# Statistical Model and Method in Ensuring Validity of HIV Data

M.A. Shah

#### ABSTRACT

Human Immune deficiency Virus (HIV)/ Acquired Immune Deficiency Syndrome (AIDS) is spreading very fast over the globe. The urban infrastructure in Bangladesh is most developed in the three cities, Dhaka, Chittagong and Sylhet attracting large foreign investments and greater volumes of commerce and trade. These three cities are also experiencing an increasing influx of people from across the country. Keeping pace with the rapid urbanization, HIV is also going up. But conventional surveys, conducted so far, seems underestimated the number of HIV infected cases. Proper estimation of HIV infected people is, therefore, essential to effectively address the issue. That purpose Pure Birth process model has been formulated to estimate the HIV for the age group 15-49 years.

Key words: AIDS, HIV, Poisson process, pure birth process model.

#### INTRODUCTION

Acquired Immune Deficiency Syndrome (AIDS) has become one of the most serious epidemics or rather pandemic which has spread to every continent. AIDS is the full-blown disease of Human-Immune Deficiency Virus (HIV) infection. It has already claimed the lives of more than 23 million and to this 3 million of deaths are added every year. The number of HIV infected people is increasing with a galloping pace and the situation is quite alarming posing a challenge to the mankind. According to WHO report 2002, an estimated 42 million people throughout the globe are currently living with HIV.

It was estimated that at the end of 2001 there were 4.2 million people living with HIV/AIDS in South Asia - Afghanistan, Bangladesh, Bhutan, India, Maldives, Nepal, Pakistan and Sri Lanka. Much is now known about the disease in South Asia. Recently, a report on the AIDS cases came out in the daily newspaper and according to that report, more than 13 thousand people are carrying HIV in Bangladesh and the experts have classified the AIDS situation as concentrated epidemic. Ironically, the government was downplaying the actual AIDS cases, just informing only 282 and taking credit from the

international community. But in reality, the actual figure would be far higher than the reported one. Due to stigma and fear of discrimination, HIV infected people are so scared to come forward with their diseases and ask for treatment and health care. It can make people hide their HIV status amidst fear of rejection from their loved ones. Although the HIV prevalence rate is still low in Bangladesh, behavioral patterns suggest that the number of people infected with HIV could reach epidemic proportions unless concerted efforts are undertaken to prevent it.

The first HIV-positive person diagnosed in Bangladesh was in 1989. Since then it is increasing steadily to become approximately 7,500 in 1994 and to become 11,000 in 2005. According to International Center for Diarrhoeal Disease Research, Bangladesh (ICDDRB) the number of cases of HIV/AIDS has increased to 1495 at the end of 2008. However UNAIDS estimates that the number of people living with HIV in the country may be as high as 12,000, which is within the range of the low estimate by UNICEF's State of the World's Children Report 2009. Although the number of HIV/AIDs cases are increasing steadily over the years, the estimates are ambiguous. The overall prevalence

#### Authors' Information:

Professor Dr. M A Shah, Professor, Department of Statistics, University of Rajshahi, Rajshahi, Bangladesh. Mobile: 01556-312554, E-mail:azs61@yahoo.com

of HIV in Bangladesh is less than 1%. According to government sources, there were 363 reported HIV-positive cases as of December 2003, Of them, 12.3% are adolescents and youth (aged 15-24) and 3.7% are infants and children. However, it is believed that the actual number of cases is much higher. According to UNAIDS, by the end of 2001 the estimated number of adults in Bangladesh living with HIV, irrespective of whether or not they had developed symptoms of AIDSs, was 13,000, More recent data of National AIDS/STD program demonstrate that new HIV, new AIDS and new AIDS deaths in 2010 were 343, 231 and 37 respectively. The underlying causes of the HIV/AIDS epidemic include poverty, gender inequality and high mobility of the population. All of which are commendably high in Bangladesh. Bangladesh is a densely populated country with its 30% population living in urban area. Continuous influx of people into the urban areas from across the country making the cities unsafe and unhealthy for future living. Emigration to other countries for employment is also very common, particularly amongst younger people, largely to the Middle East, followed by Singapore and Malaysia.

Principal metropolitan cities of Bangladesh are Dhaka, Chittagong and Sylhet. Dhaka, the capital of Bangladesh, is one of the major cities of South Asia. Dhaka, along with its metropolitan area has a population of over 103,56500, making it the largest city in Bangladesh. It is the ninth largest city in the world. Chittagong has a population of over 39,20222, making it the second largest city in the country and is a major commercial and industrial center as much of Bangladesh's export and import passes through the port of Chittagong. According to a report by International Institute Environment and Development, Chittagong is among the ten fastest growing cities in the world. Sylhet is another major city in north-Bangladesh and approaching eastern population of 23,70000 people according to the Geo Names geographical database. The city, however is currently known for its business boom - being one of the richest cities in Bangladesh, with new investments in hotels,

shopping malls and luxury housing estates, brought mainly by expatriates living in Europe. The results of the 4<sup>th</sup> round of the National Serological Surveillance has shown an alarming increase in HIV rates among the injectable drug users (IDU) in Dhaka. In this group of people, prevalence has jumped from 1.7 per cent in 2002 to 4 per cent in 2003. This is just short of the 5 per cent mark required for a situation to be identified as a concentrated epidemic.

Laxmi et al3 and Bashiru et al4 used statistical models for estimating the HIV rate for three different states of India. Luboobi, 5 Anderson and and May7 have associates<sup>6</sup> and Anderson investigated the HIV in deterministic approach. Anderson<sup>8</sup> have also considered vaccination strategies in age-structured populations. Most of the models so far investigated were within the framework of a deterministic environment, However, real environment is full of fluctuations. Further, the human behavior is stochastic rather than deterministic. Only very recently, it has been felt that stochasticity is the common feature of all the processes evolving time and space, be they related to natural science, social science, engineering and health science.9-12 The purpose of the present study is, therefore, to formulate a stochastic model for estimating the number of HIV infected persons in Dhaka, Chittagong and Sylhet divisions of Bangladesh,

### MATHEMATICAL FORMULATION

In many situations the objective of an analysis consists of merely observing the number of units that enter the system. The model in which only the addition are counted and no subtractions take place are called Pure birth models. As such, the Pure birth models are not of much importance so far as their applicability to real life situation is concerned, but these are very important in the understanding of completely random arrival problems. Every new HIV infection can be treated as an addition to the HIV/AIDS family. Let be the rate of infection and be the initial population of the system. Thus the Pure birth process can be considered as statistical model for the analysis as follows.

We now wish to determine the probability on n additions in a time interval of length t, denoted by  $P_n(t)$ . Clearly, n will be an integer greater than or equal to zero. To do so, we shall first develop the differential equations governing the process in three different situations. For n>0, there may be three mutually exclusive ways of having n units at time  $t+\delta t$ .

- There are n units in the system at time t and no birth takes place during time interval δt. Hence, there will be n units at time t+δt.
- (ii) Alternatively, there are (n-1) units in the system at time t, and one event takes place during δt. Hence there will remain n units in the system at time t+δt.
- (iii) Further, there are (n-2) units in the system at time t, and two events takes place during δt. Hence there will remain n units in the system at time t+δt.

Therefore, the probability of those two combined events explained in (i) will be

$$=P_n(t),[1-\lambda_n(t)+O(\delta t)],$$

the probability of those two combined events explained in (ii) will be

$$=P_{n-1}(t).[\lambda_{n-1}(t)+O(\delta t)],$$

and finally, the probability of those two combined events explained in (iii) will be

$$=P_{n=2}(t).[\lambda_{n=2}(t) + O(\delta t)]$$

Now, adding these three probabilities, we get the probability of events at time  $t + \delta t$ ,

$$\begin{split} P_{_{B}}(t+\delta t) &= P_{_{B}}(t) \cdot [1-\lambda_{_{B}}(t) + O(\delta t)] + P_{_{B-1}}(t) \cdot [1-\lambda_{_{B-1}}(t) + O(\delta t)] + P_{_{B-2}}(t) \cdot [1-\lambda_{_{B-2}}(t) + O(\delta t)] \\ &= P_{_{B}}(t) \cdot [1-\lambda_{_{B}}(t)] + P_{_{B-1}}(t) \cdot [1-\lambda_{_{B-1}}(t) + P_{_{B-2}}(t) \cdot [1-\lambda_{_{B-2}}(t)] + O(\delta t) \end{split}$$

The model is studied in considering three possible independent events  $n,\ n\text{-}1$  and n-2. Since the probability of more than one or two events in  $\delta t$  is assumed to be negligible, other alternatives do not exist. Assuming that more than one person can not be infected simultaneously for the same source at the same time, and then  $O(\delta t) \to 0$ .

$$P_{n}(t+\delta t) - P_{n}(t) = [-\lambda_{n}(t)P_{n}(t) + \lambda_{n-1}(t)P_{n-1}(t) + \lambda_{n-2}(t)P_{n-2}(t)] + O(\delta t)$$

Dividing both sides by  $\delta t$  and then taking limit  $\delta t$  $\rightarrow 0$ , the first derivative results

$$\frac{d}{dt}P_{n}(t) = P_{n}(t) = -\lambda_{n}(t)P_{n}(t) + \lambda_{n-1}(t)P_{n-1}(t) + \lambda_{n-2}(t)P_{n-2}(t)$$
(1)

Since,  $E(N = n) = \lambda n$ , Eq.(1) becomes

$$P_n(t) = -n\lambda(t)P_n(t) + (n-1)\lambda(t)P_{n-1}(t) + (n-2)\lambda(t)P_{n-2}(t)$$
 for  $n \ge 2$  (2)

### Solution Process:

In order to solve the Eq.(2), we have used the probability generating function (p.g.f) method,

$$P(u,t) = \sum_{n=1}^{\infty} P_n(t)u^n$$
(3)

Multiplying Eq.(2) by  $u^n$  and summing over n, we have

$$\sum P_n(t)u^n = -\lambda \sum nP_n(t)u^n + \lambda \sum (n-1)P_{n-1}(t)u^n + \lambda \sum (n-2)P_{n-2}(t)u^n \quad (4)$$

so that

$$\frac{\partial}{\partial t}P(u, t) = \sum P(t).u^n$$
 and  $\frac{\partial}{u}P(u, t) = \sum nP_n(t).u^{n-1}$  (5)

Multiplying Eq.(5) by  $\lambda u$ ,  $\lambda u^2$  and  $\lambda u^3$  in succession we have

$$\lambda u \left(\frac{\partial P}{\partial u}\right) = \lambda \sum n P_n(t) u^n$$
 (5a)

$$\lambda u^2 \left( \frac{\partial P}{\partial u} \right) = \lambda \sum_n nP_n(t)u^{n+1}$$
 (5b)

$$\lambda u^3 \left( \frac{\partial P}{\partial u} \right) = \lambda \sum n P_n(t) u^{n+2}$$
 (5c)

From the very beginning we have considered three consecutive possibilities, namely  $P_n(t)$ ,  $P_{n-1}(t)$  and  $P_{n-2}(t)$  we may assume in Eq.(5) and Eq.(5b),  $n \to (n-1)$ . And in Eq.(5a) and Eq.(5c),  $n \to (n-2)$  we have

$$\lambda u^2 \left( \frac{\partial P}{\partial u} \right) = \lambda \sum_{n=1}^{\infty} (n-1) P_{n-1}(t) u^n$$
 (5d)

$$\lambda u^{3} \left( \frac{\partial P}{\partial u} \right) = \lambda \sum (n-2) P_{n-2}(t) u^{n}$$
 (5e)

Using Eq.(5), Eq.(5a), Eq.(5d) and Eq.(5e) in Eq.(4) we have

$$\frac{\partial}{\partial t}P(u,t) = -\lambda u \frac{\partial P}{\partial u} + \lambda u^2 \frac{\partial P}{\partial u} + \lambda u^3 \frac{\partial P}{\partial u}$$

or

$$\frac{\partial}{\partial t}P(u,t) = -\lambda u(1-u-u^2)\frac{\partial}{\partial u}P(u,t) = 0$$
 (6)

With the help of Lagrange's linear equation we have,

$$\frac{dt}{1} = \frac{dP}{0}$$
(6a)

and

$$\frac{dt}{1} = \frac{du}{\lambda u(1 - u - u^2)}$$
(6b)

Eq.(6a) gives,

$$P = c_1$$
 (6c)

where c is a constant of integration.

Further, Eq.(6b)

$$\int \underline{A} dt = \int \frac{du}{u(1-u-u^2)}$$
(6d)

Therefore solving by partial fraction we have

$$\int \lambda dt = -\int \frac{1}{u} du + \frac{2}{5 - \sqrt{5}} \int \frac{1}{u + \frac{1 - \sqrt{5}}{2}} du + \frac{2}{5 + \sqrt{5}} \int \frac{1}{u + \frac{1 + \sqrt{5}}{2}} du$$

$$\lambda t = \log \left[ c \left( \frac{1}{u} \right) \left( u + \frac{1 - \sqrt{5}}{2} \right)^{\frac{2}{5 - \sqrt{5}}} \left( u + \frac{1 + \sqrt{5}}{2} \right)^{\frac{2}{5 - \sqrt{5}}} \right]$$

$$e^{Bt} = P\left(\frac{1}{u}\right)\left(u + \frac{1 - \sqrt{5}}{2}\right)^{\frac{2}{5 - \sqrt{5}}}\left(u + \frac{1 + \sqrt{5}}{2}\right)^{\frac{2}{5 - \sqrt{5}}},$$

Using P = c (constant), we have

$$P(u,t) = e^{Bt} \left[ \left( \frac{1}{u} \right) \left( u + \frac{1 - \sqrt{5}}{2} \right)^{\frac{2}{5 - \sqrt{5}}} \left( u + \frac{1 + \sqrt{5}}{2} \right)^{\frac{2}{5 - \sqrt{5}}} \right]^{-1}$$
(7)

It is mentionable that the rate of infection and initial population are  $\lambda$  and u respectively.

Therefore,

$$P_{x}(u, t) = P(u, t)/100$$
 (8)

and

$$P_2(u, t) = u \cdot P_1(u, t)/100$$
 (9)

This form of Birth Process Model can be used to estimate the number of infected persons for the time interval t when the rate of infection  $\lambda$  and the initial population u are known.

## Illustration:

TABLE |(a): Probability for the possible values n=0,1 for the Classical Process

Time	n values		Total Probability	
	0	1	Total Probability	
1	0.90955301	0,08622744	0,9957805	
2	0.8272867	0.1568569	0.9841435	
3	0.7524611	0.2140044	0.9664655	
4	0.6844032	0.2595312	0.9439344	
5	0.6225010	0.2950717	0.9175727	
6	0.5661977	0.3220600	0.8882577	
7	0.5149868	0.3417525	0.8567393	

TABLE (b): Probability for the possible values n=0,1,2 for the Classical Process

Time		Total		
	0	1	2	Probability
1	0.909553008	0.086227444	0.004087267	0.9998677
2	0.82728667	0.15685686	0.01487034	0.9990139
3	0,75246108	0,21400445	0,03043207	0.9968976
4	0,68440324	0,25953118	0,04920815	0,9931426
5	0.62250103	0,29507171	0.06993347	0,9875062
6	0.56619768	0.32206004	0.09159581	0.9798535
7	0.5149868	0.3417525	0.1133959	0.9701351

TABLE 8(c): Probability for the possible values n=0,1,2,3 for the Classical Process

Time	n values				Total
	0	1	2	3	Probability
1	0.90955300	0.08522744	0.0040872	0.00012916	0,9999959
2	0.82728667	0.15685686	0.0148703	0.00093982	0.9999537
3	0.75246108	0.21400444	0.9997826	0.03043207	0.9997826
4	0.68440324	0.25953118	0.0492081	0.00522004	0.9993626
5	0.62250103	0,29507171	0.0699334	0.01104972	0.9985559
5	0,56619768	0,32205004	0.0915958	0.01736693	0.9972205
7	0,51498681	0,34175245	0.1133958	0,02508369	0,9952188

## Caption of Tables and Figures

When rate =  $\lambda$  = 0.079361, the probability that none and single AIDS cases are presented in Table 1(a). For n=0,1,2 and n=0,1,2,3, the corresponding probabilities are shown in Tables 1(b) & 1(c) for different times. It has been observed that the number of independent cases are increases during, δt the probability values are decreases and holds the Poisson property. So that we can not ignore the importance of two/three or more AIDS cases by epoch 't'. In our theoretical development we have considered (n-2) events by epoch t and two events between t and  $(t+\delta t)$ . The above tabular results are also represents that the probabilities are gradually decreasing provided the total probabilities are close to one. The Figures 1(a)-(c), 2(a)-(c) and 3(a)-(c) represent the conception as narrated in the above tables. It is to be noted that we have examined in details for the Chittagong division only when t=1,2,3.

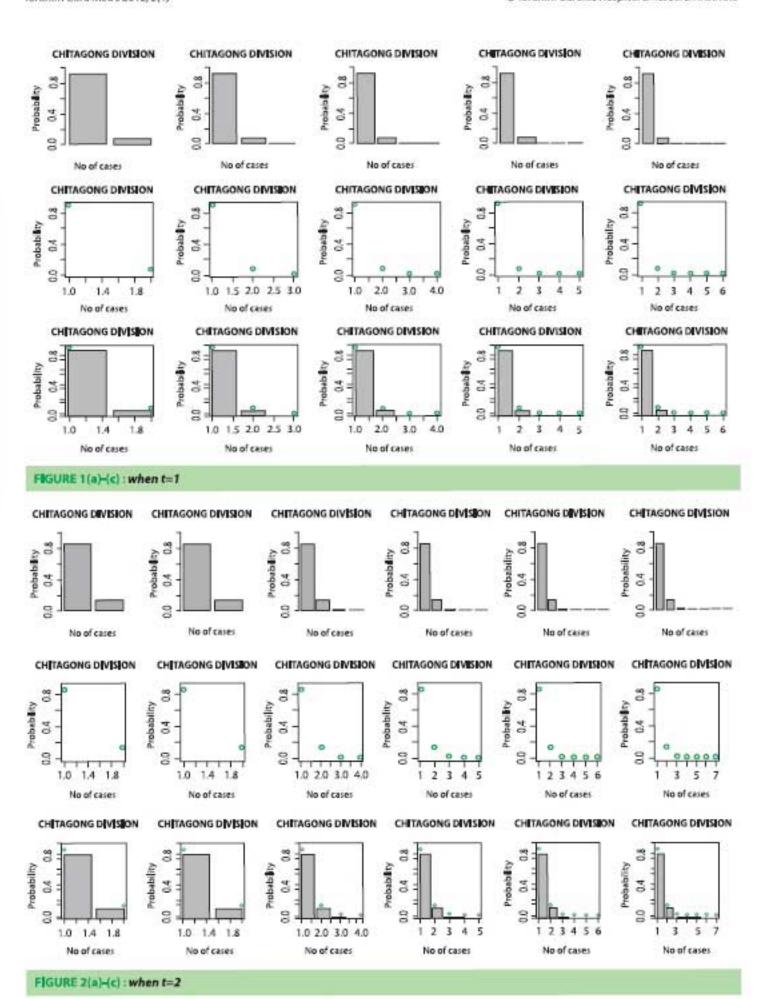
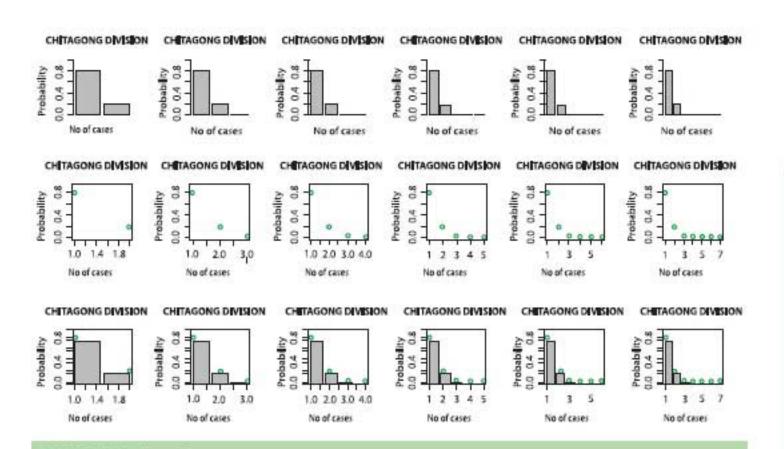


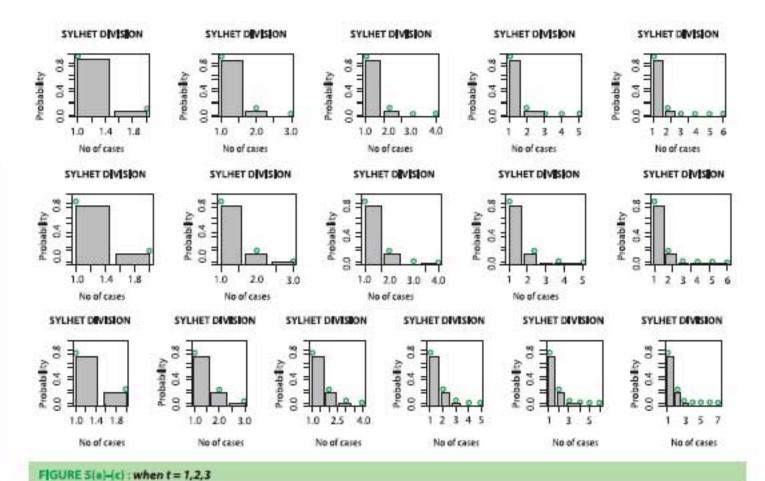
FIGURE 3(a)-(c): when t=3



Using R-language we have determined the dependence of two or more independent cases for Dhaka and Sylhet divisions of

Bangladesh and compared with the existing classical process and are presented in the Figures 4(a)-(c) and 5(a)-(c).

DHAKA DIVISION DHAKA DIVISION DHAKA DIVISION DHAKA DIVISION DHAKA DIVISION Probability 0.8 Probability. 0.8 0.8 Probability 0.8 Probability 5 0.8 Probability 3 0.4 0.4 0.4 0.4 8 0.0 00 0.0 0.0 1.4 2.0 3.0 1.0 2.0 3.0 4.0 2 3 4 1.0 1.8 5 2 3 4 5 6 1.0 No of cases DHAKA DIVISION DHAKA DIVISION DHAKA DIVISION DHAKA DIVISION DHAKA DIVISION Probability 0.8 0.8 0.8 0.8 0.8 Probability Probability Probability Probability 6 0.4 0.4 0.4 0.4 80 9 90 90 90 1.0 2.0 3.0 2.5 4.0 3 5 1.0 1.6 1.0 3 5 No of cases DHAKA DIVISION DHAKA DIVISION DHAKA DIVISION DHAKA DIVISION DHAKA DIVISION DHAKA DIVESION 8 8 0 0.8 8 8 Probability 8 1 Probability Probability Probability Probability Probability 0.4 0.4 0.4 0.4 0.4 0.4 1.4 1.8 2.0 2.5 1.0 1.0 3.0 1.0 4.0 2 3 4 3 5 3 5 No of cases FIGURE 4(a)-(c): when t = 1,2,3



## Classical and Proposed Methods:

Pr{No AIDS case observed}=Pr{N(t)=0,i.e.,n=0} have calculated in the Tables-2 for the increasing values of time parameter, t=1,2,3,4,5,6,7. We would like to point out here that our proposed model gives a far better description as compared to the classical model. The model would not be mere mathematical artifacts, but rather it will cover a much larger spectrum of the birth process.

TABLE 2:

Time t	Meti	nods
iiiie t	Classical	Proposed
1	0.9237064	0,9236605
2	0.8532335	0,8531911
3	0.7881373	0.7880981
4	0.7280074	0.7279713
5	0.6724651	0.6724317
6	0.6211604	0.6211295
7	0.5737698	0.5737413

TABLE 3: Predicted number of HIV infection  $P_3(z,t)$  in Dhaka for the age-group 15-49.

Time	Dhaka		
t	$P_1(z,t)$	$P_2(z,t)$	
1	0.010994	62,6249	
2	0.012088	68,8524	
3	0.01329	75.69918	
4	0.014611	83.22681	
5	0.016064	91.503	
6	0,017662	100,6022	
7	0.019418	110.6062	
8	0.021349	121.605	
9	0.023472	133,6976	
10	0,025806	146,9927	

## RESULTS AND DISCUSSIONS

As mentioned in the introduction the rate of HIV infection represented by  $\lambda$ , the time interval t stands for one year and population infected by HIV  $P_{2}$ . Thus for the age group (15–49 years) of the population of Dhaka division,  $\lambda$ =0.094802 and t=1. So that Equation (7) results, P(z,t)=1.09944

and the corresponding value of  $P_1(z,t)=0.010994$  and  $P_2(z,t)=62.6249$ , is the number of infected persons from the normal population. According to the nature of the problem and size of the population, using Pure Birth process model the rate of HIV infected persons  $P_2(z,t)$ , predicted for next ten years for inhabitants of Dhaka city are shown in Table-3. Therefore,  $62.6249\approx63$  persons will be infected in a year (t=1) for the population of Dhaka for the age group (15-49 years) and the results are presented in Table-4.

TABLE 4: Predicted number of HIV infection P<sub>1</sub>(t) for the age-group 15-49 with respect to time (t) in Dhaka.

Time 1 2 3 4 5 6 7 8 9 10 HIV 63 69 76 83 92 101 111 122 134 147

Further,  $\sum P_2(t) = 998$ . It implies that after 10 years there will be about 1000 more cases of HIV in the Dhaka division only and is presented in Figure-6.

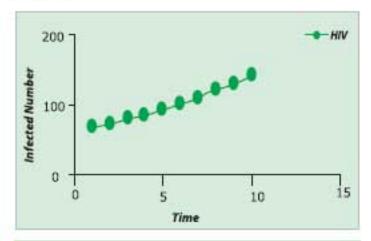


FIGURE 6: Predicted Number of HIV Infection in Dhaka within a Ce cade.

Again for another two districts Chittagong and Sylhet,  $\lambda = 0.079361$  &  $\lambda = 0.092185$ . So for the age group (15-49), P(z,t) = 1.082593 and P(z,t) = 1.096566. Therefore,  $38.19605 \approx 38$  and  $63.04522 \approx 63$  persons will be infected in a year (t = 1) for the inhabitants of Chittagong and Sylhet and the figures are presented in Table-5.

TABLE 5: Predicted number of HIV infection (P<sub>2</sub>(z,t)) in Chittagong and Sylhet for the age-group 15-49.

Time	Chittagong		Sylhet	
t	$P_1(z,t)$	$P_2(z,t)$	$P_1(z,t)$	$P_2(z,t)$
1	0.010826	38.19605	0.010966	63.04522
2	0.01172	41.35084	0.012025	69.13332
3	0.012688	44.76619	0.013186	75.80934
4	0.013736	48.46364	0.014459	83.13004
5	0.014871	52,46648	0.015855	91,15769
6	0,016099	56,79992	0,017386	99,96054
7	0,017429	61,49129	0,019065	109,6135
8	0,018868	66,57014	0,020907	120,1985
9	0.020426	72.06848	0.022925	131.8058
10	0.022114	78.02094	0.025139	144.5339

TABLE 6: Predicted number of HIV infection for the age-group 15-49 with respect to time (t) in Chittagong.

Time 1 2 3 4 5 6 7 8 9 10 HIV 38 41 45 48 52 57 61 67 72 78

TABLE 7 : Predicted number of HIV infection for the age-group 15-49 with respect to time (t) in Sylhet.

Time 1 2 3 4 5 6 7 8 9 10 HIV 63 69 76 83 91 100 110 120 132 145

Finally,  $\sum P_2(t) = 599$  and  $\sum P_2(t) = 989$ , assert that after 10 years there will be around 559 and 989 more cases of HIV cases in Chittagong and Sylhet divisions of Bangladesh and are presented in Figure-7-8.

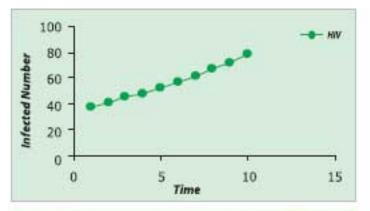


FIGURE 7 : Predicted Number of HIV Infection in Chittagong witin a Decade

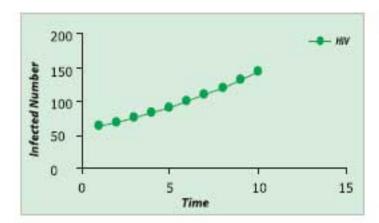


FIGURE 8: Predicted Number of HIV Infection in Sylhet within a Decade

### CONCLUSIONS

It has been observed that the expected number of HIV infection in Dhaka, Chittagong and Sylhet are 998, 559 and 989 respectively for the age group 15-19. However, the total number of infections in Chittagong is lower than those in Dhaka and Sylhet, Categorically the estimated HIV rate is much higher in Dhaka, the capital city of Bangladesh as compared to the second largest commercial city Chittagong. Bangladesh is in the unique position to succeed where several other developing countries have not succeeded to keep the HIV epidemic from expanding beyond the current level by initiating comprehensive and strategically viable preventative measures, avoiding a gradual spread of HIV infection from high-risk groups to the general population. Due to the limited access to voluntary counseling and testing services, very few Bangladeshis are aware of their HIV status. Although still considered to be a low prevalence country, Bangladesh remains extremely vulnerable to HIV epidemic. Providing real data on HIV dealing with district-wise independently following infrastructure concepts, parameters can be easily estimated and can be used in perfect decision making.

#### REFERENCES

- Hossain MS. AIDS/HIV situation in Bangladesh: A Looming Threat, Scholars Journals 2005; 15, Personal communication to journal@scholarsbangladesh.com
- Belal MA, Rahman AM, Mahmud H. National AIDS/STD Programme, Ministry of Health and Family Welfare, Dhaka, Banagladesh 2010. Available at: http://www.bdnasp.net.
- Laxmi, Kaushik and Lal. Rate of spread of Human Immune Deficiency: Case studies of Haryana, Manipur and Jammu & Kashmir, Indian Journal of Applied Statistics 2007;11.
- 4- Bashiru, K.A. and Olowofeso, O.E. Statistical model for estimating the rate of spread of Human Immune. Deficiency: A case study of Ondo Kingdom, Research Journal of Applied Sciences 2007;2(10):1025-30.
- Luboobi LS. A three age group model for HIV/AIDS epidemic and effects of medical/social interventions, Math. Comput. Modeling 1994;19:91-105.
- Anderson RM, May RM, Johnson AM. A preliminary study of the transmission dynamics of the immunodeficiency virus (HIV): The causative agents of the AIDS, IMA J Math Applied Med Bio 1986;3:229-63.
- Anderson RM, May RM. Transmission dynamics of HIV infection, Nature 1987;326:37-142.
- Anderson, Roy. 'The Transmission Dynamics of Sexually Transmitted Diseases: The Behavioral Component." In: Sexual Behavior and Networking: Anthropological and Socio-Cultural Studies on the Transmission of Hiv, edited by Tim Dyson, 23-48. Liege Belgium: International Union for the Scientific Study of Population, 1992.
- Goel NS, Richter-Dyne N. Stochastic models in Biology, Academic Press, London, 1974.
- Bailey NTJ. The Elements of Stochastic Processes with Applications to the Natural Sciences. Wiley, New York, 1964.
- Medhi J. Stochastic Processes, Second Edition, Wiley Eastern Limited, 1994.
- Olagunju SO, Bashiru KA, Olowofeso OE. Statistical Model for Estimating the Rate of Spread of Human Immune– Deficiency: A Case Study of Ondo Kingdom, Research Journal of Applied Sciences 2007;2(100):1025-30.