SOME FEATURES OF $\alpha$-T$_0$ SPACES IN SUPRA FUZZY TOPOLOGY

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ABSTRACT

Four concepts of T$_0$ supra fuzzy topological spaces are introduced and studied in this paper. The workers also established some relationships among them and studied some other properties of these spaces.

Key words: Fuzzy topology, Supra fuzzy topology

INTRODUCTION

The fundamental concept of a fuzzy set was introduced by Zadeh (1965) to provide a foundation for the development of many areas of knowledge. Chang (1968) and Lowen (1976) developed the theory of fuzzy topological spaces using fuzzy sets. Mashhour et al. (1983) introduced supra topological spaces and studied s-continuous functions and s’-continuous functions. They also gave the concept of $\alpha$-T$_0$ fuzzy topological spaces. In 1987, Abd EL-Monsef et al. introduced the fuzzy supra topological spaces and studied fuzzy supra continuous functions and characterized a number of basic concepts. Ali (1993) made some remarks on $\alpha$-T$_0$, $\alpha$-T$_1$ and $\alpha$-T$_2$ fuzzy topological spaces. In this paper, the present workers studies some features of $\alpha$-T$_0$ spaces and obtained certain characterizations in supra fuzzy topological spaces. As usual I = [0, 1] and I$_1$ = [0, 1).

Definition: For a set X, a function $u : X \rightarrow [0,1]$ is called a fuzzy set in X. For every $x \in X$, $u(x)$ represents the grade of membership of x in the fuzzy set u. Some authors say that u is a fuzzy subset of X. Thus a usual subset of X, is a special type of a fuzzy set in which the range of the function is {0, 1} (Zadeh 1965).

Definition: Let X be a nonempty set and A be a subset of X. The function $1_A : X \rightarrow [0,1]$ defined by $1_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$ is called the characteristic function of A. The present authors also write $1_x$ for the characteristic function of {x}. The characteristic functions of subsets of a set X are referred to as the crisp sets in X (Zadeh 1965).

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Example: Suppose $X$ is real number $\mathbb{R}$ and the fuzzy set of real numbers much greater than 5 in $X$ that could be defined by the continuous function $U: X \rightarrow [0,1]$ such that

$$u(x) = \begin{cases} 
0 & \text{if } x \leq 5 \\
\frac{x - 5}{50} & \text{if } 5 < x < 55 \\
1 & \text{if } x \geq 55 
\end{cases}$$

Definition: Let $X$ be a non empty set and $t$ be the collection of fuzzy sets in $I^X$. Then $t$ is called a fuzzy topology on $X$ if it satisfies the following conditions:

(i) $1, 0 \in t$,
(ii) If $u_i \in t$ for each $i \in \Lambda$, then $\bigcup_{i \in \Lambda} u_i \in t$.
(iii) If $u_1, u_2 \in t$ then $u_1 \cap u_2 \in t$.

If $t$ is a fuzzy topology on $X$, then the pair $(X, t)$ is called a fuzzy topological space (fts, in short) and members of $t$ are called t-open (or simply open) fuzzy sets. If $u$ is open fuzzy set, then the fuzzy sets of the form $1-u$ are called t-closed (or simply closed) fuzzy sets (Chang 1968).

Definition: Let $X$ be a non empty set and $t$ be a collection of fuzzy sets in $I^X$ such that

(i) $1, 0 \in t$,
(ii) If $u_i \in t$ for each $i \in \Lambda$, then $\bigcup_{i \in \Lambda} u_i \in t$.
(iii) If $u_1, u_2 \in t$ then $u_1 \cap u_2 \in t$.
(iv) All constant fuzzy sets in $X$ belong to $t$.

Then $t$ is called a fuzzy topology on $X$ (Lowen 1976).

Definition: Let $X$ be a non empty set. A subfamily $t^*$ of $I^X$ is said to be a supra topology on $X$ if and only if

(i) $1, 0 \in t^*$,
(ii) If $u_i \in t^*$ for each $i \in \Lambda$, then $\bigcup_{i \in \Lambda} u_i \in t^*$.

Then the pair $(X, t^*)$ is called a supra fuzzy topological spaces. The elements of $t^*$ are called supra open sets in $(X, t^*)$ and complement of supra open set is called supra closed set (Mashhour et al. 1983).

Example: Let $X = \{x, y\}$ and $u, v \in I^X$ are defined by $u(x) = .8, u(y) = .6$ and $v(x) = .6, v(y) = .8$. Then we have $w(x) = (u \cup v)(x) = .8, \ w(y) = (u \cap v)(y) = .8$ and $k(x) = .
Cartesian product $u \times Y$ respectively, then the topology associated with $T$ is a supra fuzzy topology on $X \times Y$ if and only if every fuzzy topology on $X \times Y$ is supra fuzzy topology but the converse is not always true.

**Definition:** Let $(X, t)$ and $(X, s)$ be two topological spaces. Let $t'$ and $s'$ be associated supra topologies with $t$ and $s$, respectively, and $f : (X, t') \rightarrow (Y, s')$ be a function. Then the function $f$ is a supra fuzzy continuous function if the inverse image of each supra-open subset $V$ of $Y$ is a supra-open subset of $X$, i.e., if for any $v \in s', f^{-1}(v) \in t'$. The function $f$ is called supra fuzzy homeomorphic if and only if $f$ is supra bijective and both $f$ and $f^{-1}$ are supra fuzzy continuous (Mashhour et al. 1983).

**Definition:** Let $(X, t')$ and $(Y, s')$ be two supra topological spaces. If $U_1$ and $U_2$ are two supra fuzzy subsets of $X$ and $Y$, respectively, then the Cartesian product $U_1 \times U_2$ is a supra fuzzy subset of $X \times Y$ defined by $(u_1 \times u_2)(x, y) = \min [u_1(x), u_2(y)]$, for each pair $(x, y) \in X \times Y$ (Azad 1981).

**Definition:** Suppose $\{X_i, i \in \Lambda\}$ be any collection of sets and $X$ denote the Cartesian product of these sets, i.e., $X = \Pi_{i \in \Lambda} X_i$. Here $X$ consists of all points $p = \langle a_i, i \in \Lambda \rangle$, where $a_i \in X_i$. For each $j_0 \in \Lambda$, the authors defined the projection $\pi_{j_0} : X \rightarrow X_{j_0}$ by $\pi_{j_0}(\langle a_i, i \in \Lambda \rangle) = a_{j_0}$. These projections are used to define the product supra topology (Wong 1974).

**Definition:** Let $\{X_\alpha\}_{\alpha \in \Lambda}$ be a family of nonempty sets. Let $X = \Pi_{\alpha \in \Lambda} X_\alpha$ be the usual product of $X_\alpha$'s and let $\pi_\alpha : X \rightarrow X_\alpha$ be the projection. Further, assume that each $X_\alpha$ is a supra fuzzy topological space with supra fuzzy topology $t'_\alpha$. Now the supra fuzzy topology generated by $\{\pi_i^{-1}(b_\alpha) : b_\alpha \in t'_\alpha, \alpha \in \Lambda\}$ as a sub basis, is called the product supra fuzzy topology on $X$. Thus if $w$ is a basis element in the product, then there exist $\alpha_1, \alpha_2, \ldots, \alpha_n \in \Lambda$ such that $w(x) = \min \{b_\alpha(x) : \alpha = 1, 2, 3, \ldots, n\}$, where $x = (x_\alpha)_{\alpha \in \Lambda} \in X$ (Wong 1974).

**Definition:** Let $(X, T)$ be a topological space and $T'$ be associated supra topology with $T$. Then a function $f : X \rightarrow R$ is lower semi continuous if and only if $\{x \in X : f(x) > \alpha\}$ is open for all $\alpha \in R$ (Abd EL-Monsef et al. 1987).

**Definition:** Let $(X, T)$ be a topological space and $T'$ be associated supra topology with $T$. Then the lower semi continuous topology on $X$ associated with $T$ is $\omega(T') = \{\mu : X \rightarrow [0, 1], \mu \text{ is supra lsc}\}$. If $\omega(T') : (X, T') \rightarrow [0, 1]$ be the set of all lower semi continuous (lsc) functions. We can easily show that $\omega(T')$ is a supra fuzzy topology on $X$ (Ming et al. 1980).

Let $P$ be the property of a supra topological space $(X, T')$ and $FP$ be its supra fuzzy topological analogue. Then $FP$ is called a ‘good extension’ of $P’ “if and only if the statement $(X, T')$ has $P$ if and only if $(X, \omega(T'))$ has $FP’ holds good for every supra topological space $(X, T')$. 

(u \cup v)(x) = .6, \ k(y) = (u \cup v)(y) = .6. If we consider $t'$ on $X$ generated by $\{0, u, v, w, 1\}$, then $t'$ is supra fuzzy topology on $X$ but $t'$ is not fuzzy topology. Thus we see that every fuzzy topology is supra fuzzy topology but the converse is not always true.
Definition: A fuzzy topological space \((X, t)\) is said to be fuzzy \(T_0\) if and only if (i) for all \(x, y \in X\) with \(x \neq y\), there exists \(u \in t\) such that \(u(x) = 1, u(y) = 0\) or \(u(x) = 0, u(y) = 1\), (ii) for all \(x, y \in X\) with \(x \neq y\), there exists \(u \in t\) such that \(u(x) < u(y)\) or \(u(y) < u(x)\) (Ali 1987).

\(\alpha-T_0(I), \alpha-T_0(II), \alpha-T_0(III)\) AND \(T_0(IV)\) SPACES IN SUPRA FUZZY TOPOLOGY

Definition: Let \((X, t)\) be a fuzzy topological space and \(t^*\) be associated supra topology with \(t\) and \(\alpha \in I_1\). Then

(a) \((X, t^*)\) is an \(\alpha - T_0\) (i) space if and only if for all distinct elements \(x, y \in X\), there exists \(u \in t^*\) such that \(u(x) = 1, u(y) \leq \alpha\) or there exists \(v \in t^*\) such that \(v(x) \leq \alpha, v(y) = 1\).

(b) \((X, t^*)\) is an \(\alpha - T_0\) (ii) space if and only if for all distinct elements \(x, y \in X\), there exists \(u \in t^*\) such that \(u(x) = 0, u(y) > \alpha\) or there exists \(v \in t^*\) such that \(v(x) > \alpha, v(y) = 0\).

(c) \((X, t^*)\) is an \(\alpha - T_0\) (iii) space if and only if for all distinct elements \(x, y \in X\), there exists \(u \in t^*\) such that \(0 \leq u(x) \leq \alpha < u(y) \leq 1\) or there exists \(v \in t^*\) such that \(0 \leq v(y) \leq \alpha < v(x) \leq 1\).

(d) \((X, t^*)\) is a \(T_0\) (iv) space if and only if for all distinct elements \(x, y \in X\), there exists \(u \in t^*\) such that \(u(x) \neq u(y)\).

Lemma: Suppose \((X, t)\) is a topological space and \(t^*\) is associated supra topology with \(t\) and \(\alpha \in I_1\). Then the following implications are true:

(a) \((X, t^*)\) is an \(\alpha - T_0\) (i) implies \((X, t^*)\) is an \(\alpha - T_0\) (iii) implies \((X, t^*)\) is \(T_0\) (iv).

(b) \((X, t^*)\) is an \(\alpha - T_0\) (ii) implies \((X, t^*)\) is \(\alpha - T_0\) (iii) implies \((X, t^*)\) is \(T_0\) (iv).

Also, these can be shown in a diagram as follows:

[Diagram showing the implications]

Proof: Suppose that \((X, t^*)\) is an \(\alpha - T_0\) (i). Let \(x\) and \(y\) be any two distinct elements in \(X\). Since \((X, t^*)\) is an \(\alpha - T_0\) (i) for \(\alpha \in I_1\), by definition, there exists \(u \in t^*\) such that \(u(x) = 1, u(y) \leq \alpha\) which shows that \(0 \leq u(y) \leq \alpha < u(x) \leq 1\). Hence by definition (c), \((X, t^*)\) is an \(\alpha - T_0\) (iii).

Suppose \((X, t^*)\) is an \(\alpha - T_0\) (iii). Then, for \(x, y \in X\) with \(x \neq y\), there exist \(u \in t^*\) such that \(0 \leq u(x) \leq \alpha < u(y) \leq 1\), i.e., \(u(x) \neq u(y)\), hence by definition, \((X, t^*)\) is an \(\alpha - T_0\) (iv).
Let \((X, t')\) is \(\alpha\)-\(T_0\) (ii). Then, for \(x, y \in X\) with \(x \neq y\), there exists \(u \in t'\) such that \(u(x) = 0\) and \(u(y) \geq \alpha\), which implies \(0 \leq u(x) \leq \alpha < u(y) \leq 1\). Hence, by definition, \((X, t')\) is \(\alpha\)-\(T_0\) (iii) and hence \((X, t')\) is \(\alpha\)-\(T_0\) (iv). Therefore, the proof is complete.

The non-implications among \(\alpha\)-\(T_0\) (i), \(\alpha\)-\(T_0\) (ii), \(\alpha\)-\(T_0\) (iii) and \(T_0\) (iv) are shown in the following examples, i.e., the following examples show that:

(a) \(T_0\) (iv) does not imply \(\alpha\)-\(T_0\) (iii), so, not imply \(\alpha\)-\(T_0\) (i) and \(\alpha\)-\(T_0\) (ii).

(b) \(\alpha\)-\(T_0\) (iii) does not imply \(\alpha\)-\(T_0\) (i) and \(\alpha\)-\(T_0\) (ii).

(c) \(\alpha\)-\(T_0\) (i) does not imply \(\alpha\)-\(T_0\) (ii).

(d) \(\alpha\)-\(T_0\) (ii) does not imply \(\alpha\)-\(T_0\) (i).

Example: Let \(X = \{x, y\}\) and \(u \in t^X\) is defined by \(u(x) = 0.4\), \(u(y) = 0.7\). Let the supra fuzzy topology \(t'\) on \(X\) generated by \([0, u, 1, \text{Constants}]\). Then for \(\alpha = 0.8\), we can easily show that \((X, t')\) is \(T_0\) (iv) but \((X, t')\) is not \(\alpha\)-\(T_0\) (iii), so, not \(\alpha\)-\(T_0\) (i) and \(\alpha\)-\(T_0\) (ii).

Example: Let \(X = \{x, y\}\) and \(u \in t^X\) be defined by \(u(x) = 0.5\), \(u(y) = 0.9\). Let the supra fuzzy topology \(t^*\) on \(X\) generated by \([0, u, 1, \text{Constants}]\). For \(\alpha = 0.7\), we have \(0 \leq u(x) \leq 0.7 < u(y) \leq 1\). Thus according to the definition, \((X, t^*)\) is \(\alpha\)-\(T_0\) (iii) but \((X, t^*)\) is not \(\alpha\)-\(T_0\) (i). Also it can be easily shown that \((X, t')\) is not \(\alpha\)-\(T_0\) (ii).

Example: Let \(X = \{x, y\}\) and \(u \in t^X\) be defined by \(u(x) = 1\), \(u(y) = 0.5\). Consider the supra fuzzy topology \(t^*\) on \(X\) generated by \([0, u, 1, \text{Constants}]\). For \(\alpha = 0.7\), we have \(u(x) = 1\) and \(u(y) \leq \alpha\). Thus according to the definition \((X, t^*)\) is \(\alpha\)-\(T_0\) (i) but \((X, t^*)\) is not \(\alpha\)-\(T_0\) (ii).

Example: Let \(X = \{x, y\}\) and \(u \in t^X\) be defined by \(u(x) = 0\), \(u(y) = 0.8\). Let the supra fuzzy topology \(t^*\) on \(X\) generated by \([0, u, 1, \text{Constants}]\). For \(\alpha = 0.4\), it can easily show that \((X, t^*)\) is \(\alpha\)-\(T_0\) (ii) but \((X, t^*)\) is not \(\alpha\)-\(T_0\) (i). This completes the proof.

Lemma: Let \((X, t')\) be a supra fuzzy topological space and \(\alpha, \beta \in t'\) with \(0 \leq \alpha \leq \beta < 1\), then

(a) \((X, t')\) is \(\alpha\)-\(T_0\) (i) implies \((X, t')\) is \(\beta\)-\(T_0\) (i).

(b) \((X, t')\) is \(\beta\)-\(T_0\) (ii) implies \((X, t')\) is \(\alpha\)-\(T_0\) (ii).

(c) \((X, t')\) is \(0\)-\(T_0\) (ii) if and only if \((X, t')\) is \(\alpha\)-\(T_0\) (iii).

Proof: Suppose that \((X, t')\) is a supra fuzzy topological space and \((X, t')\) is \(\alpha\)-\(T_0\) (i). We have to show that \((X, t')\) is \(\beta\)-\(T_0\) (i). Let any two distinct elements \(x, y \in X\). Since \((X, t')\) is \(\alpha\)-\(T_0\) (i), for \(\alpha \in I_1\), there is \(u \in t'\) such that \(u(x) = 1\), and \(u(y) \leq \alpha\). This implies that \(u(x) = 1\), and \(u(y) \leq \beta\), since \(0 \leq \alpha \leq \beta < 1\). Hence by definition, \((X, t')\) is \(\beta\)-\(T_0\) (i).
Suppose that \((X, t')\) is \(\beta - T_0\) (ii). Then, for \(x, y \in X\) with \(x \neq y\), there exist \(u \in t'\) such that \(u(x) = 0\) and \(u(y) > \beta\), which implies \(u(x) = 0\) and \(u(y) > \alpha\), since \(0 \leq \alpha \leq \beta < 1\). Hence we have \((X, t')\) is \(\alpha - T_0\) (ii).

**Example:** Let \(X = \{x, y\}\) and \(u \in I^X\) be defined by \(u(x) = 1, u(y) = 0.6\). Let the supra fuzzy topology \(t^*\) on \(X\) generated by \(\{0, u, 1, \text{Constants}\}\). Then by definition, for \(\alpha = 0.5\) and \(\beta = 0.8\); \((X, t^*)\) is \(\beta - T_0\) (i) but \((X, t')\) is not \(\alpha - T_0\) (i).

**Example:** Let \(X = \{x, y\}\) and \(u \in I^X\) be defined by \(u(x) = 0, u(y) = 0.65\). Let the supra fuzzy topology \(t^*\) on \(X\) generated by \(\{0, u, 1, \text{Constants}\}\). Then by definition, for \(\alpha = 0.45\) and \(\beta = 0.75\); \((X, t^*)\) is \(\alpha - T_0\) (ii) but \((X, t')\) is not \(\beta - T_0\) (ii).

In the same way, it can be proved that \((X, t')\) is \(0 - T_0\) (ii) if and only if \((X, t^*)\) is \(0 - T_0\) (iii).

**Theorem:** Let \((X, T)\) be a topological space and \(T^*\) be associated supra topology with \(T\) and \(\alpha \in I_1\). Suppose that the following statements:

1. \((X, T^*)\) be a \(T_0\) – space.
2. \((X, \omega (T^*))\) be an \(\alpha - T_0\) (i) space.
3. \((X, \omega (T^*))\) be an \(\alpha - T_0\) (ii) space.
4. \((X, \omega (T^*))\) be an \(\alpha - T_0\) (iii) space.
5. \((X, \omega (T^*))\) be a \(T_0\) (iv) space.

Then the following implications are true:

(a) \((1) \Rightarrow (2) \Rightarrow (4) \Rightarrow (5) \Rightarrow (1)\).

(b) \((1) \Rightarrow (3) \Rightarrow (4) \Rightarrow (5) \Rightarrow (1)\).

**Proof:** Suppose \((X, T^*)\) is a \(T_0 – \) topological space. We have to prove that \((X, \omega (T^*))\) is \(\alpha - T_0\) (i) space. Suppose \(x\) and \(y\) are two distinct elements in \(X\). Since \((X, T^*)\) is \(T_0\) there is \(U \in T^*\) such that \(x \in U, y \notin U\). By the definition of \(\text{lsc}\), we have \(1_U \in \omega(T^*)\) and \(1_U(x) = 1, 1_U(y) = 0\). Hence we have \((X, \omega (T^*))\) is \(\alpha - T_0\) (i) space. Also we have \((X, \omega (T^*))\) is \(\alpha - T_0\) (ii) space. Further, it is easy to show that \((2) \Rightarrow (4), (3) \Rightarrow (4)\) and \((4) \Rightarrow (5)\). We therefore prove that \((4) \Rightarrow (1)\).

Suppose \((X, \omega (T^*))\) be a \(T_0\) (iv) space. We have to prove that \((X, T^*)\) is \(T_0 – \) space. Let \(x, y \in X\) with \(x \neq y\). Since \((X, \omega (T^*))\) is \(T_0\) (iv), there is \(u \in \omega(T^*)\) such that \(u(x) < u(y)\) or \(u(x) = u(y)\). Suppose \(u(x) < u(y)\). Then for \(r \in I_1\), such that \(u(x) < r < u(y)\). We observe that \(x \notin u^{-1}(r, 1]\), \(y \in u^{-1}(r, 1]\), and by definition of \(\text{lsc}\), \(u^{-1}(r, 1] \in T^*\). Hence \((X, T^*)\) is \(T_0 – \) space.

Thus it is seen that \(\alpha – T_0\) (p) is a good extension of its topological counter part (p = i, ii, iii, iv)
Theorem: Let \((X, t')\) be a supra fuzzy topological space, \(\alpha \in I_t\) and \(I_d(t') = \{u^{-1}(\alpha, 1) : u \in t'\}\) then

(a) \((X, t')\) is an \(\alpha-T_0\) (i) implies \((X, I_d(t'))\) is \(T_0\).
(b) \((X, t')\) is an \(\alpha-T_0\) (ii) implies \((X, I_d(t'))\) is \(T_0\).
(c) \((X, t')\) is an \(\alpha-T_0\) (iii) if and only if \((X, I_d(t'))\) is \(T_0\).

Proof: (a) Let \((X, t')\) be a supra fuzzy topological space and \((X, t')\) be \(\alpha - T_0\) (i).
Suppose \(x\) and \(y\) be any two distinct elements in \(X\). Then, for \(\alpha \in I_t\), there exists \(u \in t'\) such that \(u(x) = 1\), \(u(y) \leq \alpha\). Since \(u^{-1}(\alpha, 1) \in I_d(t')\), \(y \not\in u^{-1}(\alpha, 1)\) and \(x \in u^{-1}(\alpha, 1)\), we have that \((X, I_d(t'))\) is \(T_0\) space. Similarly, (b) can be proved.

(c) Suppose that \((X, t')\) is \(\alpha - T_0\) (iii). Let \(x, y \in X\) with \(x \neq y\), then for \(\alpha \in I_t\), there exists \(u \in t'\) such that \(0 \leq u(x) \leq \alpha < u(y) \leq 1\). Since \(u^{-1}(\alpha, 1) \in I_d(t')\), \(y \not\in u^{-1}(\alpha, 1)\) and \(x \in u^{-1}(\alpha, 1)\), so these implies \((X, I_d(t'))\) is \(T_0\) space.

Conversely, suppose that \((X, I_d(t'))\) be \(T_0\) space. Let \(x, y \in X\) with \(x \neq y\), then there exists \(u^{-1}(\alpha, 1) \in I_d(t')\) such that \(x \in u^{-1}(\alpha, 1)\) and \(y \not\in u^{-1}(\alpha, 1)\), where \(u \in t'\). Thus, we have \(u(x) > \alpha, u(y) \leq \alpha\), i.e., \(0 \leq u(y) \leq \alpha < u(x) \leq 1\), and hence by definition, \((X, t')\) is \(\alpha - T_0\) (iii) space.

Example: Let \(X = \{x, y\}\) and \(u \in t^p\) be defined by \(u(x) = 0.7, u(y) = 0\). Suppose the supra fuzzy topology \(t'\) on \(X\) generated by \{0, 1, Constants\}. Then by definition, for \(\alpha = 0.5\), \((X, t')\) is not \(\alpha-T_0\) (i) and \((X, t')\) is not \(\alpha - T_0\) (ii). Now \(I_d(t') = \{X, \emptyset, \{x\}\}\). Then we see that \(I_d(t')\) is a supra topology on \(X\) and \((X, I_d(t'))\) is a \(T_0\) space. This completes the proof.

Theorem: Let \((X, t')\) be a supra fuzzy topological space, \(A \subseteq X\) and \(t'_{A} = \{u \cap A : u \in t'\}\), then

(a) \((X, t')\) is \(\alpha - T_0\) (i) implies \((A, t'_{A})\) is \(\alpha - T_0\) (i).
(b) \((X, t')\) is \(\alpha - T_0\) (ii) implies \((A, t'_{A})\) is \(\alpha - T_0\) (ii).
(c) \((X, t')\) is \(\alpha - T_0\) (iii) implies \((A, t'_{A})\) is \(\alpha - T_0\) (iii).
(d) \((X, t')\) is \(T_0\) (iv) implies \((A, t'_{A})\) is \(T_0\) (iv).

Proof: (c) Suppose that \((X, t')\) is \(\alpha - T_0\) (iii). Let \(x, y \in A\) with \(x \neq y\), so that \(x, y \in X\), as \(A \subseteq X\). Then, for \(\alpha \in I_t\), there exists \(u \in t'\) such that \(0 \leq u(x) \leq \alpha < u(y) \leq 1\). For \(A \subseteq X\), we have \(u \cap A \in t'_{A}\) and \(0 \leq (u \cap A)(x) \leq \alpha < (u \cap A)(y) \leq 1\) as \(x, y \in A\). Hence, by definition, \((A, t'_{A})\) is \(\alpha - T_0\) (iii).

Similarly, we can prove (a), (b) and (d).

Theorem: Suppose that \((X_i, t'_i), i \in \Lambda\) be supra fuzzy topological spaces and \(X = \prod_{i \in \Lambda} X_i\) and \(t'\) be the product supra fuzzy topology on \(X\), then
(a) \( \forall i \in \Lambda, (X_i, t_i') \) is \( \alpha - T_0(i) \) if and only if \( (X, t') \) is \( \alpha - T_0(i) \).

(b) \( \forall i \in \Lambda, (X_i, t_i') \) is \( \alpha - T_0(ii) \) if and only if \( (X, t') \) is \( \alpha - T_0(ii) \).

(c) \( \forall i \in \Lambda, (X_i, t_i') \) is \( \alpha - T_0(iii) \) if and only if \( (X, t') \) is \( \alpha - T_0(iii) \).

(d) \( \forall i \in \Lambda, (X_i, t_i') \) is \( \alpha - T_0(iv) \) if and only if \( (X, t') \) is \( T_0(iv) \).

**Proof:** (a) Suppose that \( \forall i \in \Lambda, (X_i, t_i') \) is \( \alpha - T_0(i) \). Let \( x, y \in X \) with \( x \neq y \), then \( x \neq y_i \) for some \( i \in \Lambda \). Then for \( \alpha \in I_i \), there exists \( u_i \in t_i' \) such that \( u_i(x_i) = 1 \) and \( u_i(y_j) \leq \alpha \). But we have \( \pi_i(x) = x_i \) and \( \pi_i(y) = y_j \). Thus \( u_i(\pi_i(x)) = 1 \) and \( u_i(\pi_i(y)) \leq \alpha \), i.e., \( (u_i \circ \pi_i)(x) = 1 \) and \( (u_i \circ \pi_i)(y) \leq \alpha \). It follows that there exists \( (u_i \circ \pi_i) \in t' \) such that \( (u_i \circ \pi_i)(x) = 1 \), \( (u_i \circ \pi_i)(y) \leq \alpha \). Hence by definition, \( (X, t') \) is \( \alpha - T_0(i) \).

Conversely, suppose that \( (X, t') \) is \( \alpha - T_0(i) \) space. We have to show that \( (X_i, t_i') \) \( i \in \Lambda \) is \( \alpha - T_0(i) \). Let \( a_i \) be a fixed element in \( X_i \) and \( A_i = \{ x \in X = \Pi_i \in \Lambda X_i : x_j = a_j \text{ for some } i \neq j \} \). Thus \( A_i \) is a subset of \( X \) and hence \( (A_i, t_i') \) is also a subspace of \( (X, t') \).

Since \( (X, t') \) is \( \alpha - T_0(i) \), \( (A_i, t_i') \) is also \( \alpha - T_0(i) \). Now we have \( A_i \) is homeomorphic image of \( X_i \). Thus, we have \( (X_i, t_i') \). \( i \in \Lambda \) is \( \alpha - T_0(i) \).

Similarly, (b), (c) and (d) can be proved.

**Theorem:** Let \( (X, t') \) and \( (Y, s') \) be two supra fuzzy topological spaces and \( f : X \to Y \) be a one-one, onto and open map, then

(a) \( (X, t') \) is \( \alpha - T_0(i) \) implies \( (Y, s') \) is \( \alpha - T_0(i) \).

(b) \( (X, t') \) is \( \alpha - T_0(ii) \) implies \( (Y, s') \) is \( \alpha - T_0(ii) \).

(c) \( (X, t') \) is \( \alpha - T_0(iii) \) implies \( (Y, s') \) is \( \alpha - T_0(iii) \).

(d) \( (X, t') \) is \( T_0(iv) \) implies \( (Y, s') \) is \( T_0(iv) \).

**Proof:** (b) Suppose \( (X, t') \) is \( \alpha - T_0(ii) \). Then for \( y_1, y_2 \in Y \) with \( y_1 \neq y_2 \), there exist \( x_1, x_2 \in X \) with \( f(x_1) = y_1 \), \( f(x_2) = y_2 \), since \( f \) is onto, and thus \( x_1 \neq x_2 \) as \( f \) is one-one. Again, since \( (X, t') \) is \( \alpha - T_0(ii) \), for \( \alpha \in I_i \), there exists \( u \in t' \) such that \( u(x) = 0 \), \( u(y) > \alpha \).

Now, \( f(u)(y_1) = \{ \text{ Sup } u(x_i) : f(x_i) = y_1 \} \)

\( = 0 \), otherwise.

\( f(u)(y_2) = \{ \text{ Sup } u(x_i) : f(x_i) = y_2 \} \)

\( > \alpha \), otherwise.

Since \( f \) is open, \( f(u) \in s' \) as \( u \in t' \). We observe that there exists \( f(u) \in s' \) such that \( f(u)(y_1) = 0 \), \( f(u)(y_2) > \alpha \). Hence by definition, \( (Y, s') \) is \( \alpha - T_0(ii) \).

Similarly, (a), (c) and (d) can be proved.

**Theorem:** Let \( (X, t') \) and \( (Y, s') \) be two supra fuzzy topological spaces and \( f : X \to Y \) be continuous and one-one map, then
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(a) $(Y, s^*)$ is $\alpha$-T$_0$(i) implies $(X, t^*)$ is $\alpha$-T$_0$(i).
(b) $(Y, s^*)$ is $\alpha$-T$_0$(ii) implies $(X, t^*)$ is $\alpha$-T$_0$(ii).
(c) $(Y, s^*)$ is $\alpha$-T$_0$(iii) implies $(X, t^*)$ is $\alpha$-T$_0$(iii).
(d) $(Y, s^*)$ is T$_0$(iv) implies $(X, t^*)$ is T$_0$(iv).

Proof: (c) Suppose $(Y, s^*)$ be $\alpha$-T$_0$(iii). Then, for $x_1, x_2 \in X$ with $x_1 \neq x_2$, we have, $f(x_1) \neq f(x_2)$ in Y, since $f$ is one-one. Also, since $(Y, s^*)$ is $\alpha$-T$_0$(iii), for $\alpha \in I_1$, there exists $u \in s^*$ such that $0 \leq u(f(x_1)) \leq \alpha < u(f(x_2)) \leq 1$. This implies that $0 \leq f^{-1}(u)(x_1) \leq \alpha < f^{-1}(u)(x_2) \leq 1$, since $u \in s^*$ and $f$ is continuous, then $f^{-1}(u) \in t^*$. Thus, there is an $f^{-1}(u) \in t^*$ such that $0 \leq f^{-1}(u)(x_1) \leq \alpha < f^{-1}(u)(x_2) \leq 1$. Hence, by definition, $(X, t^*)$ is $\alpha$-T$_0$(iii).

Similarly, (a), (b) and (d) can be proved.

REFERENCES

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