

# Statistical Model Based Breast Tumor Classification in Contourlet Transform Domain

Shahriar Mahmud Kabir, Md. Sayed Tanveer, ASM Shihavuddin and Mohammed Imamul Hassan Bhuiyan

**Abstract**— Determination of breast tumors from B-Mode Ultrasound (US) image is a perplexing one. Researches employing statistical modeling such as Nakagami, Normal Inverse Gaussian (NIG) distributed parametric images in this classification task have already explored but experimentation of those statistical models on contourlet transformed coefficient image in breast tumor classification task has not reported yet. The proposed method is established by considering 250 clinical cases from a publicly available database. In this database each clinical case exists as \*.bmp format. In the preprocessing step firstly, the ultrasound B-Mode image is binarized to detect the lesion contour. Then contourlet transformation is employed. These contourlet sub band coefficients are shown to be modeled effectively by Nakagami and NIG distributions. These Nakagami and NIG parametric images are obtained by estimating the parameters of those prior statistical distributions locally. Few shape and statistical features are chosen according to their effectiveness on those parametric images. The benign and malignant breast tumors are classified utilizing these features with different classifiers such as the support vector machine, k-nearest neighbors, fitted binary classification decision tree, binary Gaussian kernel classification model, linear classification models for binary learning with high-dimensional etc. It is observed that classification performance of NIG statistical model based parametric version of contourlet coefficient images gained better accuracy than those of Nakagami statistical model.

**Index Terms**—Breast Cancer, Contourlet transform, Nakagami distribution, NIG distribution.

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## I. INTRODUCTION

**B**reast cancer in women in a developing country like Bangladesh will predict to be an alarming important cause of death in the next few decades. The Bangladesh Bureau of Statistics revealed that cancer is the sixth leading cause of death. The estimated cancer-related death rates in Bangladesh by International Agency for Research on Cancer, to be 7.5% in 2005 and 13% in 2030. The two principal causes are lung and oral cancer in males and breast and cervical cancer are in females [1]. The floundering of breast cancer occurrence in Bangladesh is estimated to be 22.5 per 100000 females of all ages; where the women, aged between 15-44 years, breast cancer has the highest prevalence 19.3 per 100000 compared to any other type of cancer [2].

Now a day's various imaging techniques are employed in medical image processing such as mammogram, MRI imaging etc. However, ultrasonography has become a most promising tool to determine and classify breast tumors. Ultrasound is particularly a sound waves having frequencies higher than human audible frequencies (>20,000 Hz). The B-Mode image (brightness mode) displays the acoustic impedance of a two-dimensional cross-section of tissue [3].

P. Mohana Shankar compared two Rayleigh and Nakagami parametric images where the author showed Nakagami image had better result than Rayleigh image using cumulative density function (*cdf*). However, the author established a specified formula using Nakagami parameters to classify benign and malignant tumors and had a satisfactory result [4]. Yin-Yin Liao used B-scan and Nakagami ultrasonic images to classify breast tumors. In that paper the B-Mode images were used to calculate the standard deviation of the shortest distance for contour feature analysis. The average Nakagami parameters inside tumor region were estimated in Nakagami imaging. The clinical diagnostic accuracy is showed 81.7% and Nakagami parameter had a diagnostic accuracy of 80% which is quite satisfactory [5]. Wei-Chih Shen developed a computer-aided diagnostic

(CAD) system where the author specified few shape features and Posterior Acoustic Features were employed [6]. Their experimentation concluded of 265 images where 174 were benign and 91 were malignant and the performance report is quite satisfactory. A novel approach employing empirical mode decomposition (EMD) in digital wavelet transform (DWT) domain is represented for classification of breast tumors with a high degree of classification accuracy in [7].

The proposed system is verified with the biopsy result of 250 clinical cases where 100 are benign and 150 are malignant cases. Those benign and malignant B-Mode US best frame images are predefined and separately stored by a radiologist in this database. The objective is to study the classification accuracy of the contourlet transformed Nakagami and NIG distributed breast masses in a comparative analysis. Firstly, the B-Mode image is binarized, which is subjected to contourlet transformation. Afterward, Nakagami and NIG parametric images are estimated from those contourlet coefficients. In the proposed system a marker-based auto segmentation method is employed to detect the lesion contour. The feature extraction process consists of only ten but effective statistical and shape features those are studied on Nakagami and NIG parametric images. Later, the classification accuracies are achieved using five classifiers such as the support vector machine (SVM), k-nearest neighbors (KNN), fitted binary classification decision tree (BCDT), binary Gaussian kernel classification model (BGKC), linear classification models for binary learning with high-dimensional (BLHD) etc.

This paper is organized as the section II represent a short introduction of contourlet transformation. The Nakagami and NIG probability density functions (*pdfs*) are summarized in Section III. The proposed methodology is depicted in Section IV. The experimental results are revealed in Section V and the conclusion is in Section VI.

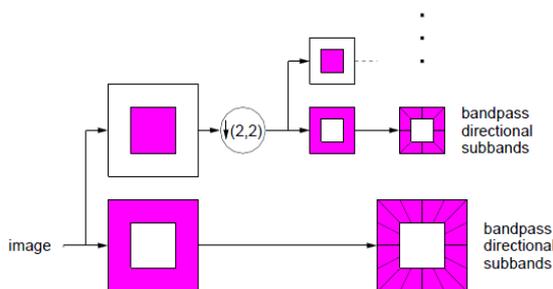


Fig. 1. A conceptual view of contourlet filter bank.

## II. CONTOURLET TRANSFORM DOMAIN

The uniqueness of the contourlet transformation is a filter bank that has the ability to decouple the multiscale and directional decompositions proposed in [8]. The conceptual set up of this multi-resolution transform domain is shown in Fig.1. The Multiscale decomposition is executed using a Laplacian pyramid and subsequent directional decomposition is executed using a directional filter bank those included in the decoupling operation [9].

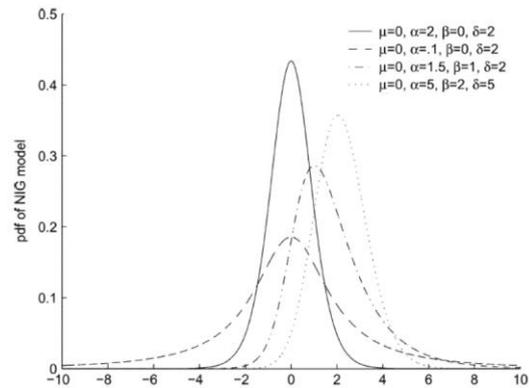


Fig. 2. Examples of the *pdfs* residing to the NIG model.

## III. THE NAKAGAMI AND NIG *PDFS*

The Nakagami parametric image is obtained from the *pdf* of the corresponding contourlet coefficient under the Nakagami statistical model is given in (1):

$$f(r) = \frac{2m^m r^{2m-1}}{\Gamma(m)\Omega^m} \exp\left(-\frac{m}{\Omega} r^2\right) U(r) \quad (1)$$

Here,  $r$  denotes the possible values for the random variable  $R$  of the backscattered envelopes. Nakagami parameter  $m$  is a shape parameter.

The Normal Inverse Gaussian (NIG) is obtained from the *pdf* of the corresponding contourlet coefficient under the NIG statistical model is given in (2):

$$NIG_{pdf} = \frac{\alpha \delta K_1(\alpha \sqrt{\delta^2 + (x-\mu)^2})}{\pi \sqrt{\delta^2 + (x-\mu)^2}} e^{\delta \gamma + \beta(x-\mu)} \quad (2)$$

Where,  $K_j$  denotes a modified Bessel function of the third kind.  $\mu$  is the location parameter,  $\alpha$  controls the tail heaviness,  $\beta$  is asymmetry parameter,  $\delta$  is scale parameter and  $\gamma = \sqrt{\alpha^2 - \beta^2}$ . Examples of the *pdfs* residing in the NIG model are shown in Fig. 2.

## IV. METHODOLOGY

In our proposed method, all the image processing tasks have been carried out using a workstation with an Intel 10th Generation core i7 processor with 6 cores, 2.4 GHz, 16 GB RAM using MATLAB R2020a (MathWorks Inc, USA) software. The step by step execution is summarized in Fig. 3.

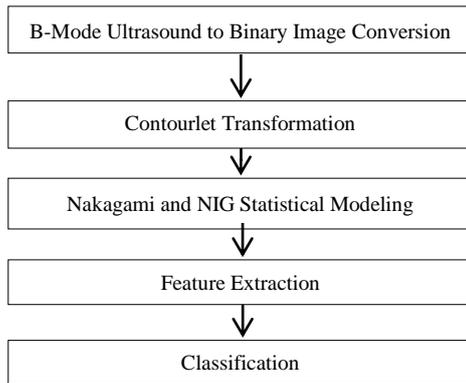


Fig. 3. Proposed methodology for breast tumor classification using classifier.

In this proposed method, 250 B-Mode US images are considered from a publicly available database (Mendeley Data). This database is contributed by Paulo Sergio Rodrigues, available at (<https://data.mendeley.com/datasets/wmy84gzngw/1>) [10]. In this database, there are 100 fibroadenoma (benign) cases and 150 malignant cases are existed. All the images are stored in \*.bmp format. In pre-processing step, as the best US frame is chosen previously and stored by a radiologist in this database, the imprecise lesion boundary region is marked manually in the extracted B-Mode image using MATLAB function ‘imfreehand’. This marker-based auto segmentation method has the capability to avoid over segmentation. The B-Mode image is reduced in pixels according to the manually selected lesion boundary region as 13 pixels are considered outside of highest most, lowest most, left most and right most lesion region. This resized B-Mode image is converted to binary image using MATLAB function ‘imbinarize’ (e.g. Fig. 4b). Few MATLAB function are utilized to determine the boundary region of lesion region of interest (ROI) such as ‘bwboundaries’ and ‘visboundaries’ etc [11]. Example of lesion boundary detection from a malignant mass is shown in Fig. 4. Later, the estimated binary images are subjected to contourlet transformation using MATLAB functions ‘pdfbdec’ which act as a pyramidal directional filter bank and ‘pdfb2vec’ which is used to convert the output of the pyramidal directional filter bank into a vector form. The Unitarian Rule is applied to measure the lesion contour of the contourlet sub-band coefficient images (e.g. Fig. 5). A very few selective contourlet sub-bands are considered in this study to minimize the computational time duration, those can be chosen having maximum resolution in their respective pyramidal decomposition. Those are pyramidal decomposition level-2 with directional

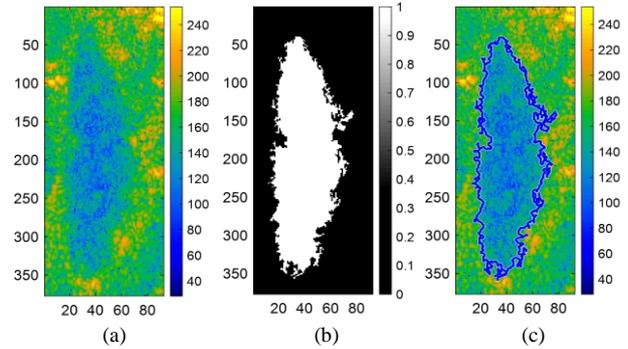


Fig. 4. Example of segmentation through image binarization, (a) B-Mode image, (b) Corresponding Binary image (c) Lesion contour detection.

decomposition level-8 (P2D8), similarly P3D16 and P4D32 etc. It is noted that, the prior contourlet sub-bands are contains maximum resolution in their respective pyramidal multiscale decomposition level. In addition, the. The Nakagami parameter image (e.g. Fig. 5b) is obtained from the pdf of the corresponding contourlet coefficient under the Nakagami statistical model is given in (1). The Nakagami parametric image is based on the Nakagami parameter map, which is constructed following the method depicted in [5], [12]. The NIG parameter image (e.g. Fig. 5c) is based on the NIG parameter map. It has four parameters  $\mu$ ,  $\alpha$ ,  $\beta$  and  $\delta$ .  $\alpha$  and  $\delta$  are measured from the estimation process of Normal Inverse Gaussian (NIG) parameter estimation process letting  $\beta \rightarrow 0$ . The process to obtain NIG parameter image is as same as described for Nakagami parameter image in [12]. Lesion boundary regions of the contourlet transformed Nakagami and NIG images are determined as same coordinates as determined in B-Mode US image (e.g. Fig. 5).

It should be noted that, a large number of ultrasound features does not certainly guarantee the precise classification of breast Tumors. Most of the time, it worsens the performance of the classifier. Moreover, often it needed a high configuration system for computation. In this work, a small number of effective features are chosen based on the criterion used to effective capability in both Nakagami and NIG parametric images. The effectiveness of feature utilization is summarized in terms of variance ( $\sigma$ ) in Table-1 where it is seen that in NIG statistical model-based images the range of feature variances is smaller than that of Nakagami statistical model-based images. In Comparison, few researches were conducted using Nakagami images and had an accuracy of 80% to 81.7%, while accuracy of 92.389% obtained using NIG parametric image is significantly better.

TABLE 1  
The variances ‘ $\sigma$ ’ values in feature extraction from both Nakagami and NIG parametric images

Features with References	Contourlet Transformed Nakagami Image		Contourlet Transformed NIG Image	
	Benign	Malignant	Benign	Malignant
Shape Class [6]	$1.61 \leq \sigma \leq 1.96$	$1.24 \leq \sigma \leq 1.46$	$0.021 \leq \sigma \leq 0.03$	$0.006 \leq \sigma \leq 0.012$
Orientation Class [6]	$0.45 \leq \sigma \leq 0.99$	$1.30 \leq \sigma \leq 1.77$	$1.51 \leq \sigma \leq 1.66$	$2.98 \leq \sigma \leq 3.04$
Margin Class [6]	$0.46 \leq \sigma \leq 1.08$	$0.83 \leq \sigma \leq 1.44$	$0.66 \leq \sigma \leq 0.79$	$1.65 \leq \sigma \leq 1.79$
Lesion Boundary Class [6]	$0.22 \leq \sigma \leq 0.48$	$0.06 \leq \sigma \leq 0.26$	$0.12 \leq \sigma \leq 0.14$	$0.35 \leq \sigma \leq 0.44$
Echo Pattern Class [6]	$0.86 \leq \sigma \leq 1.87$	$1.87 \leq \sigma \leq 2.92$	$0.14 \leq \sigma \leq 0.18$	$0.24 \leq \sigma \leq 0.32$
Texture [14]	$1.03 \leq \sigma \leq 1.39$	$0.75 \leq \sigma \leq 0.97$	$0.029 \leq \sigma \leq 0.038$	$0.005 \leq \sigma \leq 0.015$
Taller Than Wide [15], [16]	$1.45 \leq \sigma \leq 2.68$	$2.82 \leq \sigma \leq 6.38$	$0.21 \leq \sigma \leq 0.36$	$0.68 \leq \sigma \leq 0.85$
Tilted Ellipse Radius [17]	$799 \leq \sigma \leq 3443$	$296 \leq \sigma \leq 1333$	$896 \leq \sigma \leq 3180$	$422 \leq \sigma \leq 1154$
Tilted Ellipse Perimeter [17]	$0.83 \leq \sigma \leq 1.03$	$0.91 \leq \sigma \leq 1.07$	$0.89 \leq \sigma \leq 0.98$	$0.92 \leq \sigma \leq 0.99$
Tilted Ellipse Area [17]	$1.23 \leq \sigma \leq 2.01$	$0.99 \leq \sigma \leq 1.68$	$1.09 \leq \sigma \leq 1.58$	$0.91 \leq \sigma \leq 1.14$
Tilted Ellipse Compactness [17]	$0.93 \leq \sigma \leq 1.72$	$0.90 \leq \sigma \leq 1.46$	$0.91 \leq \sigma \leq 1.08$	$0.98 \leq \sigma \leq 1.19$
Mean $klv$ [18]	$1.81 \leq \sigma \leq 2.52$	$1.31 \leq \sigma \leq 2.08$	$1.33 \leq \sigma \leq 1.93$	$1.12 \leq \sigma \leq 1.42$
Mean $klb$ [19]	$2.59 \leq \sigma \leq 3.83$	$1.91 \leq \sigma \leq 2.87$	$1.85 \leq \sigma \leq 2.22$	$1.78 \leq \sigma \leq 2.09$

### V. EXPERIMENTAL RESULTS

In this proposed method, each clinical case is subjected to contourlet transformation in the prior specified three sub-band coefficients (i.e.  $250 \times 3 = 750$  coefficient images). The best fit tilted ellipses are simulated for measuring geometric features. Five different classifiers are performed for classification purpose. In this context the whole dataset is randomly divided into several groups for training and testing purposes. The 10-fold cross validation process is applied in such a manner that 10% of total clinical cases are chosen as the testing set meanwhile the remaining 90% clinical cases are chosen as the training set. The process is continued until training and testing of all clinical cases are accomplished. The achieved results are compared with the pathological result given in the database which is regarded as the standard. In Table-2, the values of accuracy rates are gathered for both Nakagami and NIG distributions and it is noticed that NIG distribution attained highest classification accuracy rate like 96.38% using KNN classifier. It is also observed that the NIG *cdf* follows the empirical *cdf* more precisely than Nakagami *cdf* in *pp*-plot shown in Fig. 6 which is simulated using (3), (4) from [13]

$$F_e(x)^t = \frac{2}{\pi} \arcsin(\sqrt{F_e(x)}) \quad (3)$$

$$F_a(x)^t = \frac{2}{\pi} \arcsin(\sqrt{F_a(x)}) \quad (4)$$

Instead of scrutinizing a huge number of features, a set of non-redundant features can be more effective in classification purpose. So, our proposed method revealed a small number of effective features those are shown to be sufficient to have a satisfactory result.

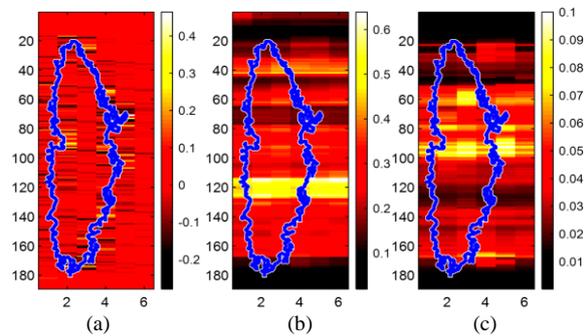


Fig. 5. Example of parametric imaging with lesion contour (a) Contourlet coefficient image decomposed at pyramid level-4 with directional level-32 (P4D32), (b) Corresponding Nakagami parametric image (c) Corresponding NIG parametric image.

### VI. CONCLUSION

In this paper the Normal Inverse Gaussian (NIG) distribution is presented as an appropriate distribution for classification of breast tumor by statistical modelling of contourlet coefficient images. In comparison with Nakagami distribution, the NIG distribution showed comparatively better performance in prior statistical and shape features. Among the other multi-resolution transform domains, the contourlet transform domain is chosen because it has a property like directional decomposition in various dimensions. Moreover, when the pyramidal decomposition levels climbing upward, its directionalities get more verities. This study is perceived on a small data of 250 clinical cases from a publicly available database. A vast study is needed considering a wide range of patient’s data with more than 10 contourlet sub-bands to establish this proposed method. In upcoming works, statistical

model-based breast tumor classification will be experienced on Dual-Tree Complex Wavelet Transform (DT-CWT) and Curvelet Transform domains etc.

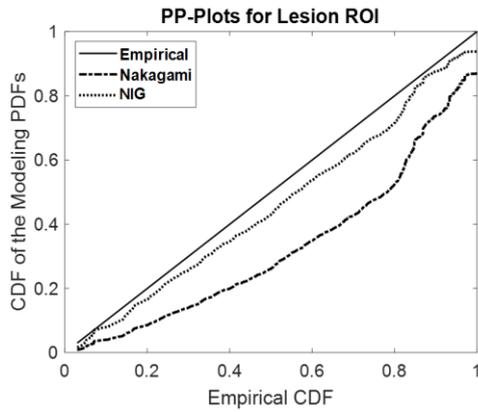


Fig. 6. Example of percentile probability plots (*pp-plots*) of Nakagami, NIG and empirical Cumulative density functions (cdfs). Here, it is shown that the NIG *cdf* follows the empirical *cdf* more precisely than Nakagami *cdf*.

TABLE 2

The performance indices using different classifiers

Classifiers	Accuracy Rate%	
Nakagami	SVM	84.779%
	KNN	85.168%
	BCDT	83.592%
	BGKC	84.413%
	BLHD	84.003%
NIG	SVM	91.552%
	KNN	<b>92.389%</b>
	BCDT	91.294%
	BGKC	91.331%
	BLHD	92.112%

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