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LEVEL SEPARATION ON FUZZY PAIRWISE T_0BITOPOLOGICAL SPACE

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ABSTRACT

In this paper, we introduce five notions of level separation in T_0 fuzzy bitopological spaces. We establish some relation among them. Also, we find relations between fuzzy topological spaces and corresponding fuzzy bitopological spaces in such spaces. Further, we prove that all these definitions satisfy "good extension" property. Finally, we prove that all these notionsare hereditary, productive and projective, moreover we observe that all concepts are preserved under one-one, onto and continuous mapping.

Keywords: Fuzzy bitopologicalspace, FP-Continuous, FP-Open, FP-Closed

INTRODUCTION

The fundamental concept of fuzzy set introduced by Zadeh (1965) provided a natural foundation for building new branches. Chang (1968) introduced the concept of fuzzy topological spaces and there after many fuzzy topologists have contributed various forms of separation axioms to the theory of fuzzy bitopological spaces. Kandil (1991) introduced the concept of fuzzy bitopologicalspaces and since then may concepts in classical topology have been extended to fuzzy bitopological spaces. FP $–T_0$ separation axioms has been introduced by Choubey (1995), Kandiland El-Shafee (1991, 1995), Safia *et al.* (in two ways) and Kandil *et al.* We add five more notions to this list; our definitions are good extension of their topological counterparts P–T_0 in a bitopological space. Our definition α-FP–T_0(j), j = i, ii, iii, iv, v are due to Choubey. This seems to be most appropriate in view of the fact that FP–T_0 fuzzy bitopological spaces in this sense, are precisely the T_0– objects in the category of 'fuzzy bitopological spaces and fuzzy pairwise continuous maps'. On comparison, it turns out that α-FP–T_0(j), j = *i, ii, iii, iv, v*. is the weakest among all. We shall prove that FP–T_0 (j) is hereditary, productive and projective.

Notations and preliminaries through this paper: X will be a nonempty set, I = [0, 1], I_0 = (0, 1], I_1 = [0, 1), I_{01} = (0, 1) and FP (resp P) stands for fuzzy pairwise (resp pairwise). The class of all fuzzy sets on a universe X will be denoted by I^X and fuzzy sets

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on X will be denoted by u, v, w, etc. Crisp subsets of X will be denoted by capital letters A, B, C etc. Fuzzy singleton will be denoted by x_r, y_r, z_r. The class of all fuzzy singleton in X is denoted by S(X). For every $x_r \in S(X)$ and $v \in I^X$, we write $x_r \in v$ iff $r \leq v(x)$, also by $\alpha(x) = \alpha$, $\forall\ x \in X$ and $\alpha \in I$, we mean the constant mapping on X with value α, 1_A denote the characteristic mapping of $A \subseteq X$.

PRELIMINARIES

Definition 2.1. For $u \in I^X$ we define

(a) $u_\alpha = \{x : x \in X \text{ and } \alpha \leq u(x)\}$ as the weak α - cut of u, where $\alpha \in (0, 1]$, the weak 1– cut is called the kernel of u and it is denoted by ker(u).

(b) $u_\alpha^+ = \{x : x \in X \text{ and } u(x) > \alpha\}$ as the strong α - cut of u, where $\alpha \in [0, 1)$, the strong
0-cut of u is called the support of u and is denoted by supp (u).

(c) $hgt(u) = \sup_{x \in X} u(x)$ as the height of u (Nouh 1996).

Lemma 2.2. Let $u \in I^X$ and $\{ u_s : s \in S \} \subseteq I^X$ then

(a) $(\cup_{s=1}^{n} u_s)_n = \cup_{s=1}^{n} (u_s)_\alpha$, $\alpha \in [0, 1)$

(b) $(\cap_{s \in S} u_s)_\alpha = \cap_{s \in S} (u_s)_\alpha$, $\alpha \in (0, 1]$

The representation (Decomposition) theorem states that a fuzzy set can be decompossed into a family of ordinary subset of the unique, namely its weak or strong α - cut:

$u = \cup_{\alpha \in (0\,,\,1]}(\alpha \cap 1_u)$, $\forall\ u \in I^X$(Nouh 1996).

Definition 2.3. Let (X, *T*) be an ordinary topological space. The set of all lower semicontinuous functions from (X, *T*) into the closed unit interval equipped with the usual topology constitative a fuzzy topology associated with (X, *T*) and is denoted as (X, $\omega(T)$) (Lowen 1976 and Weiss 1975).

Definition 2.4. A fuzzy bitopological space (X, *s*, *t*) is called p-bitopology generated iff there exist two ordinary topologies *S* and *T* on X such that $\omega(S) = s$ and $\omega(T) = t$ (Kandil 1995).

Definition 2.5. A fuzzy topological space (X, *t*) is called fuzzy – T_0 if $\forall$ x, y $\in$ X, x $\neq$ y, $\exists$ u $\in$ t such that $u(x) \neq u(y)$.

Definition 2.6. A bitopological space (X, *S*, *T*) is called pairwise – T_1(i) (in short , P -T_0(i)) if for all x, y $\in$ X, x $\neq$ y, there exist $U \in S \cup T$ such that $x \in U$, $y \notin U$ or $x \notin U$, $y \in U$ (Kandil 1991).

Definition 2.7. A fuzzy bitopological space (X, *s*, *t*) is called

(a) FP - T_0*(i)* if for all x, y ∈ X, x ≠ y then there exist u ∈ *s* ∪ *t* such that u(x) ≠ u(y).

(b) FP - T_0*(ii)* iffor all x, y ∈ X, x ≠ y then there exist u ∈ *s* ∪ *t* such that u(x) = 1, u(y) = 0 or u(x) = 0, u(y) =1.

Further, let A ⊆ X and s_A = {u/A : u ∈ *s*} , t_A = {v/A : v ∈ *t*} denoted the subspace topology on A induced by s_A, t_A. Then (A, s_A, t_A) is called subspace of (X, *s*, *t*) with the underlying set A.

A fuzzy bitopological property P is called hereditary if each subspace of a fuzzy bitopological space with property, also has property P (Srivastava 1987).

Definition 2.8. Let {(X_i, s_i, t_i), i∈Λ} be a family of fuzzy bitopological space. Then the space (∏X_i, ∏s_i, ∏t_i) is called product fuzzy bitopological space of the family {(X_i, s_i, t_i), i∈Λ}, where ∏s_i, ∏t_i respectively denote the usual product fuzzy topologies of the families {∏s_i : i ∈ Λ} and {∏t_i : i ∈ Λ} of the fuzzy topologies on X.

A fuzzy topological property P is called productive if the product of fuzzy bitopological sspace of a family of fuzzy bitopological space, each having property P, has property P.

A property P in a fuzzy bitopological space is called projective if for a family of fuzzy bitopological space {(X_i, s_i, t_i), i ∈ Λ}, the product fuzzy bitopological space (∏X_i, ∏s_i, ∏t_i) has property P implies that each coordinate space has property P (Wong 1974).

Definition 2.9. A mapping *f* : (X, *s*) → (Y, *t*) from a fuzzy topological space (X, *s*) into another fuzzy topology (Y, *t*) is said to be

(i) Continuous if and only if for every v ∈ *t*, ⇒f^{-1}(v) ∈ *s*.

(ii) Open if and only if for each open fuzzy set u in (X, *s*), ⇒*f*(u) is open in (Y, *t*).

(iii) Closed if and only if for each closed fuzzy set u in (X, *s*), ⇒ *f*(u) is closed in (Y, *t*).

(iv) The function *f*is called fuzzy homeomorphism if and only if *f*is bijective and both *f* and f^{-1} are fuzzy continuous (Chang 1968).

Definition 2.10. A mapping *f* : (X, *s*, *t*) → (X, s_i, t_i) from a fuzzy bitopological (X, *s*, *t*) into another fuzzy bitopological space (X, s_i, t_i) is said to be

(i) FP – continuous if and only if *f* : (X, *s*) → (X, s_i) and *f* : (X, *t*) → (X, t_i) are both continuous.

(ii) FP – open if and only if $f : (X, s) \rightarrow (X, s_i)$ and $f : (X, t) \rightarrow (X, t_i)$ are both open.

(iii) FP – closed if and only if $f : (X, s) \rightarrow (X, s_i)$ and $f : (X, t) \rightarrow (X, t_i)$ are both closed.

(iv) FP – homomorphism if and only if f is bijective, FP-continuous and $f : (X, s_i, t_i) \rightarrow (X, s, t)$ is FP – continuous (Mukherjee 2002).

DEFINITION AND PROPERTIES Of FP-T_0 SPACES

We mention here five possible definition of fuzzy pairwise T_0 (in short FP – T_0) bitopological space, where $\alpha \in [0, 1)$.

Definition 3.1. A fuzzy bitopological space (X, s, t) is called

(a) FP – T_0*(i)* iff for all $x, y \in X$ with $x \neq y$ there exist $u \in s \cup t$ such that $u(x) = 1$, $u(y) = 0$ or $u(x) = 0$, $u(y) = 1$.

(b) α - FP – T_0*(ii)* iff for all $x, y \in X$ with $x \neq y$ there exist $u \in s \cup t$ such that $u(x) = 1$, $u(y) \leq \alpha$ or $u(x) \leq \alpha$, $u(y) = 1$.

(c) α - FP – T_0*(iii)* ifffor all $x, y \in X$ with $x \neq y$ there exist $u \in s \cup t$ such that $u(x) = 0$, $u(y) > \alpha$ or $u(x) > \alpha$, $u(y) = 0$.

(d) α - FP – T_0*(iv)* ifffor all $x, y \in X$ with $x \neq y$ there exist $u \in s \cup t$ such that $0 \leq u(x) \leq \alpha < u(y) \leq 1$, or $0 \leq u(y) \leq \alpha < u(x) \leq 1$.

(e) FP – T_0*(v)* iff for all $x, y \in X$ with $x \neq y$ there exist $u \in s \cup t$ such that $u(x) > u(y)$ or $u(x) < u(y)$.

Theorem 3.2. Suppose (X, s, t) be a fuzzy bitopological space and $(X, s \cup t)$ be a fuzzy topological space then (X, s, t) is α - FP – T_0(j) if and only if $(X, s \cup t)$ is α - T_0(j), for j = *i, ii, iii, iv, v*.

Proof: Suppose (X, s, t) is α - FP – T_0*(ii)* then obvious $(X, s \cup t)$ is α - T_0*(ii)*, for $\alpha \in I_1$, since $s \subseteq s \cup t$ and $t \subseteq s \cup t$.

Conversely, suppose that $(X, s \cup t)$ is α - T_0*(ii)* and $x, y \in X$, with $x \neq y$, then there exist $u \in s \cup t$ such that $u(x) = 1$, $u(y) \leq \alpha$ or $u(x) \leq \alpha$, $u(y) = 1$. Hence it is clear that (X, s, t) is α - FP – T_0*(ii)*.

Further, one can easily verify that

(X, s, t) is FP – T_0*(i)* $\Leftrightarrow$ $(X, s \cup t)$ is T_0*(i)*.

(X, s, t) is α - FP – T_0*(iii)* $\Leftrightarrow$ $(X, s \cup t)$ is α - T_0*(iii)*.

(X, *s, t*) is α - FP – T_0*(iv)* $\Leftrightarrow$ (X , $s \cup t$) is α - T_0*(iv).*

(X, *s, t*) is FP – T_0*(v)* $\Leftrightarrow$ (X, $s \cup t$) is T_0*(v).*

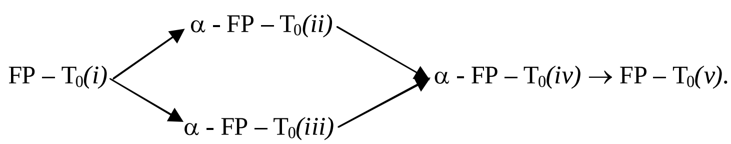
Theorem 3.3. Let (X, *s*) and (X, *t*) be two fuzzy topological spaces and (X, *s*, *t*) be its corresponding bitopological space. Then (X, *s*, *t*) is α-FP-T_0(j) does not imply (X, *s*) and (X, *t*) are α-T_1(j), for j = *i, ii, iii, iv, v*.

Example 3.4. Let X= {x, y} and *s* be the fuzzy topology on X generated by {u} $\cup$ {Constants}, where u(x) = 1, u(y) = 0. Again let *t* be the fuzzy topological space on X generated by {Constants}. For any value of $\alpha \in I_1$, the fuzzy bitopological space (X, *s, t*) is α-FP-T_0(j) . But the fuzzy topological space (X, *t*) is not α-T_0(j), for j = *i, ii, iii, iv, v*.

Theorem 3.5. If the fuzzy topological space (X, *s*) and (X, *t*) are both α-T_0(j), then the corresponding fuzzy bitopological space (X, *s, t*) is also α-FP-T_1(j), for j = *i* , *ii, iii, iv, v*.

Proof: The proof is obvious.

Theorem 3.6. Let (X, *s*, *t*) be a fuzzy bitopological space. Then we have the following implication.



Proof: Suppose (X, *s, t*) be a α - FP –T_0*(ii)* space. We shall prove that (X, *s, t*) is α - FP –T_0(iv) . Let x, y $\in$ X with x $\neq$ y. Since (X, *s, t*) is α - FP – T_0*(ii)*, for $\alpha \in I_1$ then $\exists$ u $\in s \cup t$ such that u(x) = 1, u(y) $\leq \alpha$ or u(x) $\leq \alpha$, u(y) = 1. We clear that $0 \leq u(y) \leq \alpha < u(x) \leq 1$ or $0 \leq u(x) \leq \alpha < u(y) \leq 1$. Hence it is clear that (X, *s, t*) is α - FP –T_1*(iv)* space.

Further, one can easily verify that

FP - T_0*(i)* $\Rightarrow$ α - FP - T_0*(ii).*

FP - T_0*(i)* $\Rightarrow$ α - FP - T_0*(iii).*

α - FP - T_0*(iii)* $\Rightarrow$ α - FP - T_0*(iv).*

α - FP - T_0*(iv)* $\Rightarrow$ FP - T_0*(v).*

None of the reverse implication are true, it can be seen through by the following counter example.

Example 3.7. Let X = {x, y} and *s* be the fuzzy topology on X generated by {u} $\cup$ {Constants}, where u(x) = 0.8, u(y) = 0.1 again let *t* be the fuzzy topology on X generated by $\{v\} \cup \{\text{Constants}\}$, where $v(x) = 0.7$, $v(y) = 0.2$. For $\alpha = 0.6$, We see that the fuzzy bitopological space (X, s, t) is α - FP - T_0(iv). But the fuzzy bitopological space (X, s, t) is not α - FP – T_0*(iii)*, is not α - FP – T_0*(ii)* and is not FP – T_0*(i)*.

Example 3.8. Let $X = \{x, y\}$ and s be the fuzzy topology on X generated by $\{u\} \cup \{\text{Constants}\}$, where $u(x) = 1$, $u(y) = 0.2$, again let t be the fuzzy topology on X generated by $\{v\} \cup \{\text{Constant}\}$, where $u(x) = 1$, $u(y) = 0.1$, For $\alpha = 0.8$. We see that the fuzzy bitopological space (X, s, t) is α - FP – T_0*(ii)*, but the fuzzy bitopological space (X, s, t) is not FP – T_0*(i)* and is not α - FP – T_0*(iii)*.

Example 3.9. Let $X = \{x, y\}$ and s be fuzzy topology on X generated by $\{u\} \cup \{\text{Constants}\}$, where $u(x) = 0.8$, $u(y) = 0$, again let t be the fuzzy topology on X generated by $\{v\} \cup \{\text{Constants}\}$, where $v(x) = 0.7$, $v(y) = 0$. For $\alpha = 0.4$ it is clear that the fuzzy bitopological space (X, s, t) is α - FP – T_0*(iii)*, but the fuzzy bitopological space (X, s, t) is not FP – T_0*(i)* and is not α - FP – T_0*(ii)*.

Example 3.10. Let $X = \{x, y\}$ and s be the fuzzy topology on X generated by $\{u\} \cup \{\text{Constants}\}$, where $u(x) = 0.5$, $u(y) = 0.2$, again let t be the fuzzy topology on X generated by $\{v\} \cup \{\text{Constants}\}$, where $v(x) = 0.6$, $v(y) = 0.3$. For $\alpha = 0.7$ it is clear that the fuzzy bitopological space (X, s, t) is FP – T_0(v), but the fuzzy bitopological space (X, s, t) is not FP – T_0*(i)*, is not α - FP – T_0*(ii)*, is not α - FP – T_0*(iii)* and is not α - FP – T_0*(iv)*.

Theorem 3.11. If (X, s, t) is a fuzzy bitopological space and $0 \le \alpha \le \beta < 1$ then

(a) α - FP – T_0*(ii)* $\Rightarrow$ β - FP – T_0*(ii)*.

(b) β - FP – T_0*(iii)* $\Rightarrow$ α - FP – T_0*(iii)*.

(c) 0 - FP – T_0*(iii)* $\Leftrightarrow$ 0 – FP – T_0*(iv)*.

The reverse implications are not true in general.

Proof: Suppose (X, s, t) be α - FP – T_0*(ii)*. We shall prove that (X, s, t) is β - FP – T_0*(ii)*. Let $x, y \in X$ with $x \neq y$. Since (X, s, t) is α - FP - T_0*(ii)*, for $\alpha \in I_1$ then $\exists\, u \in s \cup t$ such that $u(x) = 1$, $u(y) \le \alpha$ or $u(x) \le \alpha$, $u(y) = 1$. Suppose $u(x) = 1$, $u(y) \le \alpha$ then $u(x) = 1$, $u(y) \le \beta$ as $0 \le \alpha \le \beta < 1$. Hence the fuzzy bitopological (X, s, t) is β - FP – T_0*(ii)*.

Example 3.12. Let $X = \{x, y\}$ and s be the fuzzy topology on X generated by $\{u\} \cup \{\text{Constants}\}$, where $u(x) = 1$, $u(y) = 0.4$ again let t be the fuzzy topology on X generated by $\{v\} \cup \{\text{Constants}\}$, where $v(x) = 1$, $v(y) = 0.5$. For $\alpha = 0.3$ and $\beta = 0.7$ it is clear that the fuzzy bitopological space (X, s, t) is β - FP – T_0*(ii)* but (X, s, t) is not α - FP – T_0*(ii)*.

Further, one can easily verify that

β - FP – T_0*(iii)* $\Rightarrow$ α - FP – T_0*(iii)*.

0 - FP – T_0*(iii)* $\Leftrightarrow$ 0 – FP – T_0*(iv)*.

This completes the proof.

Now, we discuss the 'good extension' property of the definitions of α - FP – T_0 ness given above. All the definitions FP – T_0*(i)*, α - FP – T_0*(ii)*, α - FP – T_0*(iii)*, α - FP – T_0*(iv)* and FP – T_0*(v)* are 'good extension' of P – T_0*(i)* as shown below.

Theorem 3.13. Let (X, *S*, *T*) be a bitopological space. Consider the following statements.

(1) (X, *S*, *T*) be a P – T_0*(i)* space.

(2) (X, ω (*S*), ω (*T*)) be an FP- T_0*(i)* space .

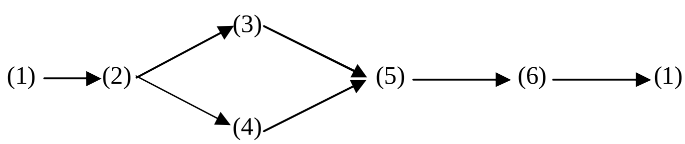
(3) (X, ω (*S*), ω (*T*)) be an α- FP- T_0*(ii)* space .

(4) (X, ω (*S*), ω (*T*)) be an α- FP- T_0*(iii)* space .

(5) (X, ω (*S*), ω (*T*)) be an α- FP- T_0*(iv)* space .

(6) (X, ω (*S*), ω (*T*)) be an FP- T_0*(v)* space .

Then the implication are true;



Proof: Let (X, *S*, *T*) be a P - T_0*(i)* space. We shall prove that (X, ω (*S*), ω (*T*)) be an FP- T_0(i) space . Suppose x, y $\in$ X with x $\neq$ y. Since (X, *S*, *T*) is P – T_0(i) then $\exists$ U $\in$ *S* $\cup$ *T* such that x $\in$ U, y $\notin$ U or x $\notin$ U, y $\in$ U, suppose x $\in$ U, y $\notin$ U but from the definition of lower semi continuous function $1_U \in \omega(S) \cup \omega(T)$ and $1_U(x) = 1$, $1_U(y) = 0$ and hence it is clear that the fuzzy bitopological space (X, ω(*S*), ω(*T*)) is an FP- T_0(i) space.

Further, it is easy to show that (2) $\Rightarrow$ (3), (2) $\Rightarrow$ (4), (3) $\Rightarrow$ (5), (4) $\Rightarrow$ (5) and (5) $\Rightarrow$ (6). We therefore prove that (6) $\Rightarrow$ (1).

Suppose (X, ω(*S*), ω(*T*)) be an FP- T_0(v) space. We shall prove that (X, *S*, *T*) be a P – T_0(i) space. Let x, y $\in$ X with x $\neq$ y. Since (X, ω(*S*), ω(*T*)) be an FP- T_0(v) space then $\exists$ u $\in \omega(S) \cup \omega(T)$ such that u(x) > u(y) or u(x) < u(y), suppose u(x) < u(y) then $\exists$ m $\in I_1$ such that u(x) < m < u(y). Then we have u^{-1}(m, 1]$\in$ *S* $\cup$ *T* and x $\notin u^{-1}$(m, 1] and y $\in u^{-1}$(m, 1]. Hence it is clear that the bitopological space (X, *S*, *T*) is P – T_0*(i)* space.

This completes the proof.

Theorem 3.14. Let (X, *s*, *t*) be a fuzzy bitopological space, and $I_\alpha(s) = \{u^{-1}(\alpha, 1] : u \in s\}$ and $I_\alpha(t) = \{v^{-1}(\alpha, 1] : v \in t\}$ then

(a) (X, s, t) is α - FP – T_0*(ii)* $\Rightarrow$ $(X, I_\alpha(s), I_\alpha(t))$ is P – T_0*(i)*.

(b) (X, s, t) is α - FP – T_0*(iii)* $\Rightarrow$ $(X, I_\alpha(s), I_\alpha(t))$ is P – T_0*(i)*.

(c) (X, s, t) is α - FP – T_0*(iv)* $\Leftrightarrow$ $(X, I_\alpha(s), I_\alpha(t))$ is P – T_0*(i)*.

The reverse implication is not true in general.

Proof: Let (X, s, t) be a α - FP - T_0*(ii)* space. We shall prove that $(X, I_\alpha(s), I_\alpha(t))$ is P – T_0(i) space. Suppose $x, y \in X$ with $x \neq y$. Since (X, s, t) is α - FP – T_0(ii), for $\alpha \in I_1$ then there exist $u \in s \cup t$ such that $u(x) = 1$, $u(y) \leq \alpha$. Again science $u^{-1}(\alpha, 1] \in I_\alpha(s) \cup I_\alpha(t)$ and it is clear that $x \in u^{-1}(\alpha, 1]$, $y \notin u^{-1}(\alpha, 1]$. Hence it is clear that the bitopological space $(X, I_\alpha(s), I_\alpha(t))$ is P – T_0*(i)*.

Next, suppose (X, s, t) be a α - FP - T_0(iii) space. We shall prove that $(X, I_\alpha(s), I_\alpha(t))$ is P – T_0*(i)* space. Let $x, y \in X$ with $x \neq y$. Since (X, s, t) is α - FP – T_0*(iii)*, for $\alpha \in I_1$ then $\exists\, u \in s \cup t$ such that $u(x) = 0$, $u(y) > \alpha$. Again science $u^{-1}(\alpha , 1] \in I_\alpha(s) \cup I_\alpha(t)$ and it is clear that $x \notin u^{-1}(\alpha, 1]$, $y \in u^{-1}(\alpha, 1]$. Hence it is clear that the bitopological space $(X, I_\alpha(s), I_\alpha(t))$ is P – T_0*(i)*.

Example 3.15. Let $X = \{x, y\}$ and $u, v, p, q \in I^X$, where u, v, p, q are defined by $u(x) = 0.9$, $u(y) = 0.1$, $v(x) = 0.2$, $v(y) = 0.8$, $p(x) = 0.2$, $p(y) = 0.5$ and $q(x) = 0.1$, $q(y) = 0.3$. Consider the fuzzy topology s on X generated by $\{p, u\} \cup \{\text{Constants}\}$ and the fuzzy topology t on X generated by $\{q, v\} \cup \{\text{Constants}\}$. For $\alpha = 0.6$, it is clear that the fuzzy bitopological space (X, s, t) is not α - FP –T_0*(ii)* and (X, s, t) is not α - FP –T_0*(iii)*. Now $I_\alpha(s) = \{X, \phi, \{x\}\}$ and $I_\alpha(t) = \{X, \phi, \{y\}\}$. Thus we see that $I_\alpha(s)$ and $I_\alpha(t)$ are topologies on X and $(X, I_\alpha(s), I_\alpha(t))$ is a bitopological space and is also P –T_0*(i)*.

Further, one can easily verify that

$$(X, s, t) \text{ is } \alpha \text{ - FP – } T_0(iv) \Leftrightarrow (X, I_\alpha(s), I_\alpha(t)) \text{ is P – } T_0(i).$$

Theorem 3.16. Let (X, s, t) be a fuzzy bitopological space, $A \subseteq X$ and $s_A = \{u/A : u \in s\}$ and $t_A = \{v/A : v \in t\}$, then

(a) (X, s, t) is FP – T_0*(i)*implies $(A\ s_A, t_A)$ is FP – T_0*(i)*.

(b) (X, s, t) is α - FP – T_0*(ii)*implies (A, s_A, t_A) is α - FP – T_0*(ii)*.

(c) (X, s, t) is α - FP – T_0*(iii)*implies (A, s_A, t_A) is α - FP – T_0*(iii)*.

(d) (X, s, t) is α - FP – T_0*(iv)*implies (A, s_A, t_A) is α - FP – T_0*(iv)*.

(e) (X, s, t) is FP – T_0*(v)*implies (A, s_A, t_A) is α - FP – T_0*(v)*.

Proof: Suppose (X, s, t) be fuzzy bitopological space and (X, s, t) is α - FP – T_0*(ii)*. We shall prove that (A, s_A, t_A) is α - FP – T_0*(ii)*. Let $x, y \in A$ with $x \neq y$, then $x, y \in X$ with $x \neq y$ as $A \subseteq X$. Since (X, s, t) is α - FP – T_0*(ii)*, for $\alpha \in I_1$ then there exist $u \in s \cup t$

such that u(x) = 1, u(y) ≤ α. For A ⊆ X, we find u/A ∈ $(s \cup t)_A$, i.e. u/A ∈ $s_A \cup t_A$ and u/A(x) = 1, u/A(y) ≤ α. Hence it is clear that the fuzzy bitopological space (X, *s*, *t*) is α - FP – T_0*(ii)*.

Similarly *(a)*, *(c)*, *(d)* and *(e)* can be proved.

Hence, we see that FP – T_0*(i)*, α - FP – T_0*(ii)*, α - FP – T_0*(iii)*, α - FP – T_0*(iv)* and FP – T_0*(v)* properties are hereditary.

Theorem 3.17. Given $\{(X_i, s_i, t_i) : i \in \Lambda\}$ be a family of fuzzy bitopological spaces. Then the product of fuzzy bitopological space $(\Pi X_i, \Pi s_i, \Pi t_i)$ is α-FP-T_0(j)iff each coordinate space (X_i, s_i, t_i) is α-FP-T_0(j), where j = *i, ii, iii, iv,v* .

Proof: Let each coordinate space (X_i, s_i, t_i), i ∈ Λ be α - FP – T_0*(ii)*. Then to show that the product space is α - FP – T_0*(ii)*. Suppose x, y ∈ X, x ≠ y again suppose $x = \prod x_i$, $y = \prod y_i$ then $x_j \neq y_j$ for some j∈Λ. Now consider x_j, y_j in X_j. Since (X_j, s_j, t_j) is α - FP – T_0(ii), for $\alpha \in I_1$ then there exist $u_j \in s_j \cup t_j$ such that $u_j(x_j) = 1$, $u_j(y_j) \leq \alpha$ or $u_j(x_j) \leq \alpha$, $u_j(y_j) = 1$. Suppose $u_j(x_j) = 1$, $u_j(y_j) \leq \alpha$. Now take $u = \prod u_j'$, where $u_j' = u_j$ and $u_i = 1$, for i ≠ j. Then u is such that u(x) = 1, u(y) ≤ α. Hence the product fuzzy bitopological space i.e. $(\Pi X_i, \Pi s_i, \Pi t_i)$ is α-FP-T_0(ii).

Conversely, let the product fuzzy bitopological space $(\Pi X_i, \Pi s_i, \Pi t_i)$ is α-FP-T_0(ii). Take any coordinate space (X_j, s_j, t_j), choose $x_j, y_j \in X_j$, $x_j \neq y_j$. Now construct x, y ∈ X such that $x = \prod x_i'$, $y = \prod y_i'$, where $x_i' = y_i'$ for i ≠ j and $x_j' = x_j$, $y_j' = y_j$. Then x ≠ y and hence there exist $u \in \Pi s_i \cup \Pi t_i$ such that u(x) =1, u(y) ≤ α or u(x) ≤ α, u(y) =1, suppose u(x) =1, u(y) ≤ α. Now u must be union ofbasic fuzzy open set say $u = \cup_{k \in K} b_k$. Thus $\cup b_k(x) =1$ and $\cup b_k(y) \leq \alpha$. Which implies that there exist at least one k such that $b_k(x) =1$, $b_k(y) \leq \alpha$. Now let $b_k = \prod v_i$, where $v_i = 1$ except for finitely many i's. So $\prod v_i(x) = 1$, $\prod v_i(y) \leq \alpha$, i.e. inf $v_i(x_i') = 1$ and inf $v_i(y_i') \leq \alpha$, which implies that $v_j(x_j) = 1$, $v_j(y_j) \leq \alpha$. Since $x_i' = y_i'$ for i ≠ j . Thus (X_i, s_i, t_i) is α - FP – T_0(ii).

Moreover, one can easily verify that

(X_i, s_i, t_i), i ∈ Λ is FP – T_0(i) ⇔ $(\Pi X_i , \Pi s_i, \Pi t_i)$ is FP-T_0(i).

(X_i, s_i, t_i), i ∈ Λ is α - FP – T_0(iii) ⇔ $(\Pi X_i, \Pi s_i, \Pi t_i)$ is α-FP-T_0(iii).

(X_i, s_i, t_i), i ∈ Λ is α - FP – T_0(iv) ⇔ $(\Pi X_i, \Pi s_i, \Pi t_i)$ is α-FP-T_0(iv).

(X_i, s_i, t_i), i ∈ Λ is FP – T_0(v) ⇔ $(\Pi X_i, \Pi s_i, \Pi t_i)$ is FP-T_0(v).

Hence we see that FP – T_0(i), α - FP – T_0(ii), α - FP – T_0(iii), α - FP – T_0(iv) and FP – T_0(v) property are productive and projective.

Theorem 3.18. Let (X, *s*, *t*) and (Y, s_1, t_2) be two fuzzy bitopological spaces and $f : X \to Y$ be a one-one , onto and FP- open map then, *(a)* (X, *s*, *t*) is FP – T_0(i) implies (Y, s_1, t_1) is FP – T_0(i).

(b) (X, *s*, *t*) is α - FP – T_0(ii) implies (Y, s_1, t_1) is α - FP – T_0(ii).

(c) (X, *s*, *t*) is α - FP – T_0(iii) implies (Y, s_1, t_1) is α - FP – T_0(iii).

(d) (X, *s*, *t*) is α - FP – T_0(iv) implies (Y, s_1, t_1) is α - FP – T_0(iv).

(e) (X, *s*, *t*) is FP – T_0(v) implies (Y, s_1, t_1) is FP – T_0(v).

Proof: *(b)*Suppose (X, *s*, *t*) be α - FP – T_0(ii). We shall prove that (Y, s_1, t_1) is α - FP – T_0(ii). Let y_1, $y_1 \in$ Y with $y_1 \neq y_2$, since *f* is onto then there exist x_1, $x_2 \in$X such that $f(x_1) = y_1$, $f(x_2) = y_2$ and $x_1 \neq x_2$ as *f* is one-one. Again since (X, *s*, *t*) is α - FP – T_0(ii), for $\alpha \in I_1$ there exist $u \in s \cup t$ such that u(x) = 1, u(y) $\leq \alpha$.

Now $f(u)(y_1) = \{\sup u(x_1) : f(x_1) = y_1\}$

$= 1$

$f(u)(y_2) = \{\sup u(x_2) : f(x_2) = y_2 \leq \alpha\}$

Science *f* is FP – open then $f(u) \in s_1 \cup t_1$.

Now it is clear that there $f(u) \in s_1 \cup t_1$ such that $f(u)(y_1) = 1$, $f(u)(y_2) \leq \alpha$. Hence it is clear that the fuzzy bitopological space (Y, s_1, t_1) is α - FP – T_0(ii).

(a), *(c)*, *(d)* and *(e)* can be proved similarly.

Theorem 3.19. Let (X, *s*, *t*) and (Y, s_1, t_2) be two fuzzy bitopological spaces and $f : X \to Y$ be FP- continuous and one-one map then,

(a) (Y, s_1, t_2) is FP- T_0(i) implies (X, *s*, *t*) is FP- T_0(i).

(b) (Y, s_1, t_2) is α - FP- T_0(ii) implies (X, *s*, *t*) is α - FP- T_0(ii).

(c) (Y, s_1, t_2) is α - FP- T_0(iii) implies (X, *s*, *t*) is α - FP- T_0(iii).

(d) (Y, s_1, t_2) is α - FP- T_0(iv) implies (X, *s*, *t*) is α - FP- T_0(iv).

(e) (Y, s_1, t_2) is FP- T_0(v) implies (X, *s*, *t*) is FP- T_0(v).

Proof:*(b)*Suppose (Y, s_1, t_2) is α - FP- T_0(ii). We shall prove that (X, *s*, *t*) is α - FP- T_0(ii). Let x_1, $x_2 \in$ X with $x_1 \neq x_2$, $\Rightarrow f(x_1) \neq f(x_2)$ as *f* is one-one. Since (Y, s_1, t_2) is α - FP- T_0(ii), for $\alpha \in I_1$ then there exist $u \in s_1 \cup t_1$ such that $u(f(x_1)) = 1$, $u(f(x_2)) \leq \alpha$ or$u(f(x_1)) \leq 1$, $u(f(x_2)) = 1$, suppose $u(f(x_1)) = 1$, $u(f(x_2)) \leq \alpha$. This implies that $f^{-1}(u)(x_1) = 1$, $f^{-1}(u)(x_2) \leq \alpha$ and $f^{-1}(u) \in s \cup t$ as *f* is FP – continuous and $u \in s_1 \cup t_1$. Now it is clear that $\exists f^{-1}(u) \in s \cup t$ such that $f^{-1}(u)(x_1) = 1$, $f^{-1}(u)(x_2) \leq \alpha$. Hence the fuzzy bitopologica space (X, *s*, *t*) is α - FP- T_0(ii).

(a), *(c)*, *(d)* and *(e)* can be proved similarly.

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