Some Approaches to the Analysis of a Group of Repeated Measurements Experiment on Mahogany Tree with Heteroscedustic Model

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ABSTRACT

Combined analysis of a group of repeated measurements experiments could play an important role in both the causes either the variance might be known or unknown. Ordinarily the variances vary from experiment to experiment depending on places and environmental conditions due to great irreconcilable inherent causes. On the other hand the variances are obviously related as a functional form of the respective error variances in different places. Relying on the functional form some tests criterion are proposed for removing any resulting bias.

Key words: Repeated measurements experiments, divergency and convergency, percentage rotatability.

INTRODUCTION

The analysis of repeated measurements experiments is a powerful experimental test procedure in the field of agricultural, biological and clinical research (Rahman, 1989; Madsen, 1977; Lana & Lubin, 1963). However, in all the cases they used in a single repeated measurement experiment. Rahman and Miah (1992) used combined analysis in more than one experimental design. They considered the variance-covariance structure among the different experiments are equal. Nevertheless, when the experiments are conducted in different places the variance and covariance matrix are not equal. In that case the analytical procedure is very much complicated. In this paper, an analytical procedure to overcome the problem through an example on Mahogany trees has been proposed.

MATERIALS AND METHODS

A repeated measurements experiment is that which has treatment structure, at least one treatment factor is not randomly assigned having more than one different size of experimental units. The required repeated measurements model can be expressed as:

\[ X_{igpt} = \mu + \alpha_g + \beta_p + (\alpha \beta)_{g} + e_{igp} + \tau_t + (\alpha \tau)_{gt} + (\beta \tau)_{pt} + (\alpha \beta \tau)_{gpt} + \epsilon_{igpt} \]

i = 1,2, ........................................ n
\[ g = 1,2, ........................................ G \]
\[ p = 1,2, ......................................... P \]
\[ t = 1,2, ......................................... T \]
Where,

\[ X_{igpt} = \text{the responses of the } i\text{th individual in the } g\text{th group on } p\text{th place at the } t\text{th occasion.} \]

\[ \mu = \text{the general mean} \]

\[ \alpha_g = \text{the effect of the } g\text{th group} \]

\[ \beta_p = \text{the effect of the } p\text{th place} \]

\[ (\alpha \beta)_{gp} = \text{the interaction effect between the } g\text{th group at the } p\text{th place} \]

\[ \varepsilon_{igp} = \text{the effect associated with the } i\text{th subject in the } g\text{th group on the } p\text{th place} \]

\[ \tau_t = \text{the effect of the } t\text{th occasion} \]

\[ (\alpha \tau)_{gt} = \text{the interaction effect between the } g\text{th group at the } t\text{th occasion} \]

\[ (\beta \tau)_{pt} = \text{the interaction effect between the } p\text{th place at the } t\text{th occasion} \]

\[ (\alpha \beta \tau)_{gpt} = \text{the interaction effect among the } g\text{th group in the } p\text{th place at the } t\text{th occasion} \]

\[ \varepsilon_{igpt} = \text{the error associated with the } i\text{th subject in the } g\text{th group on the } p\text{th place at the } t\text{th occasion.} \]

Assumption

Under the following assumptions the analysis of variance is given in Table 1.

1. \[ \varepsilon_{igp} \sim N(0, \sigma^2_{\varepsilon}) \]

2. \[ \varepsilon_{igpt} \sim N(0, s^2_{\varepsilon}) \]

3. both are independent

4. all the variances in the different places are equal i.e.

\[ \sigma^2_{e^*} = \sigma^2_{e^*} = \ldots \ldots = \sigma^2_{e^*} = \sigma^2_{e^*}; t = 1, 2, \ldots, p \]

Table 1. ANOVA

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>EMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group G-1</td>
<td>s^2 + \frac{TP}{(G-1)} Sn_g (a_g - \bar{a})^2</td>
<td></td>
</tr>
<tr>
<td>Place P-1</td>
<td>s^2 + \frac{GT}{(P-1)} S(\bar{\beta}_p - \bar{\beta})^2</td>
<td></td>
</tr>
<tr>
<td>Group×Place (G-1)(P-1)</td>
<td>s^2 + \frac{N}{(G-1)(T-1)} SS{(a\bar{\beta})_{gp} - (a\bar{\bar{\beta}})_p - (\bar{a}\bar{\beta})_g + (\bar{a}\bar{\bar{\beta}})}^2</td>
<td></td>
</tr>
<tr>
<td>Error-I P(N-G)</td>
<td>\sigma^2_{e^<em>} + \sigma^2_{e^</em>}</td>
<td></td>
</tr>
<tr>
<td>Occasion (T-1)</td>
<td>s^2 + \frac{NP}{(T-1)} S(t_t - \bar{t})^2</td>
<td></td>
</tr>
<tr>
<td>Group×Occasion (G-1)(T-1)</td>
<td>\sigma^2_{e^*} + \frac{P}{(G-1)(T-1)} \Sigma \Sigma n_g {(a\alpha)_g \alpha - (a\bar{\alpha})_g \bar{\alpha} - (\alpha\alpha)_g \bar{\alpha} + (\bar{a}\alpha)_g }^2</td>
<td></td>
</tr>
<tr>
<td>Place×Occasion (P-1)(T-1)</td>
<td>\sigma^2_{e^*} + \frac{G}{(P-1)(T-1)} \Sigma \Sigma (\beta \tau)<em>{pt} - (\beta \bar{\tau})</em>{pt} - (\bar{\beta} \tau)<em>{pt} + (\bar{\beta} \tau)</em>{pt}</td>
<td></td>
</tr>
<tr>
<td>Group×Place×Occasion (G-1)(P-1)(T-1)</td>
<td>s^2 + \frac{1}{(P-1)(T-1)(G-1)} SSS(a\beta)t^2_{gpt}</td>
<td></td>
</tr>
<tr>
<td>Error-II GT(N-G)</td>
<td>s^2</td>
<td></td>
</tr>
</tbody>
</table>
The following hypotheses are to be tested:

1. \( H_0(\text{group}) \) : \( \alpha_1 = \alpha_2 = \ldots = \alpha_g \) i.e. the group effects are equal
2. \( H_0(\text{place}) \) : \( \beta_1 = \beta_2 = \ldots = \beta_p \) i.e. the place effects are equal
3. \( H_0(\text{group} \times \text{place}) \) : The group \times place interaction effects are nil
4. \( H_0(\text{occasion}) \) : \( \tau_1 = \tau_2 = \ldots = \tau_t \) i.e. the occasion effects are equal
5. \( H_0(\text{group} \times \text{occasion}) \) : The group \times occasion interaction effects are nil
6. \( H_0(\text{place} \times \text{occasion}) \) : The place \times occasion interaction effects are nil
7. \( H_0(\text{group} \times \text{place} \times \text{occasion}) \) : The group \times place \times occasion interaction effects are nil

The above hypotheses about the main effects and the interaction effects can be tested by using F-statistic with respective degrees of freedom.

Previously the analysis considered the variances among different places are equal, which is a reasonable assumption in any case. One method for analysing data when variances are unequal simply to ignore the fact that they are unequal and calculate the same statistics. Surprisingly these tests are quite good, in particular if the variances in different places are all equal or almost equal.

If the variances in different places are not equal the usual test statistic is not valid.

Moreover, the variance structure would be a function with the respective error variances in different places, then the functional form are as follows:

1. \( \Sigma^* (\text{variance structure}) = \varphi(\Sigma^*_p) = a_0 + \sum_{p=1}^{p} a_p \Sigma^*_p \); linear functional form.
   \( \Sigma^* = \text{ Combined variance co-variance matrix} \)
2. \( \Sigma^* (\text{variance structure}) = \varphi(S^*_p) = a_0 + \sum_{p=1}^{p} a_{pp} S^*_p + S S a_{pp} S^*_p ; p \neq p' \) quadratic functional form
3. \( \Sigma^* (\text{variance structure}) = \varphi(\Sigma^*_p) = \sum_{p=1}^{p} c_p \Sigma^*_p \); linear contrast
4. \( P_{\lim} \int_{p=1}^{p} \varphi(S^*_p) dS^*_p = P_{\lim} \int_{p=1}^{p} \varphi(S^*_p) ; \text{which implies either divergency or convergency.} \)

Some test criterions are Suggested for testing

\( H_0 : \sigma_{\varepsilon^*}^{2(p)} = \sigma_{\varepsilon^*}^{2(p)} = \ldots = \sigma_{\varepsilon^*}^{2(p)} \) ; \( p = 1,2,\ldots , P \)

Test Procedure

1. If the corresponding mean sum of square is substantially large than the mean square error in different places then the linear equation does not represent the true response surface (Islam, 1992) i.e. the hypotheses is not accepted.
2. Islam (1992) showed that if the percentage rotatability is equal to or about to equal

\[
\varphi_n(D) = 100 \sqrt{\frac{P}{\sum_{p=1}^{P} \Sigma^*_{p=1}} = 100} ; \text{ then the test is not significant i.e. the null hypotheses is accepted.}
\]
3. Consider the null hypotheses

\[ H_0(\text{place}) = \sum_{p=1}^{P} c_p \Sigma_p^* = \Sigma_p^* = 0 \]

\[ H_a(\text{place}) = \sum_{p=1}^{P} c_p \Sigma_p^* \neq 0 \]

Then

\[ \sum_{p=1}^{P} c_p \Sigma_p^* \left[ \sum_{p=1}^{P} c_p \Sigma_p^*, \sum_{p=1}^{P} c_p \Sigma_p^2 \right] \]

where \( n_p \) number of sample in different places. Then the test statistic

\[ Z = \frac{1}{\sqrt{\sum_{p=1}^{P} \Sigma \Sigma \Sigma c_p \Sigma_p^2}} \]

\[ \rightarrow N(0,1) \]

the \( Z \) statistic follows standardized normal distribution.

4. If the number of places \( N > 15 \), then the test statistic would be followed:

\[ P \lim_{P \to \infty} \int_{-\infty}^{\infty} \phi(\Sigma_p^*) d\Sigma_p^* = 1 = P \lim_{p \to \infty} \int_{-\infty}^{\infty} \phi(S_p^*) ; \text{indicates null hypothesis is accepted.} \]

Numerical example

The data were derived from a group of experiments on Mahogany (Switania macrophylla) trees conducted in different stations of Jahangirnagar University, Savar, Dhaka. The different pH values of the soil in which the trees grown are 5.4, 5.6 and 5.8. The land was prepared well and planted the “Mahogany” trees at the depth of 15 to 16 cm, with a plant to plant spacing 70cm. Five trees were randomly assigned to each fertilizer in each group of different places. The time periods of the experiments were 20th November 1993 to 19th July 1994. The height of the trees were measured in every two months interval and were recorded in cm. Thus \( G=3, T=3, P=5, O=5. \) \([G=\text{group}, \ T=\text{Fertilizer}, \ P=\text{place}, \ O=\text{Occasion}]\).

The objective of the experiments was to study the effects of a level of plantation in combination with 3 levels of Nitrogen on the growth of Mahogany. The analysis techniques would be followed by Repeated Measurements experimental Design.

The error variance of the three experiments were observed as 0.06716, 0.17479 and 0.34592 in different places. These error variances were heterogeneous as was observed. But suggested test statistic (I, II, III, IV) indicated that the null hypotheses were accepted.
Thus, the usual analysis of variance is valid. The analysis of variance Table 2 is given below.

<table>
<thead>
<tr>
<th>S.V.</th>
<th>D.f.</th>
<th>M.S.</th>
<th>F.ratio</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Place</td>
<td>2</td>
<td>43.8867</td>
<td>7.5767</td>
<td>0.000313</td>
</tr>
<tr>
<td>Fertilizer</td>
<td>2</td>
<td>2.4224</td>
<td>0.4182</td>
<td>0.3389</td>
</tr>
<tr>
<td>Place &amp; Fertilizer</td>
<td>4</td>
<td>0.2758</td>
<td>0.0426</td>
<td>0.50278</td>
</tr>
<tr>
<td>Error-1</td>
<td>12</td>
<td>5.7923</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Occasion</td>
<td>4</td>
<td>238.5279</td>
<td>26.2459</td>
<td>0.000216</td>
</tr>
<tr>
<td>Place &amp; Occasion</td>
<td>8</td>
<td>0.1043</td>
<td>0.01147</td>
<td>0.82395</td>
</tr>
<tr>
<td>Fertilizer &amp; Occasion</td>
<td>8</td>
<td>9.8092</td>
<td>1.0793</td>
<td>0.09258</td>
</tr>
<tr>
<td>Place &amp; Fertilizer &amp; Occasion</td>
<td>16</td>
<td>0.01136</td>
<td>0.001249</td>
<td>0.8729</td>
</tr>
<tr>
<td>Error-2</td>
<td>48</td>
<td>9.08817</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2 showed that the place and occasion effects are highly significant. But the fertilizer effect, place and fertilizer interaction effect, place and occasion interaction effect, place and fertilizer and occasion interaction effect are insignificant. The fertilizer and occasion interaction effect is insignificant. From the analytical results, the place and occasion effects are highly significant i.e. significant variation of growth of the plant over the time and the places are present in the experiment.

**CONCLUSION**

Designed field experiments on the effect of inorganic fertilizer were performed on the planted seedlings of Mahogany, the most widely used plantation forestry species in Bangladesh. Therefore, it is very important to outline an experimental procedure to accommodate all of the plantation at a time. This experiment is very helpful for this purpose.

**LITERATURE CITED**


