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UNSTEADY FREE CONVECTIVE AND MASS TRANSFER FLOW THROUGH POROUS MEDIUM IN ROTATING SYSTEM

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ABSTRACT

An exact solution of unsteady free convective and mass transfer flow through porous medium bounded by a vertical porous plate with an oscillating free stream velocity is obtained. The entire system rotates about an axis normal to the plate. Existence of multiple Ekman boundary layer modified by the presence of free convection and mass transfer is observed near the neighbourhood of plate wall.

Key words: Porous medium, rotating system, free convective flow, and mass transfer.

1. Introduction

The flow through porous medium, under the influence of temperature differences and concentration differences, is one of the most considerable and contemporary subject, because it finds great applications in geothermy, geophysics and technology [1, 2]. Yamamoto and Iwamara [2] expressed the equations of flow through a highly porous medium. Raptis *et.al* [3,4] using the above equations studied the influences of free convection and mass transfer on the steady flow of a viscous fluid through the porous medium, which is bounded by a vertical plane surface, when the temperature and concentration on the surface is constant. Raptis *et.al* [5] also studied the influence of free convective flow on the steady flow of the viscous fluid through the porous medium, when there is a constant heat flux on the above-mentioned surface.

On the other hand, the geophysical importance of the flows in the rotating frame of reference has attracted the attention of a number of scholars. Raptis [6] analyzed the steady free convective and mass transfer flow through porous medium in presence of a rotating fluid. Later Mahato and Maiti [7] investigated unsteady free convective flow and mass transfer in a rotating porous medium. Mahato and Maiti [8] analyzed the effect of unsteady free convective flow and mass transfer during the motions of a viscous incompressible fluid in a rotating frame of references. Alam *et al*. [9] studied unsteady free convection and mass transfer flow in a rotating system with hall currents, viscous dissipation and joule heating. Later Singh *et al*. [10] studied free convection in MHD flow of a rotating viscous liquid in porous medium. Recently Singh *et al*. [11] have studied free convective MHD flow of rotating viscous fluid in a porous medium past infinite vertical porous plate. Varma *et al*. [12] studied free and force convection flow in a parallel channel bounded below by a permeable bed and rotating about an axis perpendicular to the plates under the influence of a uniform transverse magnetic field. Sarkar and Mukherjee [13] analyzed the effect of unsteady free convective flow and mass transfer during the motion of a viscous incompressible fluid through porous medium bounded by an infinite vertical porous plate in presence of heat source with variable suction under the influence of uniform magnetic field applied perpendicular to the flow of region in a rotating system.

The object of the present paper is to study the free convective and mass transfer flow of viscous fluid through a rotating porous medium bounded by a vertical porous plate subjected to a constant suction velocity in presence of constant heat flux at the plate. The temperature and concentration at the free streams are constant but the free stream velocity of the fluid vibrates about a mean constant value. The analytical expressions for velocity, temperature and concentration distribution are obtained and the results are presented graphically.

2. Mathematical analysis

We consider unsteady free convective and mass transfer flow of viscous fluid through a porous medium occupying a semi-infinite region bounded by a vertical porous plate subjected to constant suction in presence of constant heat flux at plate wall in a rotating frame of reference. The velocity of the fluid far away from the surface vibrates about a mean value with direction parallel to the plane z = 0. The temperature and species concentration at the free stream are constant. A uniform magnetic field of strength B_0 is applied in vertical upward direction. The porous medium is in fact a non-homogeneous medium, which may be replaced by a homogeneous fluid having dynamical properties equal to those of a non-homogeneous continuum. We consider that the vertical infinite porous plate rotates in unison with a viscous fluid occupying the porous region with constant angular velocity Ω about an axis which is perpendicular to the vertical plane surface Cartesian co-ordinate system is chosen such that x, y-axes, respectively, are in the vertical upward and perpendicular directions on the plane of the vertical porous surface z = 0 while z-axis is normal to it. u^+, v^+, ω^+ are the velocity components in x, y and z direction respectively. With the above frame of reference and assumptions, the physical variables, except the pressure $P$are function of z and time t only. Consequently the equation expressing the conservation of mass, momentum, energy and concentration, neglecting the heat due to viscous dissipation, which is valid for small velocities, are given by

$$\frac{\partial \omega^+}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u^+}{\partial t} - \omega^+ \frac{\partial u^+}{\partial z} - 2\Omega v^+ = \frac{\partial U^+}{\partial t} - 2\Omega V^+ - \left(\frac{\nu}{K^+} + \frac{\sigma B_0^2}{\rho}\right)\left(u^+ + U^+\right) + \nu \frac{\partial^2 u^+}{\partial z^2} + g\beta\left(T^+ - T_\infty^+\right) + g\beta^*\left(C^+ - C_\infty^+\right) \tag{2}$$

$$\frac{\partial v^+}{\partial t} - \omega^+ \frac{\partial v^+}{\partial z} - 2\Omega u^+ = \frac{\partial V^+}{\partial t} - 2\Omega U^+ - \left(\frac{\nu}{K^+} + \frac{\sigma B_0^2}{\rho}\right)\left(v^+ - V^+\right) + \nu \frac{\partial^2 v^+}{\partial z^2} \tag{3}$$

$$0=-\frac{1}{\rho}\frac{\partial P}{\partial z}-\frac{\nu}{K^{+}}\omega^{+} \tag{4}$$

$$\frac{\partial T^{+}}{\partial t}-\omega^{+}\frac{\partial T^{+}}{\partial z}=\frac{\kappa^{*^{+}}}{\rho C_{p}}\frac{\partial^{2}T^{+}}{\partial z^{2}} \tag{5}$$

$$\frac{\partial C^{+}}{\partial t}-\omega^{+}\frac{\partial C^{+}}{\partial z}=D\frac{\partial^{2}C^{+}}{\partial z^{2}} \tag{6}$$

where ν is the kinematic viscosity, t t is the time, ρ is the density, K^{+} is the permeability of the porous medium. T^{+} is the temperature and C^{+} is the concentration.

The boundary conditions relevant to the problem are

$$\left.\begin{aligned} &u^{+}=0, \quad v^{+}=0, \quad \frac{\partial T^{+}}{\partial z}=-\frac{s}{\kappa^{*^{+}}}, \quad C^{+}=C_{\infty}^{+} \quad at\ z=0\\ &u^{+}=U^{+}(t)=U_{0}(1+\varepsilon\cos\phi t), \quad v^{+}=V^{+}(t)=0, \quad T^{+}=T_{\infty}^{+}, \quad C^{+}=C_{\infty}^{+} \quad as\ z\to\infty \end{aligned}\right\} \tag{7}$$

where ϕ is the frequency of oscillation and ε is a small positive quantity.

From equation (1), we get

$$\omega^{+}=-\omega_{0} \tag{8}$$

Let equation (2) and (3) can be combined in complex form, as

$$\frac{\partial q^{+}}{\partial t}-\omega_{0}\frac{\partial q^{+}}{\partial z}+2i\Omega q^{+}=\frac{\partial Q^{+}}{\partial t}+2i\Omega Q^{+}-\left(\frac{\nu}{K^{+}}+\frac{\sigma B_{0}^{2}}{\rho}\right)\left(q^{+}-Q^{+}\right)+\nu\frac{\partial^{2}q^{+}}{\partial z^{2}}+g\beta\left(T^{+}-T_{\infty}^{+}\right)+g\beta^{*}\left(C^{+}-C_{\infty}^{+}\right) \tag{9}$$

and equations (4) and (5), using equation (8) can be written in the form as

$$\frac{\partial T^{+}}{\partial t}-\omega_{0}\frac{\partial T^{+}}{\partial z}=\frac{\kappa^{*^{+}}}{\rho C_{p}}\frac{\partial^{2}T^{+}}{\partial z^{2}} \tag{10}$$

$$\frac{\partial C^{+}}{\partial t}-\omega_{0}\frac{\partial C^{+}}{\partial z}=D\frac{\partial^{2}C^{+}}{\partial z^{2}} \tag{11}$$

We introduce the following non-dimensional quantities:

$$\eta=\frac{\omega_{0}}{\nu}z,\quad \tau=\frac{\omega_{0}^{2}t}{\nu},\quad q=\frac{q^{+}}{U_{0}},\quad K=\frac{\omega_{0}^{2}}{\nu^{2}}K^{+},\quad Gr=\frac{\nu g\beta\left(T_{w}^{+}-T_{\infty}^{+}\right)}{U_{0}\omega_{0}^{2}},\quad Gm=\frac{\nu g\beta^{*}\left(C_{w}^{+}-C_{\infty}^{+}\right)}{U_{0}\omega_{0}^{2}}$$

$$E=\frac{\Omega\nu}{\omega_{0}^{2}},\quad C=\frac{C^{+}-C_{\infty}^{+}}{C_{w}^{+}-C_{\infty}^{+}},\quad \Pr=\frac{\mu C_{p}}{\kappa^{*^{+}}},\quad Sc=\frac{\nu}{D},\quad T=\frac{T^{+}-T_{\infty}^{+}}{-\dfrac{\nu s}{\kappa^{*^{+}}\omega_{0}}},\quad M=\frac{\sigma\nu B_{0}^{2}}{\rho\omega_{0}^{2}},\quad Q=\frac{Q^{+}}{U_{0}}$$

$$\alpha=\frac{\phi\nu}{\omega_{0}^{2}}$$

Using the above stated non-dimensional quantities, the equations (9), (10) and (11) reduce to

$$\frac{\partial q}{\partial \tau}-\frac{\partial q}{\partial \eta}+2iEq=\frac{\partial Q}{\partial \tau}+2iEQ-\left(M+\frac{1}{K}\right)(q-Q)+\frac{\partial^2 q}{\partial \eta^2}+GmC+GrT \tag{12}$$

$$\frac{\partial T}{\partial \tau}-\frac{\partial T}{\partial \eta}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial \eta^2} \tag{13}$$

$$\frac{\partial C}{\partial \tau}-\frac{\partial C}{\partial \eta}=\frac{1}{Sc}\frac{\partial^2 C}{\partial \eta^2} \tag{14}$$

with boundary conditions

$$\left.\begin{aligned} &q=0, && \frac{\partial T}{\partial \eta}=-1, \quad C=1 && at \quad \eta=0 \\ &q=1+\frac{\varepsilon}{2}\left(e^{i\alpha\tau}+e^{-i\alpha\tau}\right), && T=0, \qquad C=0 && as \quad \eta\to\infty \end{aligned}\right\} \tag{15}$$

3. Solution

Let, the solutions of equations (12), (13) and (14) are assumed, respectively, as

$$q(\eta,\tau)=q_0(\eta)+\frac{\varepsilon}{2}\left\{q_1(\eta)e^{i\alpha\tau}+q_2(\eta)e^{-i\alpha\tau}\right\} \tag{16}$$

$$T(\eta,\tau)=T_0(\eta)+\varepsilon T_1(\eta)e^{i\alpha\tau}+........... \tag{17}$$

$$C(\eta,\tau)=C_0(\eta)+\varepsilon C_1(\eta)e^{i\alpha\tau}+........... \tag{18}$$

Using equations (16), (17) and (18) in equations (12), (13) and (14), we obtain following equations

$$q_0''+q_0'-(L+2iE)q_0=-\frac{Gr}{\Pr}e^{-\Pr\eta}-Gme^{-Sc\eta} \tag{19}$$

$$q_1''+q_1'-\{L+i(2E+\alpha)\}q_1=-\{L+i(2E+\alpha)\} \tag{20}$$

$$q_2''+q_2'-\{L+i(2E-\alpha)\}q_1=-\{L+i(2E-\alpha)\} \tag{21}$$

$$T_0''+\Pr T_0'=0 \tag{22}$$

$$T_1''+\Pr T_1'-i\alpha\Pr T_1=0 \tag{23}$$

$$C_0'' + ScC_0' = 0 \tag{24}$$

$$C_1'' + ScC_1' - i\alpha ScC_1 = 0 \tag{25}$$

Using (13), (14) and (15) in (12), the boundary conditions are reduced to

$$\left.\begin{array}{l} q_0 = q_1 = q_2 = 0, \quad T_0' = -1, \quad T_1' = 0, \quad C_0 = 1, \quad C_1 = 0 \quad at \quad \eta = 0 \\ q_0 = q_1 = q_2 = 1, \quad T_0 = 0, \quad T_1 = 0, \quad C_0 = 0, \quad C_1 = 0 \quad as \quad \eta \to \infty \end{array}\right\} \tag{26}$$

The solutions of equations (19) to (25), under the boundary conditions (26) and in view of (16), (17) and (18) are given by

$$q(\eta,\tau) = 1 - (1 - R_1 - R_2)e^{-R_3\eta} - R_1 e^{-\Pr\eta} - R_2 e^{-Sc\eta} + \frac{\varepsilon}{2}\left\{\left(1 - e^{-R_4\eta}\right)e^{i\alpha\tau} + \left(1 - e^{-R_5\eta}\right)e^{-i\alpha\tau}\right\} \tag{27}$$

where,

$$q_0(\eta) = 1 - (1 - R_1 - R_2)e^{-R_3\eta} - R_1 e^{-\Pr\eta} - R_2 e^{-Sc\eta} \tag{27a}$$

$$q_1(\eta) = 1 - e^{-R_4\eta} \tag{27b}$$

and

$$q_2(\eta) = 1 - e^{-R_5\eta} \tag{27c}$$

$$T(\eta) = \frac{1}{\Pr} e^{-\Pr\eta} \tag{28}$$

$$C(\eta) = e^{-Sc\eta} \tag{29}$$

The expressions for constant are given in appendix-I

4. Results and discussion

Equation (27) corresponds to the velocity distribution of free convective and mass transfer flow of viscous fluid through a rotating porous medium. The expression clearly shows the existence of thin multiple Ekman boundary layer of order $o\left(R_3^{-1}\right)$ super imposed with a boundary layer of thickness of order $o\left(Pr^{-1}\right)$ and $o\left(Sc^{-1}\right)$. It is interesting to note that Ekman boundary layer is modified by the presence of free convection and mass transfer. We also note that this layer decrease with increase of rotation parameter and magnetic parameter and increase with increase of permeability parameter.

The solution (27a) corresponds to the steady part which gives u_0 as the primary and v_0 as the secondary velocity components. The amplitude and phase difference due to these primary and secondary velocities for the steady flow are given by

$$|A_0| = \left(u_0^2 + v_o^2\right)$$

$$\theta_0 = \tan^{-1}\left(\frac{v_o}{u_0}\right)$$

where, $u_0 = \left[1 - \left\{\left(1 - P_1 - P_2\right)\cos Q_3\eta - \left(Q_2 + Q_1\right)\sin Q_3\eta\right\} e^{-P_3\eta} - P_1 e^{-\Pr\eta} - P_2 e^{-Sc\eta}\right]$

$$v_o = \left[\left\{\left(Q_2 + Q_1\right)\cos Q_3\eta + \left(1 - P_1 - P_2\right)\sin Q_3\eta\right\} e^{-P_3\eta} - Q_1 e^{-\Pr\eta} - Q_2 e^{-Sc\eta}\right]$$

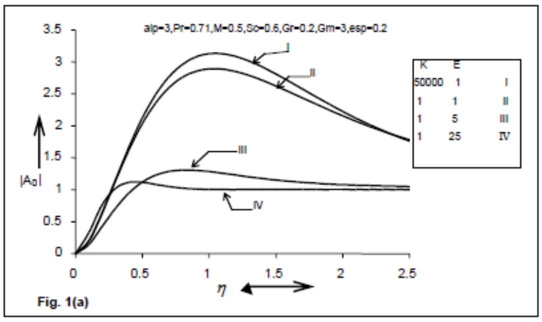


Fig. 1(a)

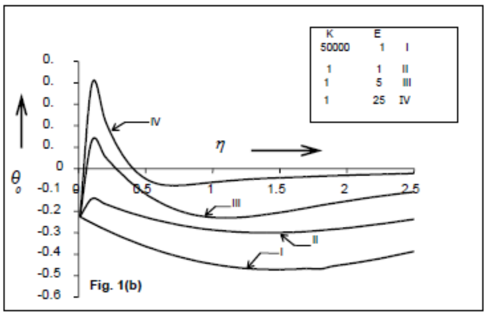


Fig. 1(b)

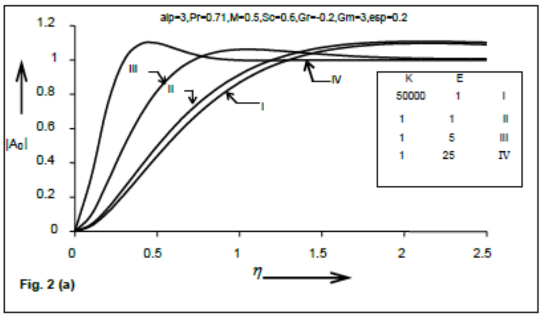


Fig. 2 (a)

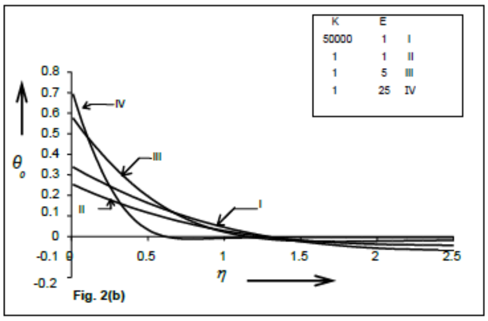


Fig. 2(b)

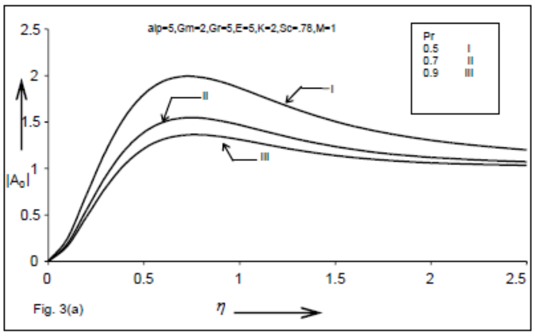


Fig. 3(a)

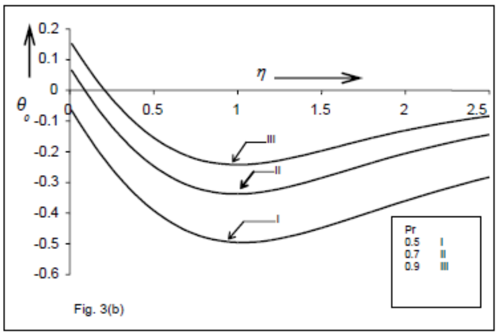


Fig. 3(b)

The amplitude of resultant velocity $|A_0|$ and the phase angle θ_0 for the steady part are shown graphically in Fig.1(a, b) and Fig.2 (a, b) for various values of the rotation parameter (E) and permeability parameter (K) for fixed values of Prandtl number (Pr), Schnidt number (*Sc*), magnetic parameter (*M*), Granshof number (*Gr*) and modified Granshof number (*Gm*). It is seen from Fig.1(a) that in case of $Gr > 0$ the Amplitude $|A_0|$ increase as *K* increases and nearly at $\eta = 2.5$ these two values coincide but opposite behaviour is seen for $Gr > 0$ and decreases with increase in rotation Fig. 1(b) θ_0 decreases with increase in *K* and increases with increasing *R* (both small and large) for $Gr > 0$ and increases as rotation parameter increases for $Gr > 0$ near the plate wall.

Variation of $|A_0|$ and θ_0 for different values of Prandtl number Pr and modified Grashof number for $Gr > 0$ are shown in Fig. 3(a, b) and 4(a, b). It is clear from these figure that amplitude decreases as Pr increases and increases as *Gm* increases but phase difference θ_0 decreases as *Gm* increases and increases as Pr increases. Numerical calculation are also made for $Gr > 0$ and shown in Graph 5(a, b) and 6(a, b).

It is essential to mention that equation (27b) and (27c) together give the unsteady part of the flow. This expression also exhibits boundary layer of thickness of order $O\left(R_4^{-1}\right)$ and order $O\left(R_5^{-1}\right)$ respectively.

The amplitude and the phase differences of shear stresses at the plate $\eta = 0$ for the steady flow can be obtained as:

$$\tau_{0r} = \left(\tau_{0x}^2 + \tau_{0y}^2\right)^{\frac{1}{2}}, \theta_{0r} = \tan^{-1}\left(\frac{\tau_{0y}}{\tau_{0x}}\right) \qquad (30)$$

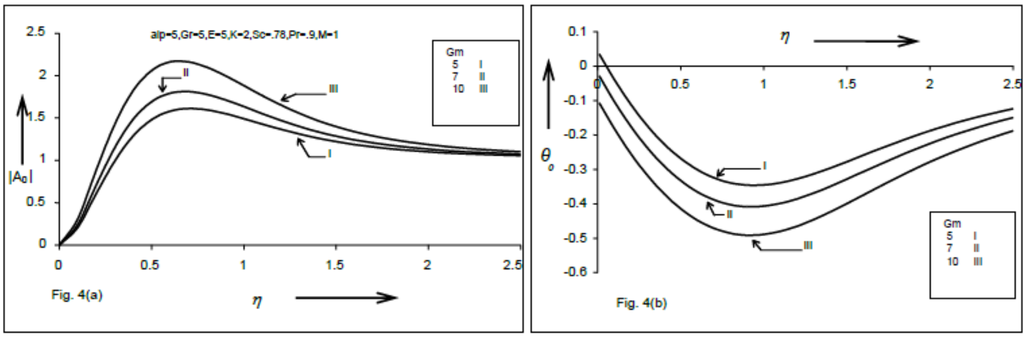
where τ_{or} and τ_{oy} are, respectively, the shear stress at the plate due to primary and secondary velocity components.

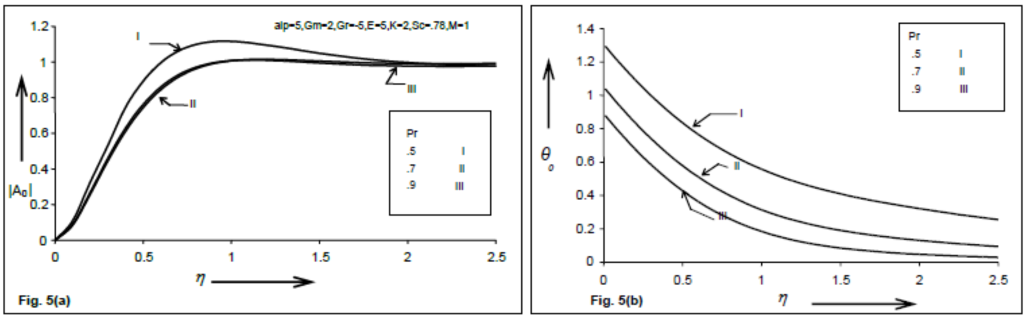
The numerical values for the resultant shear stress and the phase angle due to the shear stress are listed in Table-1.

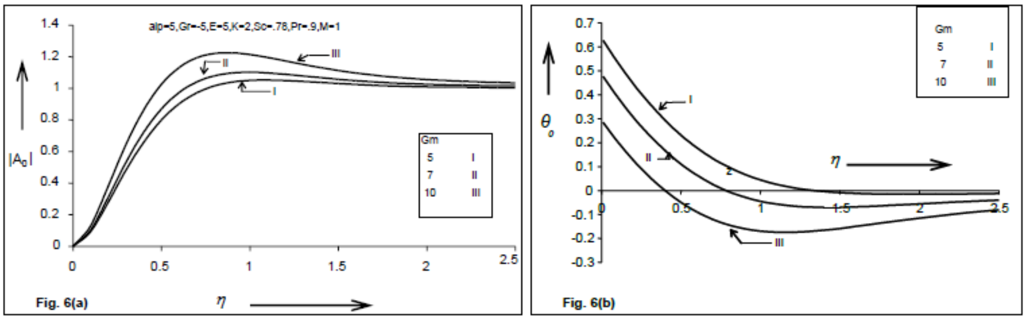
Table-1

Sl. No.	Pr	*Sc*	*K*	*E*	*M*	*Gm*	*Gr*	τ_{or}	θ_{oy}
1	0.71	0.3	1	1	0.5	5	10	15.7646	-0.5104
2	7	0.3	1	1	0.5	5	10	5.7257	-0.1937
3	0.71	0.66	1	1	0.5	5	10	15.1004	-0.4928
4	0.71	0.3	5	1	0.5	5	10	19.5276	-0.7104
5	0.71	0.3	1	5	0.5	5	10	9.3067	-0.2333
6	0.71	0.3	1	1	1	5	10	14.2511	-0.4069
7	0.71	0.3	1	1	0.5	10	10	20.2386	-0.5540
8	0.71	0.3	1	1	0.5	5	20	26.3319	-0.5768

These values clearly show that the shear stress τ_{or} increases as permeability parameter *K* increases and decreases as rotation parameter *R* increases. Also the increase in permeability parameter *K* lead to decrease in phase difference θ_{or} and the p hase difference θ_{or} increases as rotation parameter *R* increases.







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Appendix-I

$$L = M + \frac{1}{K};$$

$$R_1 = P_1 + iQ_1 = \frac{Gr}{\Pr\left\{\Pr^2 - \Pr - \left(L + 2iE\right)\right\}};$$

$$R_2 = P_2 + iQ_2 = \frac{Gm}{Sc^2 - Sc - \left(L + 2iE\right)};$$

$$R_3 = P_3 + iQ_3 = \frac{1 + \sqrt{1 + 4\left(L + 2iE\right)}}{2};$$

$$R_4 = P_4 + iQ_4 = \frac{1 + \sqrt{1 + 4\left\{L + i\left(\alpha + 2E\right)\right\}}}{2};$$

$$R_5 = P_5 + iQ_5 = \frac{1 + \sqrt{1 + 4\left\{L + i\left(2E - \alpha\right)\right\}}}{2};$$

$$P_1 = \frac{Gr\left(\Pr^2 - \Pr - L\right)}{\Pr\left\{\left(\Pr^2 - \Pr - L\right)^2 + 4E^2\right\}}; \qquad Q_1 = \frac{2GrE}{\Pr\left\{\left(\Pr^2 - \Pr - L\right)^2 + 4E^2\right\}};$$

$$P_2 = \frac{Gm\left(Sc^2 - Sc - L\right)}{\left\{\left(Sc^2 - Sc - L\right)^2 + 4E^2\right\}}; \qquad Q_2 = \frac{2GmE}{\left\{\left(Sc^2 - Sc - L\right)^2 + 4E^2\right\}};$$

$$P_3 = \frac{\sqrt{2} + \left[\left(1 + 4L\right) + \sqrt{\left(1 + 4L\right)^2 + 64E^2}\right]^{\frac{1}{2}}}{2\sqrt{2}}; \qquad Q_3 = \frac{2\sqrt{2}E}{\left[\left(1 + 4L\right) + \sqrt{\left(1 + 4L\right)^2 + 64E^2}\right]^{\frac{1}{2}}};$$

$$\tau_{0x} = \left\{\left(1 - P_1 - P_2\right)Q_3 \sin Q_3\eta + \left(Q_2 + Q_1\right)Q_3 \cos Q_3\eta\right\} e^{-P_3\eta}$$

$$+ \left\{\left(1 - P_1 - P_2\right)\cos Q_3\eta - \left(Q_2 + Q_1\right)\sin Q_3\eta\right\} P_3 e^{-P_3\eta} + P_1 \Pr e^{-\Pr\eta} + P_2 Sc e^{-Sc\eta};$$

$$\tau_{0y} = \left\{\left(1 - P_1 - P_2\right)Q_3 \cos Q_3\eta - \left(Q_2 + Q_1\right)Q_3 \sin Q_3\eta\right\} e^{-P_3\eta}$$

$$- \left\{\left(1 - P_1 - P_2\right)\sin Q_3\eta + \left(Q_2 + Q_1\right)\cos Q_3\eta\right\} P_3 e^{-P_3\eta} + Q_1 \Pr e^{-\Pr\eta} + Q_2 Sc e^{-Sc\eta};$$