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Profit Optimization of Soap Industry by using Benders' Decomposition Method

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ABSTRACT

In this paper, a large-scale linear programming problem including several parameters such as labor cost, raw material cost, machine and other cost have been formulated. Then the formulated problem has been solved by using Benders' Decomposition Method. The formulated large model is divided into master and small sub problem. In order to validate and calibrate the model, real data from a soap industry named Mega Sornali Soap and Cosmetics Industry have been collected. Soap industry is one of the most feasible business options owing to the straightforward manufacturing process involved starting a soap and detergent manufacturing business in Bangladesh. These models are solved by using AMPL. To find out the significant parameters of the model, the sensitivity analysis of different cost parameters such as labor cost, raw material cost and machine cost have been be considered. From the sensitivity analysis, the decision makers of the factory would able to find out the ranges of cost coefficients and all the resources. As a result, the company could able to see how any change can affect the profit or loss of the factory. From the numerical results, the most profitable product of the company is found to be Sornali Soap and Mega Extra Powder. On the contrary, Mega SornaliSobiMarka Soap and Mega Washing Powder are not more profitable. Further, raw material cost is the most significant sensitive cost. If the raw material cost can be decreased the profit could also beincreased. Finally, the result of the optimal solution has been represented in tabular and graphs.

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1. Introduction

Linear programming is a method to achieve the best outcome such as maximum profit or lowest cost in a mathematical model of any business organizations. Decomposition technique is one of the most commonly used techniques for solving Linear Programming Problem (LPP). If the number of decision variables and constraints are very large then it would be very difficult to solve manually. Benders' Decomposition Method

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(BDM) is a solution method for solving certain large-scale optimization problems that have special block structure.

In literature, [1] introduced a decomposition algorithm for linear programming. [2] Explained a method of decomposition for integer programs. [3] reported the partitioning procedures for solving mixed variables programming problems.[4] presented an integer programming algorithm for vehicle routing problem involving capacity and distance restrictions. They derived exact solutions for problems involving up to sixty cities. [5] Established decomposition-based pricing model for solving a large-scale MILP for an integrated fishery. They described how a fishery manager can schedule fishing trawlers to determine when and where they should go and return their caught fish to the factory. The authors of [6] presented technique for solving large nonconvex water resources management models using generalized BDM. Then [7] developed a decomposition algorithm for the design of a no simultaneous capacitated evacuation tree network. Vendor-Bayer coordination and supply chain optimization with deterministic demand function was analyzed by [8].

A mixed-integer programming techniques for decomposing IMRT fluency maps using rectangular apertures had explained by [9]. They studied the problem of minimizing the number of rectangles (and their associated intensities) necessary to decompose such a matrix. They proposed an integer programming-based methodology for providing lower and upper bounds on the optimal solution and demonstrate the efficacy of their approach on clinical data. In [10], the authors implemented the Dantzig-Fulkerson-Johnson algorithm for large travelling salesman problems. An algorithm is described for solving large-scale instances of the Symmetric Travelling Salesman Problem (STSP) to optimality. [11] Showed a Benders' decomposition algorithm for the single allocation hub location problem under congestion. The single allocation hub location problem under congestion is addressed in this article. Then a very efficient and effective generalized Benders decomposition algorithm is deployed, enabling the solution of large-scale instances in reasonable time. An approach for the locomotive and car assignment problem using Benders' Decomposition illustrated in [12]. One of the problems faced by rail transportation companies is to optimize the utilization of the available stock of locomotives and cars. They described a decomposition method for the simultaneous assignment of locomotives and cars in the context of passenger transportation.

This paper proposes an optimal formulation optimization model for Soap industry. To formulate a linear programming model that would suggest a viable product-mix to ensure optimum profit for company. This study minimizes the production cost and find out various types of effects of parameters in production period. In addition, find out the significant constraints of the company regarding cost and resources. The formulated model soles by BD method using Mathematical Programming Language (AMPL). The solution of the problem discusses briefly and carries out the optimal product for the industry.

The paper is structured as follows: in Section 2, the adaptation of the BD model for the Soap industry formulation is described and a Linear programming problem is developed, aiming at the maximization of profit. In Section 3, an effect of the model with solution and findings is worked out. Finally, Section 4 concludes the paper, highlighting the main results and introducing some research challenges for the future.

2. Formulation of the problem

In this section, it will be developed a mathematical model from this data which will be resulting into a large LPP and by applying the solving procedure of LPP and by applying the solving procedure of LPP in its production planning. It will be tried to identify its desired production rate and to answer some questions that may arise when thinking about the profit. It will be showed the impact of LPP in business planning. To understand the effect of several parameters and the profit, we propose a mathematical model that can predict the significant parameters.

A standard form of a Linear Program is:

Maximize $z = c^T x$

Subject to Constraints: $Ax \leq b$

$x \geq 0,$

Where $c \in \mathbb{R}^n$, $b \in \mathbb{R}^m$ are given vectors and $A \in \mathbb{R}^{m \times n}$ is a matrix.

The following table show the decision variables and their descriptions.

Table 2.1: List of the parameters involved in the model

Parameters	Description
X_1	The unit of Mega SornaliSobiMarka Soap (250g).
X_2	The unit of SornaliBati Soap (175g).
X_3	The unit of Sornali Soap 2015 (500g).
X_4	The unit of Sornali Soap (250g).
X_5	The unit of Mega Sornali Full Marka Soap (250g).
X_6	The unit of Mega Washing Powder (25g).
X_7	The unit of Mega Washing Powder (200g).
X_8	The unit of Mega Washing Powder (500g).
X_9	The unit of Mega Extra Powder (200g).
X_{10}	The unit of Mega Extra Powder (500g).

General mathematical form of our proposed problem as the following:

$$Maximize, Z = \sum_{n=1}^{10} C_n X_n$$

Subject to constraints,

$$\sum_{n=1}^{10} B_n^1 X_n \le b_1$$

$$\sum_{n=1}^{10} B_n^2 X_n \le b_2$$

$$\sum_{n=1}^{10} B_n^3 X_n \le b_3$$

$$0 \le X_n \le \alpha_i$$

Where b1, b2 and b3 are available labour cost, machine and other cost and raw material cost. α_i are lower and upper bound of different products.

To solve the above problem real-life data has been collected from a company named Mega Sornali Soap and Cosmetics Industries Ltd. It was established in 2015. This company produces five types of soap, three types of lemon powder and two types of mega extra powder.

In Table 2.2, presents the different types of per unit raw materials cost. Table 2.3, shows per unit raw materials cost to produce Lemon Detergent Powder. Table 2.4, display per unit raw materials cost to produce Extra Detergent Powder.

Table 2.2: Different types of raw materials cost

No.	Name	Cost (TK)/Kg
01.	Silicate	14
02.	Palm Oil	76
03.	Palm Pati	80
04.	Rice Pati	54
05.	Palm Stearing	80.50
06.	Soybean	48.50
07.	Caustic Soda	32
08.	S.L.S.(Foam Powder)	290
09.	Perfumed	1000
10.	Colour	1000

Table 2.3: Raw Materials to produce Lemon Detergent Powder

No.	Name	Cost (TK)/Kg
01.	Dolomite	5
02.	Global Salt	12
03.	Calcium Carbonet	15
04.	Soda	32
05.	Lapsa (Foam)	125
06.	Colour	4000
07.	Perfume	1000

Table 2.4: Raw materials to produce Extra Detergent Powder

No.	Name	Cost(TK)/Kg
01.	Limestone	7
02.	Soda	32
03.	Calcium Carbonet	15
04.	Global Salt	12
05.	Lapsa	125
06.	Sky White	300
07.	S. Perkel	55
08.	Perfume	1000

Table 2.5: Selling Price of Soap

No.	Name	Quantity(g)	Selling Prices Per piece (TK)
01.	Mega Sornali Sobi Marka Soap	250	11.66
02.	SornaliBati Soap	175	6.50
03.	Sornali 2015	500	20
04.	Sornali Soap	250	10.41
05.	Mega Sornali Full Marka	250	8.33

Table 2.6: Selling Price of Lemon Powder

No.	Name	Quantity(g)	Selling Prices Per piece (TK)
01.	Mega Washing Powder	25	2.5
02.	Mega Washing Powder	200	6.86
03.	Mega Washing Powder	500	14

Table 2.7: Selling Price of Mega Extra Powder

No.	Name	Quantity(g)	Selling Prices Per piece

			(TK)
01.	Mega Extra Powder	200	10.32
02.	Mega Extra Powder	500	20

Table 2.8: Price of machine

No.	Name	Price
01.	Mixer Machine	210000
02.	Sipter Machine	100000
03.	Packing Machine (Mini)	150000
04.	Packing Machine (250g, 500g)	300000

Table 2.9: Salary Structure

Post	Salary monthly (TK)
Mechanical Engineer	15000
Manager	10000
Electrician	8000
Fueling	9500
Sweeper	5000
Machine Operator	5000

Table 2.10: Some brands of foreign material

Country	Brand
Bhutan	Limestone, Dolomite
India	Lapsa
Taiwan	Foam Powder

Table 2.11: Other cost

Purpose	Cost (TK)
Oil	2250
Tools	3000
Electric Motor (5 pieces)	9000
Total	14250

In Table 2.5, represents the selling price of various companies of per unit soap. Table 2.6, defines the selling price of different types of Lemon Powder. Table 2.7, denotes the selling price of different types of Mega Extra Powder. Table 2.8, describes various types of machine price. Table 2.9, represents the salary of various employers. Table 2.10, denotes the some brands of foreign material. Finally, Table 2.11; ensure other cost to produce different types of soap. The following Table 2.12 shows the information of different soaps and their raw materials cost, labor cost and machine and other cost that has obtained the previous primary data described in Table 2.11.

Table 2.12: Product wise cost and profit

Variable	Labor cost B^1_n	Machine and other cost B^2_n	Raw material cost B^3_n	Profit for each variable
X_1	0.236	0.054	8.644	2.725
X_2	0.295	0.068	3.447	2.69
X_3	0.295	0.065	11.991	7.649
X_4	0.268	0.055	6.818	3.269
X_5	0.322	0.067	5.278	2.663
X_6	0.163	0.029	2.0	0.308
X_7	0.271	0.062	3.81	2.717
X_8	0.295	0.075	7.5	6.13
X_9	0.236	0.06	6.4	3.62
X_{10}	0.3	0.1	13.33	6.26

According to the above data the LPP problem for the Mega Sornali Soap and Cosmetics Industries Ltd is formulated as follows.

The objective function of the LPP model is:
Maximize,

$$Z = 2.725X_1 + 2.69X_2 + 7.649X_3 + 3.269X_4 + 2.663X_5 + 0.308X_6 + 2.717X_7 \\ +6.13X_8 + 3.62X_9 + 6.26X_{10}$$

Subject to:

$$0.236X_1 + 0.295X_2 + 0.295X_3 + 0.268X_4 + 0.322X_5 + 0.163X_6 + 0.271X_7 \\ +0.295\,X_8 + 0.236X_9 + 0.3X_{10} \le 60000 \tag{2.1}$$

$$0.054X_1 + 0.068X_2 + 0.065X_3 + 0.055X_4 + 0.067X_5 + 0.029X_6 + 0.062X_7 \\ +0.075\,X_8 + 0.06X_9 + 0.1X_{10} \le 850000 \tag{2.2}$$

$$8.644X_1 + 3.447X_2 + 11.991X_3 + 6.818X_4 + 5.278X_5 + 2.0X_6 + 3.81X_7 \\ +7.5X_8 + 6.4X_9 + 13.33X_{10} \le 1000000 \tag{2.3}$$

$$0 \le X_1 \le 25000 \tag{2.4}$$

$$0 \le X_2 \le 20000 \tag{2.5}$$

$$0 \le X_3 \le 20000 \tag{2.6}$$

$$0 \le X_4 \le 22000 \tag{2.7}$$

$$0 \le X_5 \le 18000 \tag{2.8}$$

$$0 \le X_6 \le 35000 \tag{2.9}$$

$$0 \le X_7 \le 21000 \tag{2.10}$$

$$0 \le X_8 \le 20000 \tag{2.11}$$

$$0 \le X_9 \le 25000 \tag{2.12}$$

$$0 \le X_{10} \le 20000 \tag{2.13}$$

Where, the objective function is to maximize the total profit where the coefficients are the profit of each product, equation (2.1) describes the labour cost where the coefficients are the labor cost required for each product. Equation (2.2) illustrates the machine cost where the coefficients are the cost of machine required for each product. Equation (2.3) represents other cost that includes tax, interest, electricity bill, fix cost etc. and the coefficients are the cost required for product production. In this study raw material cost are included with other cost that associate to the company to produce above ten products. Equations (2.4) to (2.13) are lower bound and upper bound of different products. These are the boundary constraints that illustrate the limit of the production amount of ten soaps.

2.1 Optimal solution by Bender Decomposition

In this subsection master problem is expressed. The master problems of Bender Decomposition method are as follows:

Master Problem: Maximize, $\mathrm{M} = 2.725X_1 + 2.69X_2 + 7.649X_3 + 3.269X_4 + 2.663X_5$

Subject to:

$$0 \le X_1 \le 25000 \tag{2.1.1}$$

$$0 \le X_2 \le 20000 \tag{2.1.2}$$

$$0 \le X_3 \le 20000 \tag{2.1.3}$$

$$0 \le X_4 \le 22000 \tag{2.1.4}$$

$$0 \le X_5 \le 18000 \tag{2.1.5}$$

a. *Primal sub-problem*

In this subsection primal sub-problem is generated. The primal sub-problem of Bender Decomposition method are as follows

Maximize, $P = 0.308X_6 + 2.717X_7 + 6.13X_8 + 3.62X_9 + 6.26X_{10}$

Subject to:

$$\begin{aligned} &0.163X_6 + 0.271X_7 + 0.295X_8 + 0.236X_9 + 0.3X_{10} \\ &\le 60000 - 0.236X_1 - 0.295X_2 - 0.295X_3 - 0.268X_4 + 0.322X_5 \end{aligned} \tag{2.2.1}$$

$$\begin{aligned} &0.029X_6 + 0.062X_7 + 0.075X_8 + 0.06X_9 + 0.1X_{10} \\ &\le 850000 - 0.054X_1 - 0.068X_2 - 0.065X_3 - 0.055X_4 - 0.067X_5 \end{aligned} \tag{2.2.2}$$

$$2.0X_6 + 3.81X_7 + 7.5X_8 + 6.4X_9 + 13.33X_{10} \leq 1000000 - 8.644X_1 - 3.447X_2 - 11.991X_3 - 6.818X_4 - 5.278X_5 \quad (2.2.3)$$

$$0 \leq X_6 \leq 35000 \quad (2.2.4)$$

$$0 \leq X_7 \leq 21000 \quad (2.2.3)$$

$$0 \leq X_8 \leq 20000 \quad (2.2.4)$$

$$0 \leq X_9 \leq 25000 \quad (2.2.5)$$

$$0 \leq X_{10} \leq 20000 \quad (2.2.6)$$

2.3 Dual sub-problem

Considering the data and above formation the dual problem can be derived as follows:

$$\begin{aligned}
&Minimize,\ D = \lambda_1(60000 - 0.236X_1 - 0.295X_2 - 0.295X_3 - 0.268X_4 + 0.322X_5) \\
&+\lambda_2(850000 - 0.054X_1 - 0.068X_2 - 0.065X_3 - \ 0.055X_4 - 0.067X_5) \\
&+\lambda_3(1000000 - 8.644X_1 - 3.447X_2 - 11.991X_3 - 6.818X_4 - 5.278X_5) \\
&+35000\lambda_4 + 21000\lambda_5 + 20000\lambda_6 + 25000\lambda_7 + 20000\lambda_8 \\
&= \lambda_1(60000 - 0.236*25000 - 0.295*20000 - 0.295*20000 - 0.268*22000 - 0.322*18000) \\
&+\lambda_2(850000 - 0.054*25000 - 0.068*20000 - 0.06520000 - 0.055*22000 - 0.067*18000) \\
&+\lambda_3\left(1000000 - 8.644*25000 - 3.447*20000 - 11.991*20000 - 6.818*22000 - 5.278*18000\right) \\
&+\ 35000\lambda_4 + 21000\lambda_5 + 20000\lambda_6 + 25000\lambda_7 + 20000\lambda_8 \\
&= 30600\lambda_1 + 843600\lambda_2 + 9230660\lambda_3 + 35000\lambda_4 + 21000\lambda_5 + 20000\lambda_6 + 25000\lambda_7 + 20000\lambda_8
\end{aligned}$$

Subject to:

$$0.163\lambda_1 + 0.029\lambda_2 + 2.06\lambda_3 + \lambda_4 \geq 0.308 \quad (2.3.1)$$

$$0.271\lambda_1 + 0.062\lambda_2 + 3.81\lambda_3 + \lambda_5 \geq 2.717 \quad (2.3.2)$$

$$0.295\lambda_1 + 0.075\lambda_2 + 7.5\lambda_3 + \lambda_6 \geq 6.13 \quad (2.3.3)$$

$$0.236\lambda_1 + 0.06\lambda_2 + 6.49\lambda_3 + \lambda_7 \geq 3.62 \quad (2.3.4)$$

$$\text{All } \lambda_i \geq 0 \quad (2.3.5)$$

3. Solution and results discussion

The formulated LPP has been solving using AMPL. The program consists of three parts: model file, data file and run file. After developing a model file, it must arrange a data file according to the model file. Both the model and related data file must be called in command file with proper codes. Then to obtain the output of the problem it must call command in AMPL. Then the solution can be found by run file using solver CPLEX.

After solving the LP formulated in previous section the maximum profit is obtained: 623195.5866 and the following Table 3.1 shows the values of the estimated parameters.

Table 3.1: Estimated values of the parameters

Parameter	Values
X_1	0.00
X_2	20000
X_3	20000
X_4	22000
X_5	18000
X_6	0.00
X_7	21000
X_8	20000
X_9	25000
X_{10}	4218.3

The formulated Master problem, Primal problem, and Duet problem are solved in several iterations. The estimated values of the parameters and shown in Table 3.2 as follows:

Table 3.2: Solution of the BD Method with iteration

Iteration number	Master solution	Primal solution	Dual solution
01.	X_1=25000, X_2=20000, X_3=20000, X_4=22000, X_5=18000, Master value: 394667.	X_6=35000, X_7=21000, X_8=20000, X_9=25000, X_{10}=20000; Primal value: 406137	$\lambda_1 =$ 20.8667, λ_2=λ_3=λ_4=λ_5=λ_6=λ_7=λ_8=0; Dual value 638520
02.	X_1=0, X_2=16000, X_3=20000, X_4=22000; Master value: 354757	X_5=18000, X_6=0, X_7=21000, X_8=20000, X_9=25000, X_{10}=11000; Primal value 386951	λ_1=14.327, λ_2=6.27,λ_3=λ_4= λ_5=λ_6= λ_7= λ_8=0; Dual value 563806.2
03.	X_1=0, X_2=20000, X_3=20000; Master value: 306075.265	X_4=22000, X_5=18000, X_6=0, X_7=21000, X_8=20000, X_9=25000, X_{10}=4218.3; Primal value 306075.558	λ_1=14.003, λ_2=0,λ_3=λ_4= λ_5=λ_6= λ_7= λ_8=0; Dual value 553891.8

Table 3.2 describes the estimated parameters after iteration-3 of the Master problem, primal and Dual problems. The results of Table 3.2 demonstrate that the solutions are almost identical; this means that the optimal solution is achieved**.**

The following Table 3.3 describes the comparison of the optimal results are obtained by the main problem and by BD method.

Table 3.3: Comparison of solution of main problem and BDM problem

Solution of main problem	Solution of BDM

X_1=0, X_2 = 20000, X_3 =20000, X_4 = 22000, X_5 = 18000, X_6 = 0, X_7 = 21000, X_8 = 20000, X_9 = 25000, X_{10} = 4218.3; objective z = 623195.5866	X_1=0, X_2 = 20000, X_3 =20000, X_4 = 22000, X_5 = 18000, X_6 = 0, X_7 = 21000, X_8 = 20000, X_9 = 25000, X_{10} = 4218.1; objective z = 623194.859

In the following Fig. 1.1 and Fig 1.2, show that relation among selling price, profit, and cost of numerous parameters. Fig1.1 shows that profit has negative influence with selling price. Fig 1.2 describes that labor and machine cost is very low in compare to raw materials cost.

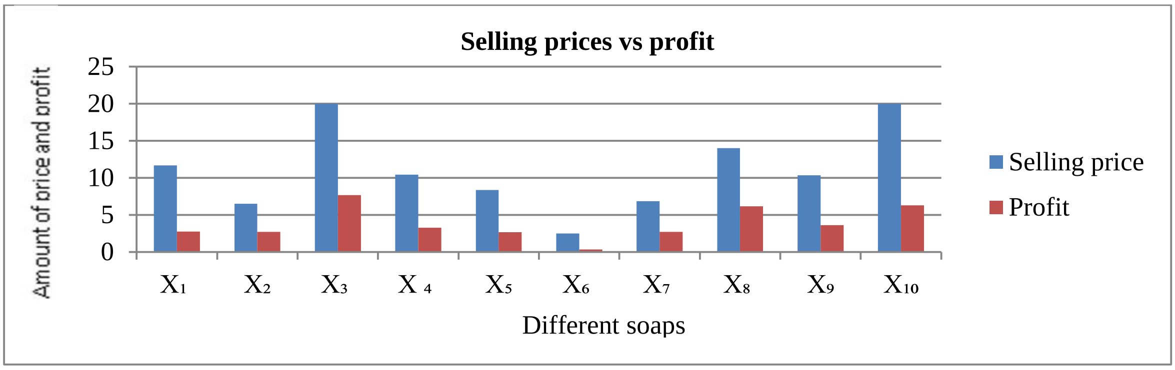

Fig 1.1: Selling price and profit

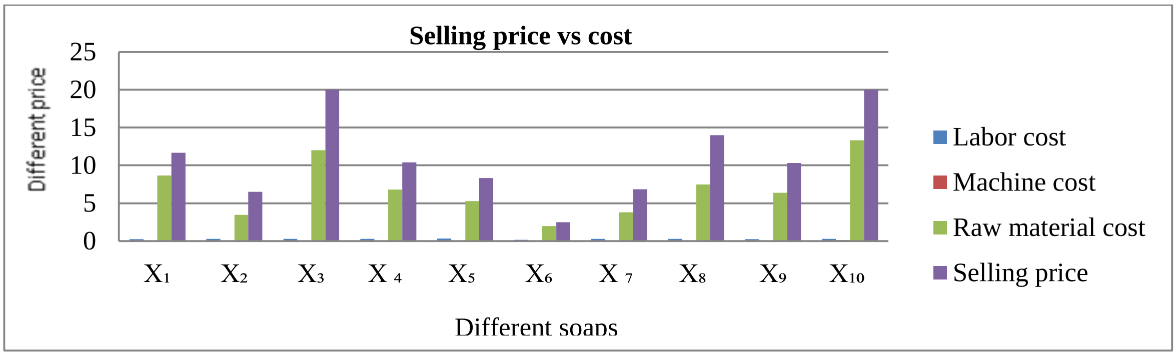

Fig 1.2: Selling price and cost

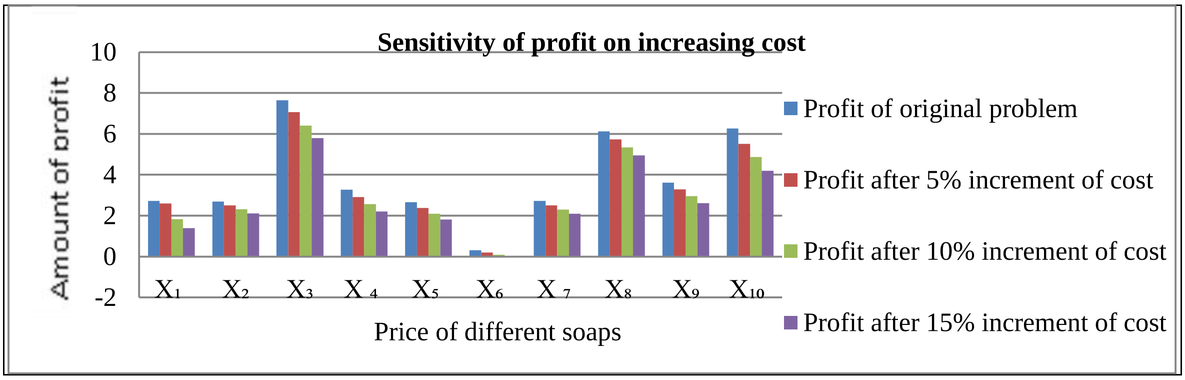

Fig1.3: Decreasing of profit by increasing cost parameters

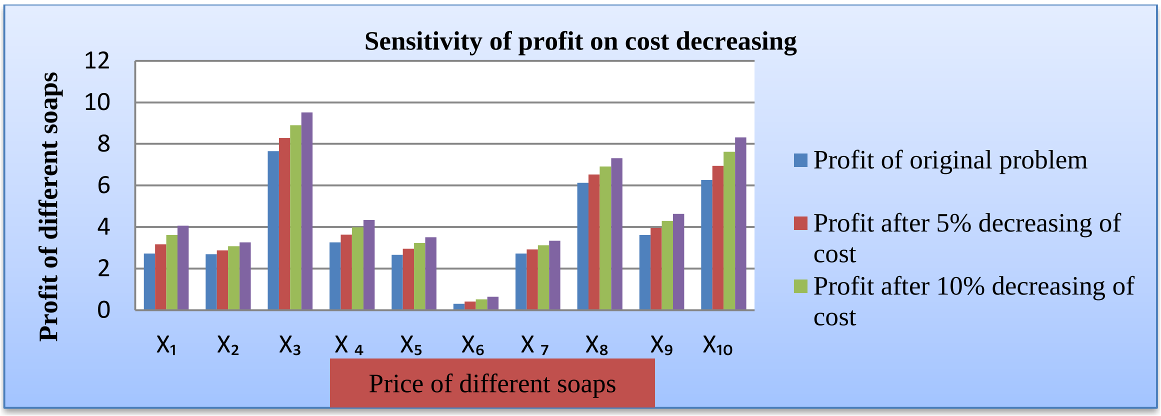

Fig 1.4: Increasing of profit by decreasing cost parameters

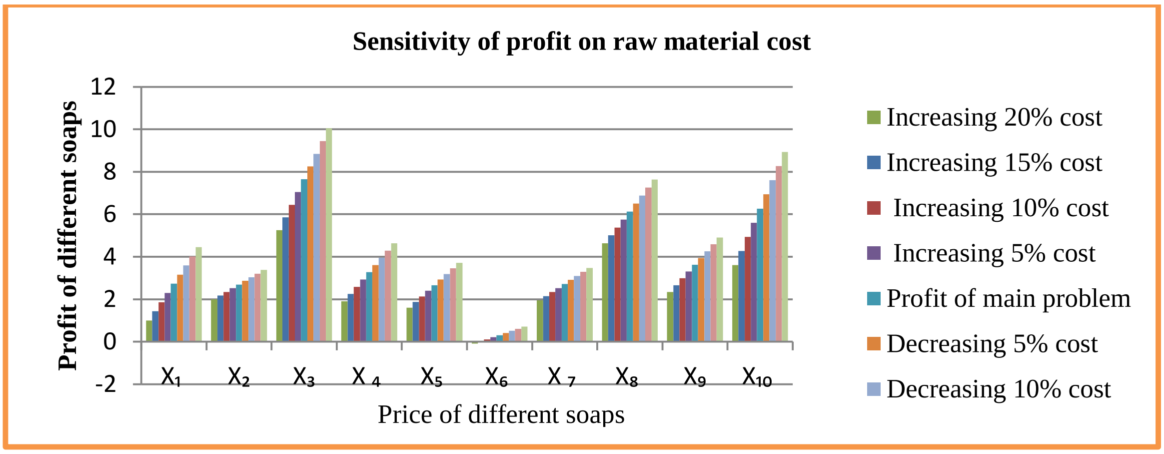

Fig 1.5: Profit analysis on raw material cost

In the Fig 1.3 and Fig 1.4 illustrate that all the cost parameters have significant effect of the profit. They demonstrate that how profit change if the cost parameters decrease. Further, if cost parameters decrease, then profit increase. From the Fig 1.5, it is shown that how profit changes if raw material cost changes. From the above Fig 1.5, if raw material cost increases profit decreases. Again, if the raw material cost decreases profit increases.Fig1.5 depicts the effects of raw materials cost on profit. Raw materials cost has significant influence on profit.

For the considered problem, the objective function is of maximization type and the objective function value gives the maximum profit. Here, the objective function value is 623195.5866. That means the maximum profit of the company is Tk 623195. From the result it is found that, 20000 unit of SornaliBati Soap, 20000 unit of Sornali Soap (2015), 22000 unit of Sornali Soap, 18000 unit of Mega Sornali Full Marka Soap, 0 unit of Mega Washing Powder (25g), 21000 unit of Mega Washing Powder (200g), 20000 unit of Mega Washing Powder (500g), 25000 unit of Mega Extra Washing Powder (200g), 4218 unit of Mega Extra Washing Powder (500g) are produced.

It is noticed from the result that, the production of product type one and six are zero. They are not so profitable. So, the company can stop to produce these two types of products. It is also noticed that production types three and ten are more profitable than other types of production. From sensitivity analysis, it has been found that if cost parameters increase by 5%, 10% and 15%, profit decrease. If cost parameters are decreased by 5%, 10%, 15% profit increase. It is also clear that labor cost and machine cost have not much effect on profit in the soap industry because labor cost and machine cost are very low in compare to other costs. Further, raw material cost is very much effective on profit. From the sensitivity analysis, it is also clear that if raw material cost can be reduced profit will be increased. It is shown that a small change can affect the profit a lot.

So, if the government can reduce tax and vat on raw materials of soap industry that is imported from abroad, this sector will become more profitable for the businessmen. If these raw materials can be produced in our country, soap industry will be more profitable in future than before. This sector can increase our GDP. It will also be able to contribute a lot to our economy. From this data, the company can easily get a clear idea about their profit, production rate and selling procedures. The main aim of any company is to maximize their gain with minimum resources. In the case of this company, they can get best profit with minimum cost. Dual variable that is shadow prices help the company to assume their profit. Now it can be said that, if the company uses mathematical modelling technique and plans about its production according to the optimal solution, obtained by computer programming, they will get an accurate idea about the cost, production rate and profit.

4. Conclusion

In this paper it is shown that the maximization of profit of Mega Sorsnali Soap and Cosmetic Industries Ltd. BDM is used to maximize profit. After obtaining the optimal result sensitivity analysis is also used to see the changes of optimal result after changing cost parameters.

Here ten types of production from the selected company have been taken into consideration. Labor cost, machine cost and raw material cost have also been taken into consideration. Then using this data an LPP is formulated. In this LPP, objective function is to maximize profit. Labor cost, machine cost, raw material cost and other cost are considered as subject to constraints. Maximum production rate that the company gave us are also taken into consideration as subject to constraints. After that this LPP is solved in AMPL. Then this problem is solved by BDM. Both solutions gave the same result. After that sensitivity analysis is discussed. Sensitivity analysis helps the company to improve their business policy. In the sensitivity analysis, we have increased cost parameters by 5%, 10% and 15%. Then we have found that profits decrease. Similarly, we have decreased cost parameters by same percentage. In this case, it is found that profits increase. Both cases are shown in the Fig 1.3 and Fig 1.4. We know that labor cost is very low in our country. We have taken machine cost also very low. But raw material cost is very high. In the Fig1.5 it is shown that if the raw material cost can be reduced this sector becomes more profitable. Like this company, applying of mathematical programming can help the owners of business organization to take correct decisions. This can identify the future production patterns and outlook resulting in the establishment of new production units, while thinking for maximizing profit and minimizing the cost of the company.

When we want to collect data the industry owners did not want to disclose their real data. In this study, we have collected data from a single industry. Future study can be done by collecting data from more industries to get better result. In future some other cost parameters such as transportation cost can be included. In this paper, there is no discussion on shadow price. In future, anyone can work on it. In future this model can be used in other industries.

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