LINEAR NUCLEAR ACOUSTIC WAVES IN DEGENERATE QUANTUM PLASMA

M. Hasan and D. M. S. Zaman

Abstract—A rigorous theoretical investigation has been made on the linear propagation of electrostatic perturbation modes of degenerate pressure driven modified nucleus-acoustic (DPDMDA) waves in a degenerate quantum plasma (DQP) system. It contains cold inertia-less degenerate electron species (DES), cold inertial non-degenerate light nucleus species (LNS) and stationary heavy nucleus species (HNS) which maintains the quasi-neutrality condition at equilibrium only. The mass density of the cold LNS provides the inertia and the cold inertia-less LNS provides the inertia and the cold inertia-less DES gives rise to the restoring force. The reductive perturbation method has been used for the study of nonlinear propagation of the DPDMDA waves. The basic features of linear waves are supervised in a theoretical manner. It has been observed that the phase speed of the DPDMDA waves changes with the change of density of the stationary HNS for both non-relativistic and ultra-relativistic DES; The NA waves with their dispersion properties which are consequential in various astrophysical and laboratory plasmas, have been broadly considered.

Index Terms—Degenerate electron, Nucleus-acoustic waves, Quantum plasma.

I. INTRODUCTION

In recent years, modern plasma physics is very much concentrated on the degenerate quantum plasma (DQP) system [1]. Through plasma physics is sometime considered as classical field, it may be quantum because of its particle’s quantum nature [2]. The DQP systems are significantly different from other plasma systems because of their extraordinarily high density particularly, for astrophysical plasma systems [3, 4, 5, 6, 7, 8] and low temperature particularly, for laboratory plasma systems [9, 10, 11, 12, 13]. They usually contain inertialess non-relativistically (NR) or ultra-relativistically (UR) degenerate electron species (DES), cold inertial non-degenerate light nucleus species (LNS) like $^4\text{He}$ [5] or $^{12}\text{C}$ or $^{16}\text{O}$ [4, 6] and heavy nucleus species (HNS) like $^{56}\text{Fe}$, $^{87}\text{Rb}$ or $^{96}\text{Mo}$ [14, 15].

The degeneracy of the cold electron species (in such DQP systems) arises due to Heisenberg’s uncertainty principle, and the uncertainty in momenta of extremely DES are infinitely large. This introduces a pressure called ‘degenerate pressure’ which depends only on the cold electron number density, but not on the electron thermal temperature. The degenerate pressure $P$ exerted by the DES is given by [16, 17, 18, 19, 20, 21, 22]

$$P = Kn_c\gamma,$$

where $n_c$ is the number density of the DES, and $\gamma$ and $K$ are

$$\gamma = \frac{5}{3}, \quad K = \frac{3}{5}\left(\frac{\pi}{3}\right)^{\frac{1}{2}}\frac{\hbar^2}{m} \approx \frac{3}{5}\Lambda_0 c, \hbar c,$$

for the non-relativistic limit [8, 17], and

$$\gamma = \frac{4}{3}, \quad K = \frac{3}{4}\left(\frac{\pi}{9}\right)^{\frac{1}{2}}\hbar c \approx \frac{3}{4}\hbar c,$$

for the ultra-relativistic limit [8, 17],

where $\Lambda_0 = \pi\hbar/m_n c$, $\hbar$ is the reduced Planck’s constant, and $m_n$ is the rest mass of the cold DES.

The DQP systems are significantly different from other plasma systems not only because they have unique properties (extra-ordinarily low density and temperature), but also because they introduce new kinds of waves. One of these kinds of waves is predicted here. The present work is attempted to find the possibility for the existence of such a new kind of waves, called here degenerate pressure-driven modified nucleus-acoustic (DPDMDA) waves, in which the inertia is provided by the mass density of the cold inertial LNS, and the restoring force is provided by the degenerate pressure of the cold inertia-less NR or UR DES, and to identify the dispersion properties of the linear DPDMDA waves propagating in such a DQP system. A linear system is a system in which the output is linearly proportional to input. For an example, an amplifier (or any other system) to an input signal

$$y = y_o \cos(\omega t)$$

can be considered, where $\omega$ is the signal frequency resulting in an output which can be expressed as output $\alpha$ input in case of small amplitude i.e the response of the system is linear to its input. In linear theory, the wave amplitude is assumed to be sufficiently small to ignore contributions of terms of second order or higher (i.e., nonlinear terms) in wave
amplitude. However, when the wave amplitude becomes larger, the linear approximation breaks down and nonlinear effects must be taken into account.

A DQP system is considered containing non-inertial DES [5] represented by the subscript \(e\), and inertial LNS (which can be \(^{3}He\) or \([4, 6]\) \(^{12}\)C or \(^{16}\)O [4, 6]) represented by the subscript \(l\), and the HNS (which can be \(^{56}\)Fe, \(^{88}\)Rb or \(^{96}\)Mo [14, 15]) represented by the subscript \(h\). Thus, at equilibrium we have \(n_0 = Z_l n_{l0} + Z_h n_{h0}\), where \(n_0\) is the number density of the DES (LNS) HNS and \(n_{e0}\) is the charge state of the NLNS (SHNS). The dynamics of the DES is describe by the balance between the electrostatic force (\(\varepsilon e\phi\)) and the degenerate pressure force (\(\partial P/\partial x\)). Thus, substituting (1) into \(n_e \partial \phi/\partial x = \partial P/\partial x\), the number density \(n_e\) of the DES can be expressed as

\[
n_e = n_{e0} \left[ 1 + \left( \frac{\gamma - 1}{\gamma} \right) \frac{e\phi}{\varepsilon_{e0}} \right]^{\frac{1}{\gamma - 1}}, \tag{5}
\]

where \(\varepsilon_{e0} = K n_{e0}^{-\gamma}\) is the energy associated with the electron degenerate pressure; \(\phi\) is the electrostatic wave potential; \(x (t)\) is the space (time) variable; \(e\) is the charge of a proton.

We note that (5) is valid for the DPDMNA waves, whose phase speed is much smaller than \(C_e\) where \(C_e = (\varepsilon_{e0}/m_e)^{1/2}\)

II. MATHEMATICAL MODEL

The dynamics of these DPDMNA waves is described by

\[
\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} (n u) = 0, \tag{6}
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = - \frac{Ze e\phi}{m_i} + \eta \frac{\partial^2 u}{\partial x^2}, \tag{7}
\]

\[
\frac{\partial^2 \phi}{\partial x^2} = 4\pi e (n_e - Z_l n_l - Z_h n_{h0}), \tag{8}
\]

where \(n_i (u)\) is the number density (fluid sped) of the cold inertial LNS \(l\) and \(\eta\) is the co-efficient of the viscous fluid of light nuclei. It may be noted here that (i) the effect of the self-gravitational field is neglected since it is inherently small in comparison with the electric field in the DQP systems under consideration; (ii) the effect of the degeneracy of the cold LNS is neglected since the DPDMNA wave phase speed is much larger than \(C_l\) where \(C_l = (\varepsilon_{l0}/m_i)^{1/2}\) and \(\varepsilon_{l0} = K n_{l0}^{-\gamma}\). \(\varepsilon_{l0}\) is the degenerate energy of the cold LNS, \(K = 3\Lambda_{cl} \hbar c/5m_i\), and \(\Lambda_{cl} = \pi \hbar /mc\); (iii) the consideration of stationary HNS is valid as long as \(\omega_{ph} \ll \omega\) where \(\omega_{ph} = (4\pi n_{h0} Z^2 Z^2 / m_h)^{1/2}\) is the heavy nucleus plasma frequency, \(\omega\) is the frequency of the DPDMNA waves, and \(m_h\) is the mass of the HNS.

To find the linear dispersion relation for the DPDMNA waves, (6)–(8) are linearized to reduce them to a set of linear equations:

\[
\frac{\partial \tilde{n}_l}{\partial t} + \frac{\partial}{\partial x} (n_{l0} \tilde{u}_l) = 0, \tag{9}
\]

\[
\frac{\partial \tilde{u}_l}{\partial t} = - \frac{Ze e\phi}{m_i} + \eta \frac{\partial^2 \tilde{u}_l}{\partial x^2}, \tag{10}
\]

\[
\frac{\partial^2 \tilde{\phi}}{\partial x^2} = 4\pi e \left( \frac{n_{e0} e\phi}{\varepsilon_{e0}} - Z_l \tilde{n}_l \right), \tag{11}
\]

where \(\tilde{n}_l, \tilde{u}_l, \) and \(\tilde{\phi}\) are the perturbed parts of the quantities involved. It is now assumed that all of these perturbed quantities are directly proportional to \(\exp(-i\omega t + ikx)\), where \(k\) is the propagation constant of the DPDMNA waves. This assumption reduces (9)–(11) to the linear dispersion relation for the DPDMNA waves:

\[
\tilde{\omega}^2 (1 + \alpha \gamma k^2) + i \tilde{\omega} \eta (1 + \alpha \gamma k^2) - k^2 \alpha \gamma = 0, \tag{12}
\]

where \(\alpha = Z_l n_{l0} / n_{e0}\) with \(\mu = 1 - \alpha\). Here, \(\omega, k\) and \(\eta\) are normalized by \(\omega_{pl}, L^{-1}\) and \(\omega_{pl} L^2\) respectively.
Fig. 3: The dispersion curves of the DPDMNA waves for $\gamma = 5/3$, $\alpha = 0.9$ and $\eta = 0.1$ (dashed curve), $\eta = 0.2$ (solid curve) and $\eta = 0.3$ (dotted curve).

where $L_q = (E_{e0}/4\pi Z n_{i0})^{1/2}$ is the modified DQP screening length; $\omega_{pl} = (4\pi n_{i0}Z^2 n_{i0}/m_i)^{1/2}$ is the light nucleus plasma frequency.

TABLE I: The basic differences between the new DPDMNA waves and the well-known IA waves.

<table>
<thead>
<tr>
<th>Properties</th>
<th>DPDMNA Waves</th>
<th>IA Waves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Restoring force</td>
<td>Degenerate pressure</td>
<td>Thermal pressure</td>
</tr>
<tr>
<td>Phase speed at $T_e = 0$</td>
<td>$\sqrt{\frac{e_0}{m_i}}$</td>
<td>$\sqrt{\frac{m_i}{e_0}}$</td>
</tr>
<tr>
<td>Existence at $T_e = 0$</td>
<td>Possible</td>
<td>Not possible</td>
</tr>
<tr>
<td>Length scale</td>
<td>$\sqrt{\frac{E_{e0}}{4\pi Z_0 n_{i0}^2}}$</td>
<td>$\sqrt{\frac{k_B T_e}{4\pi Z_0 n_{i0}^2}}$</td>
</tr>
</tbody>
</table>

(i) The existence of a new kind of nucleus-acoustic waves (named here DPDMNA waves) with viscous force propagating in DQP systems is predicted for the first time. (ii) The DPDMNA waves are new not only from the view of the inertia kind of waves), but also from the view of their phase speed and length scale as shown in Table I.

(iii) The DPDMNA waves completely disappear if the degenerate pressure associated with DES is neglected.

(iv) The phase speed $V_p$ of the DPDMNA waves decreases with the rise of the charge number density of the stationary HNS in both non-relativistic and ultra-relativistic situations of DES. This is due to the fact that as the charge number density of the stationary HNS increases, the repulsive force (acting between positive LNS and HNS) reduces frequency of the compression and rarefaction of the LNS. We finally hope that the DPDMNA waves and their dispersion properties with viscous force (identified here for the first time) should be useful in understanding the basic features of the electrostatic perturbation mode in space [3, 4, 5, 6] and laboratory [9, 10, 11] DQP systems.

Author contribution statement
Both of the authors contributed equally to the paper.

REFERENCES

M. Hasan was born in Dhaka, Bangladesh, in 1994. She received the B.Sc. (Hons.) and M.S. degrees in physics from Jahangirnagar University, Dhaka, Bangladesh. Her MS project work was mainly concerned with nuclear-acoustic waves in degenerate quantum plasmas. Her future works include nuclear-acoustic waves in self-gravitating degenerate quantum plasmas. He was a member of Jahangirnagar University Physics Club as well as Jahangirnagar University Physics Club.

D. M. S. Zaman was born in Tangail, Dhaka (Bangladesh), in 1992. He received the B.Sc. (Hons.) and M.S. (Thesis) degrees in physics from Jahangirnagar University, Savar, Dhaka-1342, Bangladesh. He successfully completed his M. S. thesis under the direct supervision of Prof. Dr. Abdullah Al Mamun. His M. S. thesis was mainly concerned with nuclear-acoustic waves in self-gravitating degenerate quantum plasmas. He was awarded "National Science and Technology Fellowship 2016-17" as a researcher.

Since May 2018, he has been a Lecturer of Physics with the Electrical and Electronics Engineering (EEE) Department, Green University of Bangladesh. He has already authored four articles in prestigious international peer-reviewed journals like European Physical Journal Plus, Chinese Physics B, Journal of the Physical Society of Japan, and High Temperature. Mr. Zaman is also working as a member of editorial board of the International Journal of Mathematical Physics. He is an honorary member of research group "SELF" (Self Education and Learning Forum).

His research interests include Medical Physics and Biophysics, Plasma Physics, and Material Science. He is now working on a project entitled "Exhaled gas detection by Rh-doped Tin (Sn5) nanocluster for prediagnosis of lung cancer: a DFT study" with Dr. Kabir Uddin Sikder, Associate Professor, Department of Physics, Jahangirnagar University, Savar, Dhaka, Bangladesh.