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Valuation of Currency Options with Stochastic Interest Rates and Uncertain Exchange Rate

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ABSTRACT

There are numerous currency models, but only a few are suitable for practical use. The Liu-Chen-Ralescu (L-C-R) model, proposed by Liu, Chen, and Ralescu, is one of them, where the exchange rate abides by an uncertain differential equation. However, the model treats interest rates as constants, which is unrealistic given the fluctuations in financial markets caused by various human-caused or natural disasters. Our preliminary intention was to explore a model that incorporates additional parameters such as log drift, log diffusion, and variance of the observed data. We discovered a new currency model, referred to as the X-W model, which was proposed by Xiao Wang in his 2019 research article. He proposed that an uncertain differential equation governs the exchange rate, while stochastic differential equations govern both domestic and foreign interest rates. Since the X-W model considers various circumstances and calculates more parameters, the option prices are assumed to be higher. We find the call and put currency option prices for the X-W model using Euler's method and trapezoidal rule in this article and then the option prices have been compared with the currency option prices obtained from the L-C-R model. Finally, we present some numerical and graphical results for both models using MATLAB coding for better observation. We were successful in finding our expected results and can finally conclude that the X-W model is a better fit for the real market.

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1. Introduction

Currency options are highly popular investment tools nowadays, mitigating potential risk against unfavorable movements in the exchange rate. A currency option is a contract that grants the client the authority to buy (call) or sell (put) a specific currency at a pre-agreed exchange rate (strike price) on the European option or before the American option on a precise date (maturity date) without any obligation. Therefore, financial mathematics also demands a discussion of the theoretical models used to determine the prices of currency options. A Chinese mathematician, Ho C. Yang, derived formulas for European currency options similar to Merton's option pricing models [1, 2]. He considered risk-free domestic and foreign interest rates and spot exchange rates. However, the actual risk-free parameters in investments are merely theoretical concepts. Interest rates rise and fall from time to time in the natural financial market. A stochastic process is a probability model that outlines the random phenomenon, like interest rates, over time. The

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stochastic differential equation (SDE) is the key to calculating the probability of such unplanned occurrences. Kazi Mehedi Mohammad and Md. Kamrujjaman used SDE to model influenza transmission with continuous and discrete time, which strengthens our understanding of influenza epidemiology, deterring methods, and possible cures [3].

Hilliard, Madura, and Tucker developed a currency option pricing model in which interest rates follow a stochastic differential equation when interest rate parity holds. They assumed domestic and foreign bond prices to have local variances depending only on time [4]. However, their model's arithmetic random walk process suggested an unrealistic shape for the instantaneous short interest rates. So, Sanjay K. Nawalkha and Donald R. Chambers attempted to include stochastic domestic and foreign interest rates by applying the Heath, Jarrow, and Morton framework for the unfluctuating forward rate volatility term structure model [5, 6].

Emergencies stemming from wars, political policies, or natural disasters can impact exchange rates. During these times, acquiring reliable statistical data about exchange rates may be burdensome, assuming that exchange rates follow a stochastic differential equation less applicable. Liu established the uncertainty theory to stand against the challenges of collecting statistical data on exchange rates affected by wars, political issues, pandemics, or natural disasters [7]. Uncertainty theory is a special branch of mathematics used to study the phenomenon of uncertain behavior. Later, he proposed the Uncertain Process and the Liu Process to describe the growth of an uncertain event [8, 9]. In 2014, Y. Liu, X. Chen, and D. A. Ralescu introduced the L-C-R model, a currency option model that considers interest rates (both domestic and foreign) as constants, while the exchange rate is subject to an uncertain differential equation [10].

However, considering the fluctuations of interest rate markets, treating interest rates as constants is unreasonable. When a huge amount of documented data about interest rates is available, the stochastic process usually describes the short interest rates. Morton has proposed the first-interest rate model under the framework of probability theory. The spot interest rate is a diffusion process, and the Wiener process is used to calculate such a stochastic process in continuous time with independent time intervals. Kazi Mehedi Mohammad, Mayesha Sharmim Tisha, and Md. Kamrujjaman used the Wiener process to analyze the stochastic behavior of contagious viruses to prevent disease outbreaks [11]. An interest rate model is a mathematical method to describe the movement of interest rates. Many other specialists in economics and mathematics have proposed other models, like the Hull-White model and the Vasicek model [12, 13]. Both models imply the Wiener process to describe the evolution of interest rates.

Xiao Wang has proposed a new currency model, known as the X-W model, in 2019. He assumes that the currency rate follows a stochastic differential equation (SDE), while the exchange rate follows an uncertain differential equation (UDE). He has employed the Vasicek model to establish the X-W model [14].

The paper is sorted in this way: in Section 2, the L-C-R model and the X-W model have been discussed here along with their respective pricing formulas. In section 3, we show our results and discussions, and lastly, in section 4, we conclude the model that will be more appropriate to use in the real financial market.

Nomenclature	
$X(t)$	risk-free domestic currency
$Y(t)$	risk-free foreign currency
$r(t)$	native (domestic) interest rate
$f(t)$	foreign interest rate
$Z(t)$	spot native currency price of a unit of foreign exchange at
K	strike price
$C(t)$	Liu process
$B(t)$	Brownian motion
μ	log-drift of $Z(t)$
σ	log-diffusion of $Z(t)$
C_E	European call currency option price
P_E	European put currency option price

2. Mathematical Model Formulation and Methodology

In this section, we introduce the two currency models we are going to compare and provide the option pricing formulas under these models.

2.1. L-C-R Model

The Liu-Chen-Ralescu (L-C-R) model is given by the following equations [10]-

$$\begin{aligned} dX(t) &= r\,X(t)dt, \\ dY(t) &= f(t)dt, \\ dZ(t) &= \mu\, Z(t)dt + \sigma\, Z(t)dC(t). \end{aligned} \tag{2.1}$$

C_E and P_E under this model are given respectively by,

$$C_E = \frac{1}{2}\,\mathbb{E}[Z(t)-K]^+e^{-rt}] + \frac{Z(0)}{2}\mathbb{E}\left[\left(1-\frac{K}{Z(t)}\right)^+ e^{-ft}\right]. \tag{2.2}$$

and

$$P_E = \frac{1}{2}\,\mathbb{E}\left[K-Z(t)\right)^+e^{-rt}] + \frac{Z(0)}{2}\mathbb{E}\left[\left(\frac{K}{Z(t)}-1\right)^+ e^{-ft}\right]. \tag{2.3}$$

2.2. X-W Model

The Vasicek model has been employed for both interest rates to establish the new model. In this model, the exchange rate is assumed to follow an uncertain differential equation. The proposed new currency model (X-W) looks like this [14]-

$$\begin{aligned} dr(t) &= a_1\big(b_1 - r(t)\big)dt + \sigma_1 dB_1, \\ df(t) &= a_2\big(b_2 - f(t)\big)dt + \sigma_2 dB_2, \\ dZ(t) &= \mu\, Z(t)dt + \sigma\, Z(t)dC(t). \end{aligned} \tag{2.4}$$

where,
σ_1 is the diffusion of $r(t)$ and σ_2 is the diffusion of $f(t)$.
a_1, a_2, b_1, b_2 are constant parameters and $B_i(t)$ and $C(t)$ are independent for $i = 1,2$.
We will denote $r(0)$, $f(0)$ and $Z(0)$ as r_0,
f_0 and Z_0 respectively.
So, C_E under the X-W model is given by,

$$\begin{aligned} C_E = &\frac{1}{2}\int_0^1 (Z_t^\alpha - K)^+ d\alpha \exp\left(-b_1T - \frac{(1-e^{-a_1t})(r_0-b)}{a_1} + \frac{\sigma_1^2}{2a_1^2}\left[T - \frac{2}{a_1}(1-e^{-a_1T}) + \frac{1}{2a_1}(1-e^{-2a_1T})\right]\right) \\ &+ \frac{Z_0}{2}\int_0^1 \left(1-\frac{K}{Z_t^\alpha}\right)^+ d\alpha \exp\left(-b_2T - \frac{(1-e^{-a_2T})(f_0-b)}{a_2} + \frac{\sigma_2^2}{2a_2^2}\left[T - \frac{2}{a_2}(1-e^{-a_2T}) + \frac{1}{2a_2}(1-e^{-2a_2T})\right]\right). \end{aligned} \tag{2.5}$$

And P_E under the X-W model is given by,

$$\begin{aligned} P_E = &\frac{1}{2}\int_0^1 (K - Z_t^\alpha)^+ d\alpha \exp\left(-b_1T - \frac{(1-e^{-a_1t})(r_0-b)}{a_1} + \frac{\sigma_1^2}{2a_1^2}\left[T - \frac{2}{a_1}(1-e^{-a_1T}) + \frac{1}{2a_1}(1-e^{-2a_1T})\right]\right) \\ &+ \frac{Z_0}{2}\int_0^1 \left(\frac{K}{Z_t^\alpha}-1\right)^+ d\alpha \exp\left(-b_2T - \frac{(1-e^{-a_2T})(f_0-b)}{a_2} + \frac{\sigma_2^2}{2a_2^2}\left[T - \frac{2}{a_2}(1-e^{-a_2T}) + \frac{1}{2a_2}(1-e^{-2a_2T})\right]\right). \end{aligned} \tag{2.6}$$

3. Results and Discussion

Let us consider a European call currency option with [14]

- Native interest rate, $r(t) = 1.2\%$,
- Foreign interest rate, $f(t) = 0.32\%$,
- Exercise price, $K = \$6.68$,
- Initial exchange rate, $Z_0 = \$6.65$,
- Mean, $\mu = 0.0065$,
- Variance, $\sigma = 0.0001$,
- Maturity date, $T = 150$ years.

Table 3.1. Call Currency Option Price (C_E).

Maturity Date (y)	L-C-R Model	X-W Model
0	0	0
15	0.5256	0.5390
30	0.9623	0.9935
45	1.3015	1.3505
60	1.5603	1.6266
75	1.7528	1.8354
90	1.8909	1.9882
105	1.9844	2.0948
120	2.0416	2.1632
135	2.0695	2.2004
150	2.0738	2.2121

The above table shows C_E under the mentioned models. As we can see, in Table 3.1, the X-W model gives higher results than the L-C-R model.

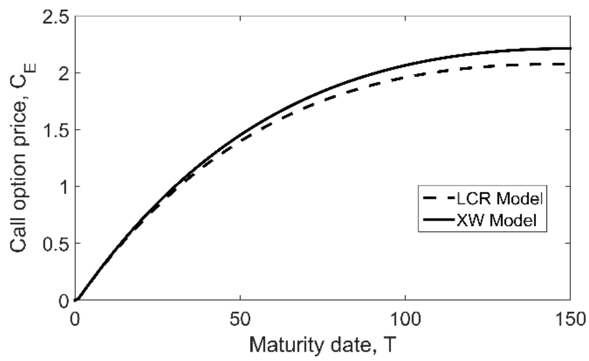


Fig. 3.1. C_E under the L-C-R model and the X-W model.

Since the X-W model involves many parameters, like rate of adjustment and volatility, a question can arise if the new model is time consuming or not. So, we compute the computational (CPU) time.

Table 3.2. CPU Time in Milliseconds for C_E.

Maturity Date (y)	L-C-R Model	X-W Model
0	5.166	6.656
15	0.020	0.014
30	0.011	0.012
45	0.011	0.012
60	0.011	0.012
75	0.019	0.019
90	0.019	0.019
105	0.011	0.012
120	0.022	0.014
135	0.011	0.012
150	0.013	0.013

As shown in Table 3.2, there is no notable difference between the CPU time of the two models. This indicates that even though the X-W model utilizes numerous parameters, it simplifies the computation of currency option prices. Similarly, we find P_E under the models. We use the same parameters as the European call currency option except the exercise price, $K = \$17$.

Table 3.3 shows the put currency option prices under the mentioned models. As we can see in Table 3.3, the X-W model gives higher results than the L-C-R model.

Table 3.3. Put Currency Option Price (P_E).

Maturity Date (y)	L-C-R Model	X-W Model
0	9.8328	9.8328
15	7.8070	8.0054
30	6.1220	6.3207
45	4.7232	4.9009
60	3.5649	3.7162
75	2.6084	2.7312
90	1.8214	1.9151
105	1.1764	1.2419
120	0.6504	0.6891
135	0.2240	0.2381
150	0	0

We show these results in the following graph.

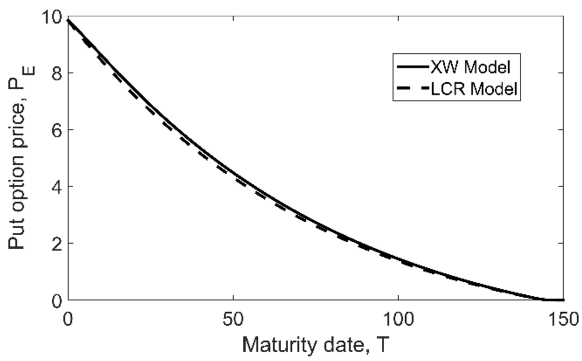


Fig. 3.2. P_E under the L-C-R model and the X-W model.

Now again, we compute the computational time and present the result in Table 3.4.

Table 3.4. CPU Time in milliseconds for P_E.

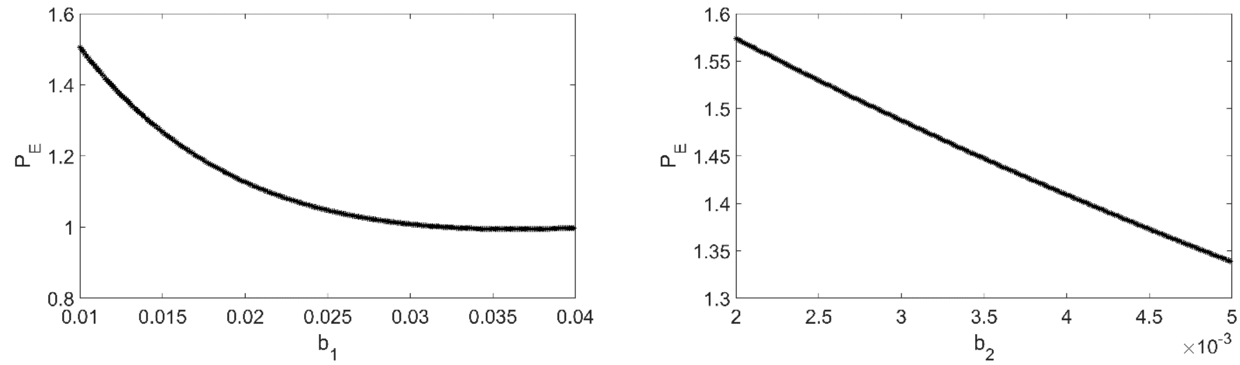
Maturity Date (y)	L-C-R Model	X-W Model
0	5.164	6.463
15	0.014	0.013
30	0.012	0.012
45	0.011	0.012
60	0.013	0.013
75	0.017	0.017
90	0.020	0.020
105	0.012	0.012
120	0.025	0.021
135	0.011	0.013
150	0.018	0.014

We find no significant difference in the computational time for the put currency option price as well. That means we can easily compute the currency option prices while considering other parameters that may affect the native and foreign interest rates.

Since the r and f of the X-W model depend on a few parameters (rate of adjustment, long-run average value, and volatility), the question of how these parameters affect the currency option price may arise. We will show the results for P_E. We have obtained the formula for P_E in equation (2.6).

Now, we have, $a_1 T + \exp(-a_1 T) - 1 > 0 \ (a_1, T > 0)$,

So, $\exp(-b_1 T - \frac{b_1}{a_1}(\exp(-a_1 T) - 1) = \exp\left(-\frac{b_1}{a_1}(a_1 T + \exp(-a_1 T) - 1\right)$ is decreasing with b_1 and so P_E is also decreasing with b_1.



(a) P_E versus b_1. (b) P_E versus b_2.

Fig. 3.3: European put currency option price against long-run average value.

Similarly, we have

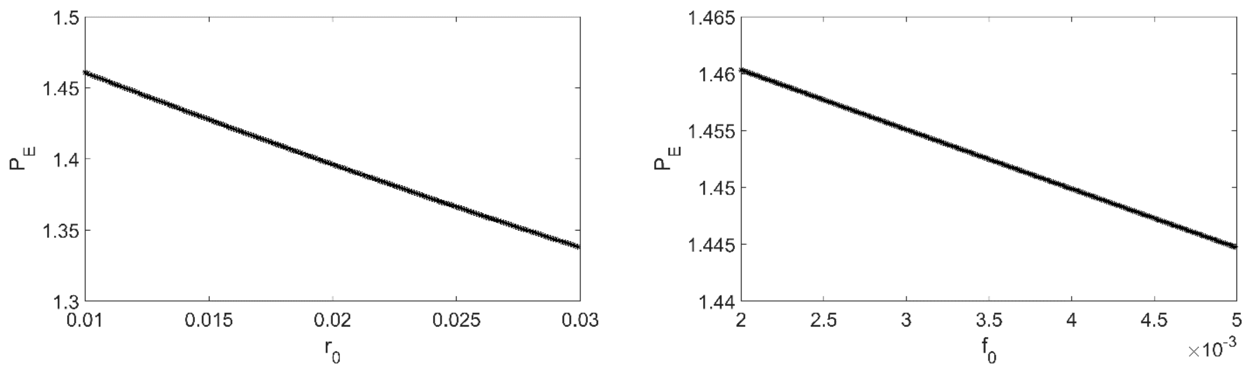
$$a_2T + \exp(-a_2T) - 1 > 0 \; (a_2, T > 0).$$

And that's why,

$\exp\left(-b_2T - \frac{b_2}{a_2}(\exp(-a_2T) - 1)\right) = \exp\left(-\frac{b_2}{a_2}(a_2T + \exp(-a_2T) - 1\right)$ is decreasing with b_2 and so P_E is also decreasing with b_2.

As we can see, if the long-run average value for either interest rate increases (which means the financial market risk is reduced), then the put currency option price decreases.

Again, $\exp(-r_0)$ and $\exp(-f_0)$ are decreasing with r_0 and f_0. Hence, P_E is decreasing with r_0 and f_0. This also makes sense because an increase in interest rates signals a decrease in the financial market as it becomes safer and so P_E decreases. Fig 3.4 shows this result.



(a) P_E versus r_0. (b) P_E versus f_0.

Fig. 3.4: European put currency option price against native and foreign interest rates.

So, P_E is decreasing with μ and Z_0. That means if the log-drift of the spot currency exchange rate increases (which makes the financial market less risky) or the initial spot price of the exchange rate increases, then the put currency option price decreases. And P_E increases with the strike price, K.

When the strike price (pre-determined exchange rate) is higher, that means the chance of the contract not being exercised is higher, which means this is risky for the put currency option holder. Therefore, the price of the put currency option rises in tandem with the strike price.

The holder of a European call currency option experiences the opposite situation. European call currency option prices decrease with the increase of the interest rates, but the currency option price is increasing with μ and Z_0 and decreasing with K. Because the payoff of C_E includes the terms,

$$Z_t^\alpha - K = \exp\left(\mu T + \sigma T \Omega^{-1}(\alpha)\right) - K,$$

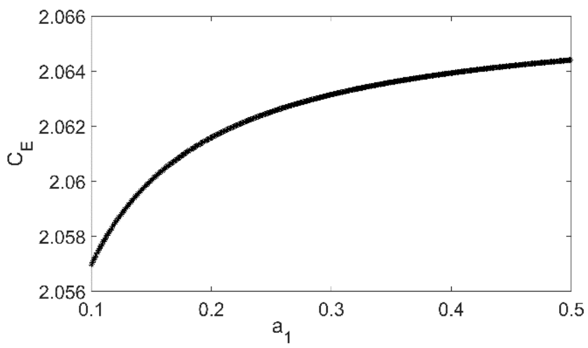
and

$$Z_0\left(1 - \frac{K}{Z_t^\alpha}\right) = Z_0 - \frac{K}{\exp\left(\mu T + \sigma T \Omega^{-1}(\alpha)\right)}.$$

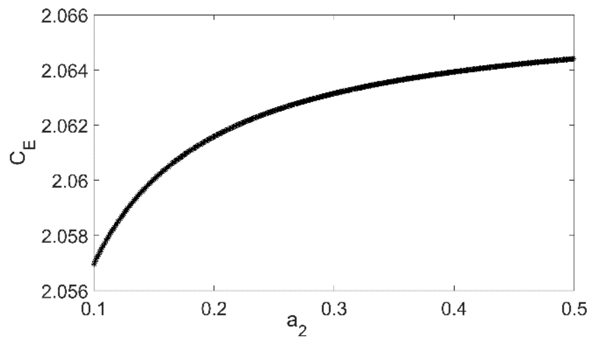
Since K, Z_0 and μ are always positive, C_E increases with Z_0 and μ while decreases with K.

If the interest rates change frequently, then the adjustment rate gets higher and the contract becomes riskier. Therefore, an increase in the domestic rate of adjustment (a_1) leads to an increase in the prices of currency options, both call and put, as the deal becomes riskier.

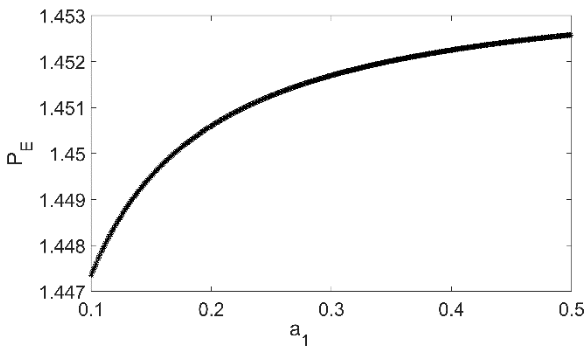
Contrarily, when the foreign rate of adjustment (a_2) increases, the option prices (both call and put) decrease, as with the increase of a_2 of the market, it has been less sensitive for the currency option holder.



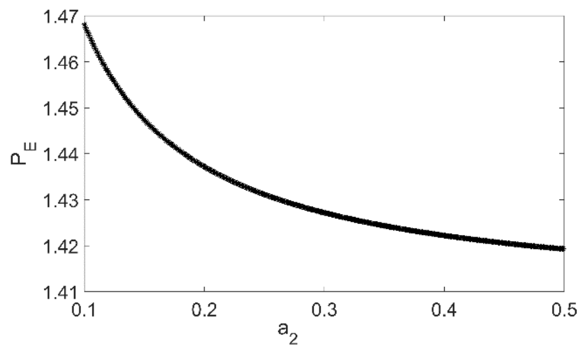
(a) C_E versus a_1.



(b) C_E versus a_2.



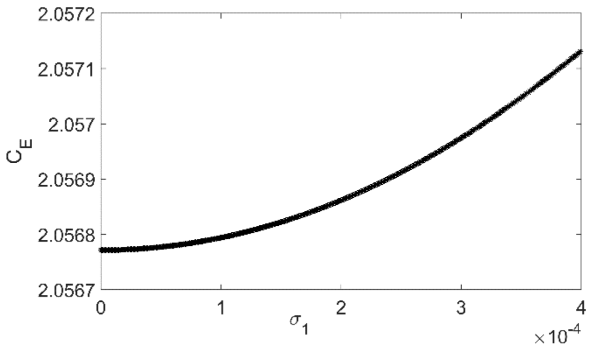
(c) P_E versus a_1.



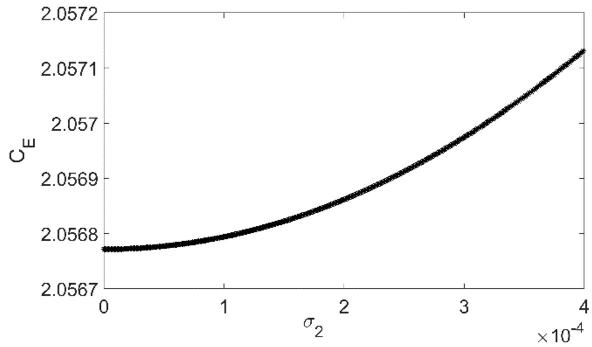
(d) P_E versus a_2.

Fig. 3.5. European currency option prices against the rate of adjustments of the interest rates.

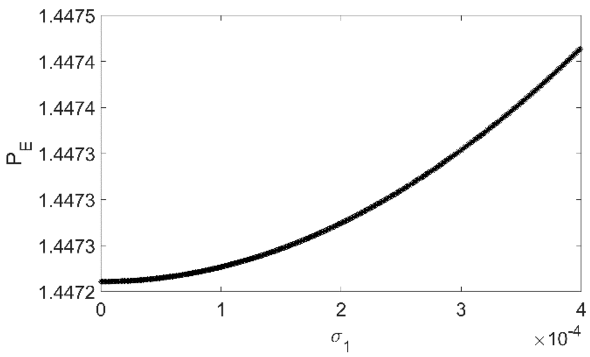
The X-W model considers both domestic and foreign volatility. When the volatility increases, the risk of the contract also increases. Naturally, this will impact the prices of the options. When both domestic and foreign volatility ($\sigma 1,\ \sigma_2$) increases, the option prices (C_E, P_E) should also rise.



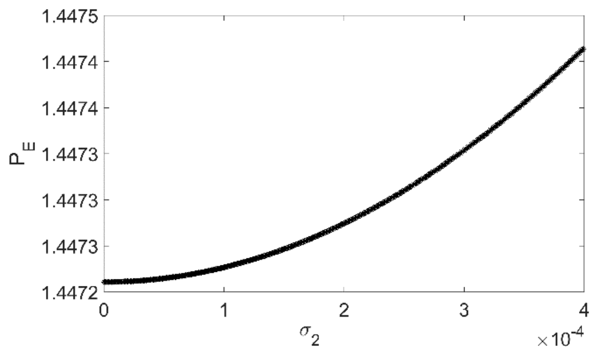
(a) C_E versus σ_1.



(b) C_E versus σ_2.



(c) P_E versus σ_1.



(d) P_E versus σ_2.

Fig. 3.6. European currency option price against the volatility of the interest rates.

From the formulas of the European call and put currency option prices given by equations (2.5) and (2.6), we see that both C_E and P_E are increasing functions of σ_1 and σ_2. That means the currency option prices increase with the volatility.

4. Conclusion

As we can see in the result and discussion section, the X-W model gives higher prices for both European call and put currency options. The new currency model assumes the interest rates (both native and foreign) to be stochastic processes influenced by volatility, rate of adjustment, and the long-run average value, whereas the L-C-R model considers none of these parameters. Even though the X-W model necessitates the calculation of several parameters, it does not require a significant amount of time. While the L-C-R model and the X-W model yield similar results for shorter maturity dates, the situation changes when the maturity date increases, leading to some failures in the L-C-R model. Ultimately, it is evident that the X-W model is a superior option for use in financial markets. We hope that this work will help with pricing currency options.

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APPENDIX A

A.1. Uncertain Distribution

Let $\xi \in \mathcal{B}$ ($\mathcal{B}$ is a set of real numbers in a topological space, which can be resulted from the operations of countable union, countable intersection, and relative complement of open sets) be an uncertain variable. Then, the uncertainty distribution $\Omega: \mathbb{R} \rightarrow [0,1]$ of ξ is defined as,

$$\Omega(x) = \mathcal{M}\{\xi \leq x\}. \quad (A.1)$$

When $\Omega(x)$ is continuous and strictly increasing with respect to x satisfying the following conditions: $0 < \Omega(x) < 1, \lim_{x\to-\infty} \Omega(x) = 0$ and$\lim_{x\to\infty} \Omega(x) = 1$, it is said to be regular [7].

A.2. Inverse Uncertainty Distribution

The inverse function $\Omega^{-1}(x)$ is called inverse uncertainty distribution of ξ, where ξ is an uncertainty variable having official uncertainty distribution $\Omega(x)$. Then ξ has an expected value,

$$E[\xi] = \int_0^1 \Omega^{-1}(a)da. \quad (A.2)$$

When f is a strictly increasing or decreasing function, then $\xi = (v)$ is an uncertain variable with inverse uncertain distribution [7].

A.3. Canonical Liu Process

An uncertain process (t) is said to be a canonical Liu process if,

1. $C(0) = 0$ and almost all sample paths are Lipschitz continuous, that is, $|C(t1) - C(t2)| \leq L|t1 - t2|$, where $L > 0$ is Lipschitz constant.
2. $C(t)$ has stationary and independent increments, and
3. Every increment $C(t + s) - C(t)$ is a normal uncertain variable which has uncertainty distribution

$$\Omega(x) = \left(1 + \exp\left(-\frac{\pi x}{\sqrt{3}t}\right)\right)^{-1}, x \in \mathbb{R}. \quad (A.3)$$

with expected value 0 and variance t^2[8].

A.4. Uncertain Differential Equation

Let us assume $C(t)$ be a Liu process, and F and G be two functions. We also assume that, an initial value X_0 is given. Now, we call the differential equation

$$dX_t = F(t, X_t)dt + G(t, X_t)dC. \quad (A.4)$$

an UDE (uncertain differential equation) with initial value X_0 [8].

A.5. α-path

Suppose that α is a number, where $0 < \alpha < 1$. If the following ODE (ordinary differential equation),

$$dX_t^\alpha = F(t, X_t^\alpha)dt + |G(t, X_t^\alpha)|\Omega^{-1}(\alpha)dt. \quad (A.5)$$

is solved by X_t^α, where,

$$\Omega^{-1}(\alpha) = \frac{\sqrt{3}}{\pi}\ln\left(\frac{\alpha}{1-\alpha}\right).$$

Then, the UDE $(A.4)$ is said to have an α- path X_t^α [8].

A.6. Brownian Motion

Let us consider, $t \geq 0$. If we put time on the horizontal axis and $B(t)$ on the vertical axis, $B(t)$ is the location of a physical particle in one dimension at each time $t,$ and is a random variable. The group of these random variables is a random process recorded by the continuous time parameter t, called Brownian motion (also called Wiener process) and is denoted by $B(t)$, with the characteristics:

a. The increment is continuous and initially, time and position are set at zero, $B(0) = 0$.
b. Increments over non-intersecting time intervals are independent random variables.
c. The increment over any time interval of length s, from any time t to time $(t + s)$ is $B(t + s) - B(t) \sim N(0, s^2)$[12].

A.7. Volatility

The statistical measure of the uncertainty about the returns for an agreed security or market index is called volatility. When volatility is higher, the security is considered to be risky.

A.8. Vasicek Model

A stochastic process is generally used to explain the short interest rate. O. Vasicek (a Czech mathematician and quantitative analyst) has proposed a model for interest rates under the foundation of probability theory. In this model, the risk-neutral process for $r(t)$ is,

$$dr(t) = a\big(b - r(t)\big)dt + \sigma dB, \quad (A.6)$$

where,

$B(t) =$ Brownian Motion,
$\sigma =$ The volatility of interest rate,
$a =$ The rate of adjustment,
$b =$ The long-run average value.

The drift factor $a(b - r(t))$ acts on behalf of the anticipated instantaneous change in the interest rate at time, t. If $dB = 0$, that means if shocks are absent, the interest rate remains constant when $r(t) = b$. If $r(t)$ is below b, then $a(b - r(t))$ is positive for positive a [13].

A.9. The rate of adjustment

The rate of adjustment is the mechanism to reduce the fluctuations of short-term interest rates. When interest rates get much higher or lower, it affects the economy. So, the rate of adjustment controls the interest rate.

A.10. The long-run average value

The long-run average value is the per unit value collected by a financial institution when all inputs are variables. Now from equation $(A.6)$ we have,

$dr + ardt = ab\,dt + \sigma dB,$
or, $e^{at}dr + ae^{at}rdt = abe^{at}\,dt + \sigma e^{at}dB,$
or, $d(e^{at}r) = abe^{at}\,dt + \sigma e^{at}dB.$
Integrating both sides with respect to s from 0 to t we have,

$\int_0^t d(e^{as}r) = \int_0^t abe^{as}\,ds + \int_0^t \sigma e^{as}dB,$
or, $e^{at}r_t - r_0 = b(e^{at} - 1) + \int_0^t \sigma e^{as}dB,$
or, $r_t = r_0e^{-at} + b\big(1 - e^{-at}\big) + \sigma e^{-at}\int_0^t e^{as}dB,$
and,

$$r_t \sim N\left(r_0e^{-at} + b\big(1 - e^{-at}\big), \frac{\sigma^2}{2a}(1-e^{-2at}\ \right)[14]. \quad (A.7)$$