Available online at https://ganitjournal.bdmathsociety.org/

GANIT: Journal of Bangladesh Mathematical Society

GANIT

GANIT *J. Bangladesh Math. Soc.* 44.2 (2024) 017–027
DOI: https://doi.org/10.3329/ganit.v44i2.78528

Bi-level Problem with Facility Allocation for Evacuation Planning

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ABSTRACT

At the time of evacuation, placement of the facilities for the support of evacuees is an important task. The proper allocation of the facilities in such a way that the reduction in the flow value due to the placement of facilities on the arcs is minimal, is another important aspect of the problem. In this paper, we introduce an evacuation planning problem with facility allocation by using bi-level formulation. The upper level problem identifies the best possible location and lower level problem finds the optimal solution in the network with facility allocation. We solve the problem with a naive approach of combinatorial optimization and the Karush-Kuhn-Tucker (KKT) transformation.

Received: August 08, 2023 **Accepted:** September 18, 2024 **Published Online:** December 30,2024

Keywords: Evacuation planning; Maximum flow; Facility allocation; Bi-level optimization

1 Introduction

Disasters, may be natural or human caused, are uncertain disruptions that may cause massive loss of human lives and infrastructures. Pre-disaster evacuations can be possible only if pre-informations are available. Otherwise, post-disaster evacuations with the shipment of the maximum amount of evacuees from the disaster zones to the safe shelters in the minimum possible time are very essential. In the meantime, settlement of the facilities to assist the evacuees at appropriate places is another important task that can be a milestone for the saving of their lives.

In network optimization, a graphical representation of the physical scenario is represented by the network where the demand point, supply point, and intersection of paths are considered as source,

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†The first, third and fourth authors are deeply shocked to report the untimely demise of young and energetic Prof. Dr. Urmila Pyakurel (second author), who passed away on April 12, 2023 at the age of 42. She was a role model Nepalese mathematician with an outstanding research career. As the draft of the paper was finalized with her active involvement, rest of the authors have decided to continue her as a co-author.

sink and intermediate nodes, respectively. Similarly, the links joining the pair of nodes are termed as arcs. The maximum flow problem concerns with shifting of the maximum amount of flow from the source node to the sink node without violating the capacity constraints on the arcs, [1, 2]. Many researchers have presented their algorithms to improve the results such as the shortest augmenting path algorithm of [3], the blocking flow algorithm of [4], the push-relabel algorithm of [5], and many more. We refer to the book of [6], survey papers of [7, 8], and the references therein for detailed illustrations of maximum flow problems and their solution strategies.

Network flow algorithms are applicable in evacuation planning, congestion minimization, distribution management and facility allocation problems. One of the technique, known as flow with intermediate storage, is introduced by Pyakurel and Dempe [9] focusing particularly on the holding of flow at intermediate nodes which is unable to reach to the destination. Considering the flow of more than one commodities, Khanal et al. [10] addressed the solution strategy associated with maximum multi-commodity flow problem by incorporating the intermediate storage facilities, [11]. In abstract network topology, Pyakurel et al. [12] proposed polynomial time solutions for maximum abstract flow by considering intermediate storage. Recently, Dhamala et al. [13] presented an algorithm to solve the generalized network flow problem with intermediate storage in lossy network. Different variant of evacuation planning problems and their solution strategies by reversing the direction of arcs can be found in [14, 15, 16, 17, 18] and the reference articles of [19].

The facility allocation problem concerns with allocation of the facilities at appropriate locations and the optimization of flow value on the facilitated network. The placement of the facility at some arcs reduces the capacity of the arc by the size of the facility and may reduce the total flow. The first location flow theory with its application in industries is introduced in [20]. Different discrete location models and algorithms with applications can be found in [21]. By incorporating the maximum flow problem with location analysis, Hamacher et al. [22] introduced single and multi-facility flow location problems. For single and multi-facility, they have presented polynomial time algorithms and polynomial time heuristics, respectively. In two-way network, the maximum static and dynamic contraflow problems with facility location are solved in [23]. The single and multiple quickest flow location problems and their solution strategies can be found in Nath et al. [24]. Recently, Dhamala et al. [25] introduced the single-source single-sink maximum static and dynamic flow location (FlowLoc) problems with storage of excess flow at intermediate shelters.

Bi-level problems are two stage optimization problems in which the first stage optimizes the overall system under the best possible decision of the second stage. It is also known as a leader-follower optimization problem in game theory with successive iterations of two players where the leader first decides a variable to optimize a given objective function and then the follower reacts optimally to the leader's decision. The first studies in bi-level optimization can be found in von Stackelberg [26]. The serious interest in this field of optimization rapidly increased after the 1970s. For more detailed illustration, we refer to the papers of Anandalingam and Friesz [27], Wen and Hsu [28], Ben-Ayed [29], Dempe and Zemkoho [30], survey papers of Colson et al. [31, 32], Dempe [33], and the books of Bard [34], Dempe [35], Dempe and Zemkoho [36].

The bi-level formulation of the facility allocation problem is a major research gap in the literature in which the allocation of facilities is made in such a way that the loss in optimal flow value due to the best placement of facilities is minimal. To deal with this problem, we formulate an upper level problem that declares the appropriate location for the facility and a lower level problem that maximizes the flow on the facilitated network. We present the solution procedure in two ways. The first is a naive approach with the combinatorial problem for selecting the arcs to allocate the facility and finding the maximum dynamic flow on the network with facility allocation. The second one is the KKT transformation approach.

The paper is organized as follows. Basic notations used throughout the paper are presented in Section 2. In Section 3, we formulate the facility allocation problem by using bi-level programming, and the solution procedures are presented in Section 4. The paper is concluded in Section 5.

2 Notations

Consider a network $\mathbb{K} = (V, A, u, s, t)$, where V represents a set of n nodes (i.e., $|V| = n$) and $A \subseteq V \times V$ represents a set of m arcs (i.e., $|A| = m$). Here, $s \in V$ and $t \in V$ are the source (origin) and sink (destination) nodes, and $I = V \setminus \{s, t\}$ represents the set of intermediate nodes. Each arc $a = (v, w) \in A$ with $head(a) = w$ and $tail(a) = v$ has a capacity function $u : A \to \mathcal{R}^+$ that limits the flow on arc. Similarly, let $L \subseteq A$ be a set of feasible locations and $d : L \to \mathcal{R}^+$ is the size of the facility that is to be placed in some arc. We denote the set of outgoing arcs from node v and incoming arcs to node v by $\delta^{out}(v)$ and $\delta^{in}(v)$, respectively. In the case of a dynamic network, two additional parameters are to be considered, one transit time function $\tau : A \to \mathcal{R}^+$ that measures the transmission time from v to w and another $\mathcal{T} = \{0, 1, ..., T\}$ to represent time horizon T in discrete time settings. Thus the dynamic network is of the form $\mathbb{K} = (V, A, u, s, t, \tau, T)$.

3 Bi-level Problem Formulation with Facility Allocation

Let x be a static flow on a network $\mathbb{K} = (V, A, u, s, t)$ which is defined as a non-negative arc flow function $x : A \to \mathcal{R}^+$. The static flow model is the network flow satisfying the conditions (3.1 - 3.4). The mathematical formulation of the maximum static flow as a linear programming problem is as follows.

$$\max \ f^* \tag{3.1}$$

such that,

$$\sum_{a \in \delta^{out}(s)} x_a = f^* = \sum_{a \in \delta^{in}(t)} x_a \tag{3.2}$$

$$\sum_{a \in \delta^{in}(v)} x_a - \sum_{a \in \delta^{out}(v)} x_a = 0 \quad \forall v \in I \tag{3.3}$$

$$0 \leq x_a \leq u_a \quad \forall a \in A \tag{3.4}$$

Equation (3.1) is an objective function that is to maximize the total flow. The total outflow from the source is presented in Equation (3.2) which must be equal to the inflow at the sink. Here, f^* denotes the total value of static $s - t$ flow that is to be maximized. The flow conservation at intermediate nodes is represented by Equation (3.3) and Equation (3.4) represents the boundedness of the flow on each arc by its capacity.

The dynamic flow within the time horizon T can be obtained by temporal repetition of the static flow along the paths as

$$Tf^* - \tau_a x_a.$$

Here τ_a is the traversal time of the flow along the arc $a = (v, w)$ so that any flow starting from the tail node v at time $\Theta \in \mathcal{T}$ reaches the head node w at $\Theta + \tau_a$.

Bi-level programming problem is a hierarchical optimization problem of two levels in which the lower level problem is among the constraints of the upper level problem. If γ and x are two variables, then the lower level problem is of the form

$$\max_{\gamma} \{h(\gamma, x) : g(\gamma, x) \leq 0\} \tag{3.5}$$

which depends on the upper level variable x. Here, h is a real valued function defined as $h : \mathbb{R}^p \times \mathbb{R}^q \to \mathbb{R}$ and $g = (g_1, \ldots, g_l)$ is a vector valued function defined as $g : \mathbb{R}^p \times \mathbb{R}^q \to \mathbb{R}^l$. Similarly, if $\Psi : \mathbb{R}^p \to 2^{\mathbb{R}^q}$

be a solution set mapping such that $x \in \Psi(\gamma)$, then the upper level optimization problem is of the form

$$\max_{\gamma,x}\{H(\gamma,x) : G(\gamma,x) \leq 0, x \in \Psi(\gamma)\}, \tag{3.6}$$

where $H : \mathbb{R}^p \times \mathbb{R}^q \to \mathbb{R}$ and $G : \mathbb{R}^p \times \mathbb{R}^q \to \mathbb{R}^k$ with $G = (G_1, \ldots, G_k)$. Here, $\Psi(\gamma)$ equals the set of (global) optimal soloutions of problem (3.5) for fixed x. We refer to Dempe [35, 37] and references therein for detailed illustrations. With the help of these formulations, we introduce the bi-level formulation of the maximum dynamic flow problem with the allocation of facility at the arcs hereafter.

At the time of disasters, proper allocation of the emergency facilities for the support of evacuees is very important. Let d represent the size of a facility that is to be placed at an appropriate arc in L. The upper level problem in network $\mathbb{K}$ is

$$\begin{aligned}
\max \quad & F(\gamma, x) && (3.7)\\
s.t. \quad & 0 \leq d \cdot \gamma_a \leq u_a \qquad \forall a \in L && (3.8)\\
& \sum_{a \in L} \gamma_a = 1 && (3.9)\\
& \gamma_a = 0 \qquad \forall a \in A \setminus L && (3.10)\\
& \gamma_a \in \{0, 1\} \qquad \forall a \in L && (3.11)\\
& x \text{ solves the lower level problem depending on } \gamma && (3.12)
\end{aligned}$$

where $\gamma = (\gamma_a)_{a\in L}$ and $x = (x_a)_{a\in L}$. Constraint in (3.8) represents that the facility is allocated at an arc with sufficient capacity. The single facility location is assured by Equation (3.9). Non-selection of arc outside of L for the facility allocation is represented by Equation (3.10), where (3.11) represents the binary variable. The upper level objective function in (3.7) is to maximize F defined by

$$F(\gamma, x) = Tf - \sum_{a\in A} \tau_a x_a + \sum_{a\in L} \gamma_a \cdot w_a$$

where for $a \in L$, w_a is a predefined reward for locating the facility on arc a. Thus, the objective of the upper level problem is to maximize the flow out from the source by appropriate allocation of the facility on the arc. Here, f is the value of static flow induced by x after placement of the facility that is to be maximized in static flow computation.

The lower level problem is to obtain the maximum flow after the reduction of the capacity at the facility allocated arc by the size of the facility as follows.

$$\begin{aligned}
\max_{f,x} \quad & Tf - \sum_{a\in A} \tau_a x_a && (3.13)\\
s.t. \quad & \sum_{a\in\delta^{in}(v)} x_a - \sum_{a\in\delta^{out}(v)} x_a = \begin{cases} -f & \text{for } v = s\\ 0 & \forall v \in I\\ f & \text{for } v = t\end{cases} && (3.14)\\
& 0 \leq x_a \leq u_a - d \cdot \gamma_a \qquad \forall a \in A && (3.15)
\end{aligned}$$

Here, the objective of lower level problem (3.13) is to maximize the dynamic flow obtained by temporal repetition of static flow. The flow conservation at intermediate nodes and non-conservation of the flow at source and sink are represented by Equation (3.14) and the boundedness of the flow on each arc is represented in (3.15).

In case of multiple facility allocation, say r facilities of size d_i, $i = 1, \ldots, r$ with $r \leq |L|$, the upper level constraint in (3.9) is to be replaced by $\sum_{i=1}^{r} \gamma_{(a,i)} = 1$ for all $a \in L$ (i.e., each facility is placed on exactly one arc) and $\sum_{a\in L} \gamma_{(a,i)} = 1,\ i = 1, ..., r$ (i.e., each facility is located at some arc). Similarly, lower level constraint in (3.15) is to be replaced by $0 \leq x_a \leq u_a - d_i\gamma_{(a,i)}$. The objective function and the rest of the constraints remain the same with replacement of γ_a to $\gamma_{(a,i)}$.

The dual formulation of the lower level problem is as follows.

$$\min \quad \sum_{a\in A} \theta_a(u_a - d \cdot \gamma_a) \tag{3.16}$$

$$s.t. \quad \lambda_v - \lambda_w + \theta_a \geq -\tau_a \qquad \forall a = (v, w) \in A \tag{3.17}$$

$$\lambda_s - \lambda_t \geq T \tag{3.18}$$

$$\theta_a \geq 0 \qquad \forall a \in A \tag{3.19}$$

$$\lambda_v \in \mathbb{R} \quad \text{(unrestricted)} \tag{3.20}$$

4 Solution Procedure

In this section, we present two approaches to solve the facility allocation problem. The first one is a naive approach which selects an arc with some strategy to place the facility, finds the maximum dynamic flow over the time horizon, and continues the process until the best solution is obtained. Another approach is the conversion of a bi-level problem to a single level one by using the Karush-Kuhn-Tucker (KKT) transformation and solving the problem by replacing the complementarity condition by big-M method with mixed-integer reformulation.

4.1 Solution by Naive Approach

For a given subset $L \subseteq A$ of possible locations, our concern here is to present a simple procedure to solve the facility allocation problem. The basic idea for this approach is from the Stackelberg leadership model of a strategic game in which the leader makes the first move and then the follower reacts sequentially for the optimal output. The leader (upper level) iteratively chooses an arc for the allocation of a facility unless the best optimal solution from the follower (lower level) is produced.

Here, we present the pseudo codes of an algorithmic framework to solve the maximum dynamic flow problem with facility allocation (see Algorithm 1). The first and second steps inside the 'for loop' of the algorithm are obtained by the upper level problem and the third one by the lower level problem. This loop runs over all arcs $a' \in A$ with $u_{a'} \geq d$ to obtain the best optimal flow MDF^{opt}.

The time complexity of the algorithm depends on the number of iterations over the arcs in L and the complexity of the maximum dynamic flow, that is, $|L| \times O(MDF)$, where $O(MDF)$ is the time complexity of maximum dynamic flow problem. As $|L| \leq m$ and maximum dynamic flow can be computed in polynomial time, the overall time complexity of the algorithm with single facility allocation is polynomial, [22]. However, solving multiple (i.e., r) facility allocation problem is a combinatorial optimization problem with complexity ${}^{|L|}P_r \times O(MDF)$, where ${}^{|L|}P_r$ represents the permutation of $|L|$ locations taken r at a time. Thus, it's time complexity is not the polynomial but exponential.

4.2 Solution by KKT Transformation

The Karush-Kuhn-Tucker (KKT) condition is one of the most commonly used approaches to solve the bi-level programming problem which is only applicable if the lower level problem is a convex optimization problem. It transfers the problem into a single level optimization problem. It is to be noted that an optimization problem where all of the constraints are convex functions, and the objective

Algorithm 1: Naive algorithm for maximum dynamic flow with facility allocation

Input : Given a dynamic network $\mathbb{K} = (V, A, u, s, t, \tau, T)$.
Output: MDF^{opt} = Maximum dynamic flow with facility allocation.

L = Set of feasible locations ($L \subseteq A$).

d = Size of facility.

$\gamma = \{0, 1\}$, a decision variable.

for $a' \in L$ with $u_{a'} \geq d$:

Assign $\gamma_{a'} = 1$ and $\gamma_a = 0 \ \forall a \in L \setminus \{a'\}$.

Assign $u_{a'} = u_{a'} - d$.

$\text{MDF}^{(a')}$ = Maximum dynamic flow after placement of facility at a'.

$\text{MDF}^{opt} = \max\{\text{MDF}^{(a')} : a' \in L \text{ and } \gamma_{a'} = 1\}$

is a convex function if minimizing, or a concave function if maximizing, is a convex optimization problem. In case, for non-convex lower level problems under some regularity condition, the lower level problem is replaced by the KKT conditions, the feasible set of the original problem is enlarged, we add local optima and stationary solutions. This, in general, implies that global optimal solutions of the transformed problem do not need to be feasible for the original problem, [38].

As both lower and upper level problems in our facility allocation model are linear, KKT transformation is possible. For this, consider the Lagrangian function of lower level problem as

$$\mathcal{L}(\gamma, x, \lambda, \theta) = Tf - \sum_{a \in A} \tau_a x_a + \sum_{v \in I} \lambda_v \left(\sum_{a \in \delta^{in}(v)} x_a - \sum_{a \in \delta^{out}(v)} x_a \right) + \sum_{a \in A} \theta_a \cdot (u_a - d \cdot \gamma_a - x_a).$$

The objective function for KKT condition is the objective of upper level problem and the KKT constraints are the constraints for lower level problem (3.14)- (3.15) together with dual constraints (3.17)-(3.20) and the complementarity constraint

$$\theta_a \cdot (u_a - d \cdot \gamma_a - x_a) = 0 \qquad \forall a \in A.$$

This yields the mathematical model with the complementarity constraints (MPCC) as follows.

$$\max_{\gamma, x, \lambda, \theta} \quad F(\gamma, x) \tag{4.1}$$

$$s.t. \quad 0 \leq d \cdot \gamma_a \leq u_a \qquad \forall a \in L \tag{4.2}$$

$$\sum_{a \in L} \gamma_a = 1 \tag{4.3}$$

$$\sum_{a \in \delta^{in}(v)} x_a - \sum_{a \in \delta^{out}(v)} x_a = \begin{cases} -f & \text{for } v = s \\ 0 & \forall v \in I \\ f & \text{for } v = t \end{cases} \tag{4.4}$$

$$0 \leq x_a \leq u_a - d \cdot \gamma_a \qquad \forall a \in A \tag{4.5}$$

$$\lambda_v - \lambda_w + \theta_a \geq -\tau_a \qquad \forall a = (v, w) \in A \tag{4.6}$$

$$\lambda_s - \lambda_t \geq T \tag{4.7}$$

$$\theta_a \cdot (u_a - d \cdot \gamma_a - x_a) = 0 \qquad \forall a \in A. \tag{4.8}$$

$$\theta_a \geq 0 \qquad \forall a \in A \tag{4.9}$$

$$\lambda_v \in \mathbb{R} \quad \text{(non-restricted)} \tag{4.10}$$

The complementarity constraints in (4.8) can be replaced by the mixed-integer reformulation using sufficiently large big-M constants M' and M'' as follows.

$$u_a - d \cdot \gamma_a - x_a \leq M'(1 - \beta_a), \qquad \theta_a \leq M''\beta_a, \qquad \beta_a \in \{0, 1\}, \ \ \forall a \in A.$$

This reformulation was introduced by Fortuny-Amat and McCarl [39] so that resulting model of single level problem can be solved by standard mixed-integer solvers. However, a major concern here is to approximate the value of big-M. Pineda and Morales [40] have shown that choosing too small big-M can result in a sub-optimal solution. Similarly, too large values of big-M may cause an infeasible solution for the original bi-level problem, [41]. As M' is an upper bound of the primal variable x_a on arcs, without loss of generality, we can set $M' = \max\{u_a : a \in A\}$. However, M'' is an upper bound of the dual variable θ_a, and tuning such a large enough constant for the dual variable is a more challenging task. The trial-and-error tuning procedure is most commonly used in literature for the upper bound of the dual variable.

Due to the linear constraints in lower level problem, an ε bound method can be used to reformulate the complementarity constraint (4.8) and solve with relaxation in the sense of Scholtes [42]. The reformulation of complementarity constraint is

$$\theta_a \cdot (u_a - d \cdot \gamma_a - x_a) \leq \varepsilon \qquad \forall a \in A,$$

where locally optimal solution of the problem is obtained for $\varepsilon \downarrow 0$, (see also in [43]). According to Scholtes [42], the idea of ε bound method is as follows: Replace constraint (4.8) as mentioned with positive ε and solve this problem. Now, decrease ε by replacing it with $\varepsilon/2$ and solve the resulting problem starting with the optimal solution in the previous step. Then, repeat the process. Here, the objective (4.1) with constraints (4.2)–(4.10) is irregular, the Mangasarian-Fromovitz constraint qualification (MFCQ) is violated. The result is that solution algorithms can in general not solve the problem (they often cannot compute a starting point).

Similarly, we can also use Lagrange duality approach of Dempe and Mehlitz [44] to reduce bilevel problem to single level. There are other approaches which are described in [33] and the ideas to replace constraint (4.8) are given and compared in [45].

Not only for a single facility allocation, KKT transformation solves the multiple facility allocation problem at the same pace.

5 Conclusions

In network optimization, the maximum flow and facility location problems have been studied in the literature. In this paper, we introduced a bi-level formulation of the maximum dynamic flow problem with facility allocation in which the upper level problem finds the best possible arc for facility allocation and the lower level problem finds the optimal flow on the facilitated network. We solved the problem one with a naive approach of combinatorial optimization and another by KKT transformation using big-M and ε bound methods. To the best of our knowledge, formulation of maximum dynamic flow with facility allocation using bi-level programming is for the first time.

Acknowledgments

The first author (Durga Prasad Khanal) thanks the German Academic Exchange Service - DAAD for Research Grants - Bi-nationally Supervised Doctoral Degrees/Cotutelle, 2021/22 and the University Grants Commission, Nepal, for the PhD research fellowship.

Data Availability

The authors have not used any additional data in this article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Funding

No any funding is available from any sources to carry out this research.

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