p-1)(12p-1)(10p-1)}{7}\right].$$

Here, by Lemma 4, the term inside the square bracket is an integer if and only if $p \neq 7$. All these complete the proof of the lemma.

It may be mentioned here that, in Lemma 6, p can be any prime except 7. Thus, Lemma 6 is supplemented by the value $SS(420) = 412$.

The next lemma deals with $SS(210p)$.

Lemma 7: Let $p \geq 2$ be a prime. Then,

$$SS(210p) = \begin{cases} 210p-4, & if\ p = 4s+1,\ s \geq 1 \\ 210p-8, & if\ p = 8t+3,\ t \geq 0 \\ 210p-9, & if\ p = 72u+31,\ u \geq 0 \\ & or, if\ p = 72v+71,\ v \geq 0 \\ 210p-11, & otherwise \end{cases}$$

Proof: Consider the expression below:

$$C(210p,4) \equiv 210p\left[\frac{(210p-1)(105p-1)(70p-1)}{4}\right].$$

Note that, if $p = 4s+1$, then

$$105p-1 = 4(105s+26),$$

This shows that $105p - 1$ is divisible by 4, so the term inside the square bracket is an integer. This establishes part (1) of the lemma.

Next, let $p = 4t+3$. The expression

$$C(210p,5) \equiv 210p\left[\frac{(210p-1)(105p-1)(70p-1)(105p-2)}{2\times 5}\right]$$

shows that $SS(210p) \neq 210p-5$ for any prime p, from the expression

$$C(210p,6) \equiv 210p\left[\frac{(210p-1)(105p-1)(70p-1)(105p-2)(42p-1)}{2\times 6}\right]$$

(since 4 divides neither $105p - 1$ nor $105p - 2$) it follows that $SS(210p) \neq 210p-6$ for any prime p, and the expression

$$C(210p,7) \equiv 210p\left[\frac{(210p-1)(105p-1)(70p-1)}{2\times 7} \times (105p-2)(42p-1)(35p-1)\right]$$

shows that $SS(210p) \neq 210p-7$ for any prime p, since by Lemma 4, the numerator of the term inside the square bracket is not divisible by 7. So, consider

$$C(210p,8) \equiv 210p\left[\frac{(210p-1)(105p-1)(70p-1)}{16} \times (105p-2)(42p-1)(35p-1)(30p-1)\right].$$

If $p = 8s+3$, then

$$105p-1 = 2(420s+157),$$

$$35p-1 = 8(35s+13),$$

so that, the term inside the square bracket is an integer if $p = 8s+3$.

Next, consider the expression:

$$C(210p,9) \equiv 210p\left[\frac{(210p-1)(105p-1)(70p-1)}{8\times 9} \times (105p-2)(42p-1)(35p-1)(30p-1)(105p-4)\right].$$

Now, the problem is to find the condition such that the term inside the square bracket is an integer. Looking at the terms in the numerator, it is clear that, one possibility is that 4 divides $35p - 1$ (in which case, $105p-1$ is divisible by 2) and 9 divides $70p-1$. By inspection, it is found that, when $p = 72s + 31$, then

$$35p-1 = 4(630s+271),$$

$$70p-1 = 9(560s+241),$$

so that $(105p-1)(35p-1)(70p-1)$ is divisible by 72. The second possibility is that 36 divides $35p - 1$. With $p = 72t+71$,

$$35p-1 = 36(70t+69),$$

so that $(105p - 1)(35p - 1)$ is divisible by 72. All these prove part (2) of the lemma.

Next, consider the expression below:

$$C(210p,10) \equiv 210p\left[\frac{(210p-1)(105p-1)(70p-1)(105p-2)}{3\times 5\times 16} \times (42p-1)(35p-1)(30p-1)(105p-4)(70p-3)\right].$$

Here, in order that the term inside the square bracket is an integer, a necessary condition is that $42p - 1$ must be divisible by 5. This leads to the Diophantine equation $42p-1 = 5\alpha$, with the solution $p = 5x + 3$ ($x \geq 0$) (see Lemma 3). The second condition that must be satisfied is that $35p - 1$ must be divisible by 8. Since, $35p-1 = 175x + 104$, it follows that $x = 8$, so that $p = 40x+3$, which violates part (2) of the lemma.

Finally, consider the following expression for $C(210p,11)$:

$$210p\left[\frac{(210p-1)(105p-1)(70p-1)(105p-2)(42p-1)}{8\times 3\times 11} \times (35p-1)(30p-1)(105p-4)(70p-3)(21p-1)\right].$$

Now, by Lemma 4, $(70p - 1)(35p - 1)(70p - 3)$ is divisible by 3. Also, it may easily be verified that $(105p - 1)(35p - 1)(21p - 1)$ is divisible by 8 if p is either of the form $p = 4s + 1$ or of the form $p = 4t + 3$.

Moreover, $p \neq 11$. Hence, the term inside the square bracket is an integer, which was intended to prove.

The lemma below deals with $SS(420p)$.

Lemma 8: Let $p \geq 2$ be a prime. Then

$$SS(420p) = \begin{cases} 420p-6, & if\ p = 6s+5,\ s \geq 0 \\ 420p-8, & if\ p = 8t+1,\ t \neq 3x+2 \\ 420p-9, & if\ p = 18u+13,\ u \neq 4y+2 \\ 420p-10, & if\ p = 40v+29,\ v \neq 3a, v \neq 9b+5 \\ 420p-11, & otherwise \end{cases}$$

Proof: The expressions

$$C(420p,4) \equiv 420p\left[\frac{(420p-1)(210p-1)(140p-1)}{4}\right],$$

$$C(420p,5) \equiv 420p\left[\frac{(420p-1)(210p-1)(140p-1)(105p-1)}{5}\right],$$

show that, for any prime p,

$$SS(420p) \neq 420p-4,\ SS(420p) \neq 420p-5.$$

So, consider the expression:

$$C(420p,6) \equiv 420p\left[\frac{(420p-1)(210p-1)(140p-1)(105p-1)(84p-1)}{6}\right].$$

Here so that the term inside the square bracket is an integer, p must be odd, and 3 must divide $140p-1$. Now, the solution of the Diophantine equation $140p-1=3\alpha$ is $p = 3x+2$. To guarantee that p is odd, x must be odd. Therefore, by writing $x = 2t+1$, the desired expression of p is obtained.

The expression

$$C(420p,7) \equiv 420p\left[\frac{(420p-1)(210p-1)(140p-1)}{7} \times (105p-1)(84p-1)(70p-1)\right]$$

shows $SS(420p) \neq 420p-7$ for any prime p. So, consider

$$C(420p,8) \equiv 420p\left[\frac{(420p-1)(210p-1)(140p-1)}{8} \times (105p-1)(84p-1)(70p-1)(60p-1)\right].$$

Here, the term inside the square bracket is an integer if and only if 8 divides $105p-1$. Thus, p must satisfy the equation $105p-1 = 8\alpha$, with the solution $p = 8t+1$ ($t \geq 2$ being any integer). Now, considering the Diophantine equation $8t+1 = 6a+5$, using Lemma 3, the solution is found to be $t = 3x+2$ ($x \geq 0$ being any integer).

Next, consider the expression:

$$C(420p,9) \equiv 420p\left[\frac{(420p-1)(210p-1)(140p-1)}{2\times 9} \times (105p-1)(84p-1)(70p-1)(60p-1)(105p-2)\right].$$

Now, note that, one of $105p-1$ and $105p-2$ is even. Thus, the term inside the square bracket is an integer if and only if 9 divides $70p-1$. This leads to the Diophantine equation $70p-1 = 9\alpha$, whose solution is $p = 9x+4$. In order to guarantee that p is odd, x is replaced by $2u+1$, to get $p = 18u+13$. To exclude common values, the Diophantine equations $18u+13 = 6a+5$, and $18u+13 = 8b+1$ are to be considered. By Lemma 3, the first equation has no solution, while the solution of the second equation is $u = 4x+2$ ($x \geq 0$ being any integer).

Now, consider the expression:

$$C(420p,10) \equiv 420p\left[\frac{(420p-1)(210p-1)(140p-1)(105p-1)}{3\times 4\times 5} \times (84p-1)(70p-1)(60p-1)(105p-2)(140p-3)\right].$$

Here, in order that the term inside the square bracket is an integer, the only possibility is that 5 divides $84p-1$ and 4 divides $105p-1$. Thus, for some integers α and β,

$$84p-1 = 5\alpha,\ 105p-1 = 4\beta,$$

with the solutions $p = 5x+4$ and $p = 4y+1$ respectively. Now, the combined equation is $5x+4 = 4y+1$, whose solution is $x = 4z+1$, so that, finally, $p = 5(4z+1)+4 = 20z+9$. Next, the equation $20z+9 = 8b+1$. This shows that z must be even. Therefore, writing $z = 2v+1$, finally, $p = 20(2v+1)+9 = 40v+29$. Considering the equations $40v+29 = 6a+5$ and $40v+29 = 18c+13$, the solutions are found to be $v = 3a$ and $v = 9b+5$ respectively, $a \geq 0$ and $b \geq 0$ being any integers.

Finally, consider the expression:

$$C(420p,11) \equiv 420p\left[\frac{(420p-1)(210p-1)(140p-1)(105p-1)}{2\times 3\times 11} \times (84p-1)(70p-1)(60p-1)(105p-2)(140p-3)(42p-1)\right].$$

Here, $p \neq 11$. Hence, the term inside the square bracket is an integer.

The lemma below finds $SS(840p)$.

Lemma 9: Let $p \geq 2$ be a prime. Then,

$$SS(840p) = \begin{cases} 840p - 9, & if\ \ p = 9s + 1,\ s \geq 0 \\ & or,\ p = 9t + 2,\ t \geq 0 \\ 840p - 10, & if\ \ p = 10s + 7, \\ & s \neq 9x + 3,\ s \neq 9y + 4 \\ 840p - 11, & otherwise \end{cases}$$

Proof: From the expressions of $C(840p, 4)$, $C(840p, 5)$, $C(840p, 6)$, $C(840p, 7)$, and $C(840p, 8)$, it can be seen that, for any prime p,

$$SS(840p) \neq 840p - 4,\ SS(840p) \neq 840p - 5,$$

$$SS(840p) \neq 840p - 6,\ SS(840p) \neq 840p - 7,$$

$$SS(840p) \neq 840p - 8.$$

So, consider the expression

$$C(840p,9) \equiv 840p\Big[\frac{(840p-1)(420p-1)(280p-1)}{9} \times (210p-1)(168p-1)(140p-1)(120p-1)(105p-1)\Big].$$

Clearly, the term inside the square bracket is an integer if and only if either 9 divides $280p - 1$ or 9 divides $140p - 1$. The resulting equations are $280p - 1 = 9\alpha$ and $140p - 1 = 9\beta$, whose solutions are $p = 9s + 1$ and $p = 9t + 2$ respectively.

Next, consider

$$C(840p, 10) \equiv 840p\Big[\frac{(840p-1)(420p-1)(280p-1)(210p-1)}{2\times3\times5} \times (168p-1)(140p-1)(120p-1)(105p-1)(280p-3)\Big].$$

Here, so that the term inside the square bracket is an integer, 5 must divide $168p - 1$; moreover, p must be odd. Now, the solution of the Diophantine equation $168p - 1 = 5\alpha$ is $p = 5x + 2$. In order to guarantee that p is odd, x is replaced by $2s + 1$ to get the desired result.

Finally, consider

$$C(840p, 11) \equiv 840p\Big[\frac{(840p-1)(420p-1)(280p-1)(210p-1)}{3\times11} \times (168p-1)(140p-1)(120p-1)(105p-1)(280p-3)(84p-1)\Big].$$

Here, since $p \neq 11$, it follows that the term inside the square bracket is an integer.

Conclusions

This paper derives the explicit forms of $SS(30p)$, $SS(60p)$, $SS(210p)$ and $SS(420p)$, where p is a prime. It is found that, surprisingly, $SS(30p)$ and $SS(60p)$ behave differently. For example, in $SS(30p)$, the minimum integer k such that $30p$ divides $C(30p, k)$ can be 4 and 7 only (depending on p), while the only possible values of the minimum k in $SS(60p)$ are 6 and 7. Again, in $SS(210p)$ (depending on p), the minimum k can only be one of four possible values, namely, 4, 8, 9, and 11, whereas the minimum k in $SS(420p)$ is five in number, namely, 6, 8, 9, 10, and 11. On the other hand, Lemma 9 shows that, the minimum k in $SS(840p)$ can be only one of three values, namely, $k = 9, 10, 11$.

The accompanying tables give the values of $SS(30p)$, $SS(60p)$, $SS(210p)$, and $SS(420p)$ for the first 200 primes, calculated on a computer, using the formula (3) for the binomial coefficients.

Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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Table 1. Values of *SS*(30*p*), *p* is a prime

p	SS(n)	p	SS(n)	p	SS(n)	p	SS(n)	p	SS(n)
1	23	173	5183	409	12263	659	19766	941	28223
2	53	179	5366	419	12566	661	19823	947	28406
3	86	181	5423	421	12623	673	20183	953	28583
5	143	191	5726	431	12926	677	20303	967	29006
7	206	193	5783	433	12983	683	20486	971	29126
11	326	197	5903	439	13166	691	20726	977	29303
13	383	199	5966	443	13286	701	21023	983	29486
17	503	211	6326	449	13463	709	21263	991	29726
19	566	223	6686	457	13703	719	21566	997	29903
23	686	227	6806	461	13823	727	21806	1009	30263
29	863	229	6863	463	13886	733	21983	1013	30383
31	926	233	6983	467	14006	739	22166	1019	30566
37	1103	239	7166	479	14366	743	22286	1021	30623
41	1223	241	7223	487	14606	751	22526	1031	30926
43	1286	251	7526	491	14726	757	22703	1033	30983
47	1406	257	7703	499	14966	761	22823	1039	31166
53	1583	263	7886	503	15086	769	23063	1049	31463
59	1766	269	8063	509	15263	773	23183	1051	31526
61	1823	271	8126	521	15623	787	23606	1061	31823
67	2006	277	8303	523	15686	797	23903	1063	31886
71	2126	281	8423	541	16223	809	24263	1069	32063
73	2183	283	8486	547	16406	811	24326	1087	32606
79	2366	293	8783	557	16703	821	24623	1091	32726
83	2486	307	9206	563	16886	823	24686	1093	32783
89	2663	311	9326	569	17063	827	24806	1097	32903
97	2903	313	9383	571	17126	829	24863	1103	33086
101	3023	317	9503	577	17303	839	25166	1109	33263
103	3086	331	9926	587	17606	853	25583	1117	33503
107	3206	337	10103	593	17783	857	25703	1123	33686
109	3263	347	10406	599	17966	859	25766	1129	33863
113	3383	349	10463	601	18023	863	25886	1151	34526
127	3806	353	10583	607	18206	877	26303	1153	34583
131	3926	359	10766	613	18383	881	26423	1163	34886
137	4103	367	11006	617	18503	883	26486	1171	35126
139	4166	373	11183	619	18566	887	26606	1181	35423
149	4463	379	11366	631	18926	907	27206	1187	35606
151	4526	383	11486	641	19223	911	27326	1193	35783
157	4703	389	11663	643	19286	919	27566	1201	36023
163	4886	397	11903	647	19406	929	27863	1213	36383
167	5006	401	12023	653	19583	937	28103	1217	36503

Table 2. Values of $SS(60p)$, p is a prime

p	SS(n)	p	SS(n)	p	SS(n)	p	SS(n)	p	SS(n)
1	53	173	10374	409	24533	659	39534	941	56454
2	113	179	10734	419	25134	661	39653	947	56814
3	173	181	10853	421	25253	673	40373	953	57174
5	294	191	11454	431	25854	677	40614	967	58013
7	412	193	11573	433	25973	683	40974	971	58254
11	654	197	11814	439	26333	691	41453	977	58614
13	773	199	11933	443	26574	701	42054	983	58974
17	1014	211	12653	449	26934	709	42533	991	59453
19	1133	223	13373	457	27413	719	43134	997	59813
23	1374	227	13614	461	27654	727	43613	1009	60533
29	1734	229	13733	463	27773	733	43973	1013	60774
31	1853	233	13974	467	28014	739	44333	1019	61134
37	2213	239	14334	479	28734	743	44574	1021	61253
41	2454	241	14453	487	29213	751	45053	1031	61854
43	2573	251	15054	491	29454	757	45413	1033	61973
47	2814	257	15414	499	29933	761	45654	1039	62333
53	3174	263	15774	503	30174	769	46133	1049	62934
59	3534	269	16134	509	30534	773	46374	1051	63053
61	3653	271	16253	521	31254	787	47213	1061	63654
67	4013	277	16613	523	31373	797	47814	1063	63773
71	4254	281	16854	541	32453	809	48534	1069	64133
73	4373	283	16973	547	32813	811	48653	1087	65213
79	4733	293	17574	557	33414	821	49254	1091	65454
83	4974	307	18413	563	33774	823	49373	1093	65573
89	5334	311	18654	569	34134	827	49614	1097	65814
97	5813	313	18773	571	34253	829	49733	1103	66174
101	6054	317	19014	577	34613	839	50334	1109	66534
103	6173	331	19853	587	35214	853	51173	1117	67013
107	6414	337	20213	593	35574	857	51414	1123	67373
109	6533	347	20814	599	35934	859	51533	1129	67733
113	6774	349	20933	601	36053	863	51774	1151	69054
127	7613	353	21174	607	36413	877	52613	1153	69173
131	7854	359	21534	613	36773	881	52854	1163	69774
137	8214	367	22013	617	37014	883	52973	1171	70253
139	8333	373	22373	619	37133	887	53214	1181	70854
149	8934	379	22733	631	37853	907	54413	1187	71214
151	9053	383	22974	641	38454	911	54654	1193	71574
157	9413	389	23334	643	38573	919	55133	1201	72053
163	9773	397	23813	647	38814	929	55734	1213	72773
167	10014	401	24054	653	39174	937	56213	1217	73014

Table 3 . Values of *SS*(210*p*), *p* is a prime

p	SS(n)	p	SS(n)	p	SS(n)	p	SS(n)	p	SS(n)
1	206	173	36326	409	85886	659	138382	941	197606
2	412	179	37582	419	87982	661	138806	947	198862
3	622	181	38006	421	88406	673	141326	953	200126
5	1046	191	40099	431	90501	677	142166	967	203061
7	1459	193	40526	433	90926	683	143422	971	203902
11	2302	197	41366	439	92179	691	145102	977	205166
13	2726	199	41779	443	93022	701	147206	983	206419
17	3566	211	44302	449	94286	709	148886	991	208099
19	3982	223	46819	457	95966	719	150981	997	209366
23	4819	227	47662	461	96806	727	152659	1009	211886
29	6086	229	48086	463	97221	733	153926	1013	212726
31	6501	233	48926	467	98062	739	155182	1019	213982
37	7766	239	50179	479	100579	743	156019	1021	214406
41	8606	241	50606	487	102259	751	157701	1031	216499
43	9022	251	52702	491	103102	757	158966	1033	216926
47	9859	257	53966	499	104782	761	159806	1039	218181
53	11126	263	55219	503	105621	769	161486	1049	220286
59	12382	269	56486	509	106886	773	162326	1051	220702
61	12806	271	56899	521	109406	787	165262	1061	222806
67	14062	277	58166	523	109822	797	167366	1063	223219
71	14901	281	59006	541	113606	809	169886	1069	224486
73	15326	283	59422	547	114862	811	170302	1087	228259
79	16579	293	61526	557	116966	821	172406	1091	229102
83	17422	307	64462	563	118222	823	172821	1093	229526
89	18686	311	65299	569	119486	827	173662	1097	230366
97	20366	313	65726	571	119902	829	174086	1103	231619
101	21206	317	66566	577	121166	839	176179	1109	232886
103	21621	331	69502	587	123262	853	179126	1117	234566
107	22462	337	70766	593	124526	857	179966	1123	235822
109	22886	347	72862	599	125779	859	180382	1129	237086
113	23726	349	73286	601	126206	863	181221	1151	241701
127	26659	353	74126	607	127461	877	184166	1153	242126
131	27502	359	75381	613	128726	881	185006	1163	244222
137	28766	367	77059	617	129566	883	185422	1171	245902
139	29182	373	78326	619	129982	887	186259	1181	248006
149	31286	379	79582	631	132499	907	190462	1187	249262
151	31699	383	80419	641	134606	911	191299	1193	250526
157	32966	389	81686	643	135022	919	192979	1201	252206
163	34222	397	83366	647	135861	929	195086	1213	254726
167	35059	401	84206	653	137126	937	196766	1217	255566

Table 4 . Values of $SS(420p)$, p is a prime

p	SS(n)	p	SS(n)	p	SS(n)	p	SS(n)	p	SS(n)
1	412	173	72654	409	171772	659	276774	941	395214
2	831	179	75174	419	175974	661	277611	947	397734
3	1249	181	76009	421	176809	673	282652	953	400254
5	2094	191	80214	431	181014	677	284334	967	406131
7	2929	193	81052	433	181852	683	286854	971	407814
11	4614	197	82734	439	184369	691	290209	977	410334
13	5451	199	83569	443	186054	701	294414	983	412854
17	7134	211	88611	449	188574	709	297770	991	416209
19	7969	223	93649	457	191932	719	301974	997	418729
23	9654	227	95334	461	193614	727	305329	1009	423772
29	12174	229	96171	463	194451	733	307851	1013	425454
31	13011	233	97854	467	196134	739	310369	1019	427974
37	15529	239	100374	479	201174	743	312054	1021	428811
41	17214	241	101212	487	204529	751	315411	1031	433014
43	18049	251	105414	491	206214	757	317929	1033	433852
47	19734	257	107934	499	209571	761	319614	1039	436371
53	22254	263	110454	503	211254	769	322972	1049	440574
59	24774	269	112974	509	213774	773	324654	1051	441409
61	25609	271	113809	521	218814	787	330531	1061	445614
67	28131	277	116329	523	219649	797	334734	1063	446449
71	29814	281	118014	541	227209	809	339774	1069	448970
73	30652	283	118851	547	229729	811	340609	1087	456529
79	33169	293	123054	557	233934	821	344814	1091	458214
83	34854	307	128929	563	236454	823	345651	1093	459051
89	37374	311	130614	569	238974	827	347334	1097	460734
97	40732	313	131452	571	239811	829	348170	1103	463254
101	42414	317	133134	577	242332	839	352374	1109	465774
103	43251	331	139009	587	246534	853	358249	1117	469129
107	44934	337	141532	593	249054	857	359934	1123	471649
109	45770	347	145734	599	251574	859	360771	1129	474172
113	47454	349	146570	601	252412	863	362454	1151	483414
127	53329	353	148254	607	254931	877	368331	1153	484252
131	55014	359	150774	613	257449	881	370014	1163	488454
137	57534	367	154129	617	259134	883	370849	1171	491809
139	58371	373	156651	619	259969	887	372534	1181	496014
149	62574	379	159169	631	265009	907	380929	1187	498534
151	63409	383	160854	641	269214	911	382614	1193	501054
157	65931	389	163374	643	270051	919	385969	1201	504412
163	68449	397	166729	647	271734	929	390174	1213	509449
167	70134	401	168414	653	274254	937	393532	1217	511134