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Intuitionistic Fuzzy R_0 Bitopological Space: An In-depth Exploration

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ABSTRACT

We propose intuitionistic fuzzy bitopological R_0 -spaces (abbreviated IFB-R_0) in this study. On intuitionistic fuzzy bitopological space, we introduce several new ideas of R_0-space. Our notions are maintained under one-one, onto, fuzzy open, and fuzzy continuous mappings, and we demonstrate that R_0 -spaces satisfy the "good extension" property.

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1. Introduction

Zadeh [1] introduced the first fuzzy set notion in 1965. Chang [2] defined fuzzy topological spaces in 1968 using this idea. Important components of fuzzy topological spaces are separation axioms [3, 4].

A wide range of separation axioms have also been offered by other academics [5, 6, 7]. About fuzzy bitopological space, which Kandil and EL-Shafee [8] initially discussed in 1991. M. S. Hossain and M. R. Amin conducted research in fuzzy bitopological spaces [9, 10].

Atanassov [11] introduced the idea of an intuitionistic fuzzy set, which is a generalization of fuzzy sets and takes into consideration both degree and non-degree membership as long as their sum does not exceed 1.

Coker [12,13] and his coworkers developed intuitionistic fuzzy topological spaces by using intuitionistic fuzzy sets. Numerous authors have written numerous articles on intuitionistic fuzzy topological space. Such as S. Bayhan and D. Coker [14] introduced fuzzy separation hypothesis in intuitionistic fuzzy topological space. D. Coker [15] also introduced fuzzy subspace in intuitionistic fuzzy topological space. Ahmed et al [16] studied on intuitionistic fuzzy R_0 space. The intent of this study is to advance intuitionistic fuzzy bitopological space. The fuzzy R_0 intuitionistic bitopological space is instance. In the current research, we develop intuitionistic fuzzy R_0 bitopological space and showed that the approach satisfies good extension property and hereditary property. The major objective of this research is to focus on theoretical development of separation axiom on intuitionistic fuzzy bitopological spaces. Also, the specific objectives are mentioned as follows:

- To define new notions on Intuitionistic fuzzy R_0 bitopological spaces.
- To study Hereditary and Good Extension properties on the notions.
- Justification of homeomorphism preserving, productive and projective properties.

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2. Intuitionistic Fuzzy R_0 Bitopological space

We convey our thoughts and assessments in the following section. Here, we are applying our definition for debating several well-known features.

Definition 2.1: *It has been referred to as an intuitionistic fuzzy bitopological space* (X, s, t) *is*

(a) *IFB-*R_0*(i) if for all* $x_1, x_2 \in X$, $x_1 \neq x_2$ *whenever* $\exists\ M = (\mu_M, \nu_M) \in (s \cup t)$ *with* $\mu_M(x_1) = 1$, $\nu_M(x_1) = 0$, $\mu_M(x_2) = 0$, $\nu_M(x_2) = 1$ *then* $\exists\ N = (\mu_N, \nu_N) \in (s \cup t)$ *such that* $\mu_N(x_1) = 1$, $\nu_N(x_2) = 0$, $\mu_N(x_1) = 0$, $\nu_N(x_1) = 1$.

(b) *IFB-*R_0*(ii) if for all* $x_1, x_2 \in X$, $x_1 \neq x_2$ *whenever* $\exists\ M = (\mu_M, \nu_M) \in (s \cup t)$ *with* $\mu_M(x_1) = 1$, $\nu_M(x_1) = 0$, $\mu_M(x_2) = 0$, $\nu_M(x_2) > 0$ *then* $\exists\ N = (\mu_N, \nu_N) \in (s \cup t)$ *such that* $\mu_N(x_2) = 1$, $\nu_N(x_2) = 0$, $\mu_N(x_1) = 0$, $\nu_N(x_1) > 0$.

(c) *IFB-*R_0*(iii) if for all* $x_1, x_2 \in X$, $x_1 \neq x_2$ *whenever* $\exists\ M = (\mu_M, \nu_M) \in (s \cup t)$ *with* $\mu_M(x_1) > 0$, $\nu_M(x_1) = 0$, $\mu_M(x_2) = 0$, $\nu_M(x_2) = 1$ *then* $\exists\ N = (\mu_N, \nu_N) \in (s \cup t)$ *such that* $\mu_N(x_2) > 0$, $\nu_N(x_2) = 0$, $\mu_N(x_1) = 0$, $\nu_N(x_1) = 1$.

(d) *IFB-*R_0*(iv) if for all* $x_1, x_2 \in X$, $x_1 \neq x_2$ *whenever* $\exists\ M = (\mu_M, \nu_M) \in (s \cup t)$ *with* $\mu_M(x_1) > 0$, $\nu_M(x_1) = 0$, $\mu_M(x_2) = 0$, $\nu_M(x_2) > 0$ *then* $\exists\ N = (\mu_N, \nu_N) \in (s \cup t)$ *such that* $\mu_N(x_2) > 0$, $\nu_N(x_2) = 0$, $\mu_N(x_1) = 0$, $\nu_N(x_1) > 0$.

α-Intuitionistic fuzzy bitopological R_0-space

Definition 2.2: *Let* $\alpha \in (0, 1)$. *An intuitionistic fuzzy bitopological space* (X, s, t) *is called*

(a) α *−IFB-*R_0*(i) if for all* $x_1, x_2 \in X$, $x_1 \neq x_2$ *whenever* $\exists\ M = (\mu_M, \nu_M) \in (s \cup t)$ *with* $\mu_M(x_1) = 1$, $\nu_M(x_1) = 0$, $\mu_M(x_2) = 0$, $\nu_M(x_2) \geq \alpha$ *then* $\exists\ N = (\mu_N, \nu_N) \in (s \cup t)$ *such that* $\mu_N(x_2) = 1$, $\nu_N(x_2) = 0$, $\mu_N(x_1) = 0$, $\nu_N(x_1) \geq \alpha$

(b) α *−IFB-*R_0*(ii) if for all* $x_1, x_2 \in X$, $x_1 \neq x_2$ *whenever* $\exists\ M = (\mu_M, \nu_M) \in (s \cup t)$ *with* $\mu_M(x_1) \geq \alpha$, $\nu_M(x_1) = 0$, $\mu_M(x_2) = 0$, $\nu_M(x_2) \geq \alpha$ *then* $\exists\ N = (\mu_N, \nu_N) \in (s \cup t)$ *such that* $\mu_N(x_2) \geq \alpha$, $\nu_N(x_2) = 0$, $\mu_N(x_1) = 0$, $\nu_N(x_1) \geq \alpha$

(c) α *−IFB-*R_0*(iii) if for all* $x_1, x_2 \in X$, $x_1 \neq x_2$ *whenever* $\exists\ M = (\mu_M, \nu_M) \in (s \cup t)$ *with* $\mu_M(x_1) > 0$, $\nu_M(x_1) = 0$, $\mu_M(x_2) = 0$, $\nu_M(x_2) \geq \alpha$ *then* $\exists\ N = (\mu_N, \nu_N) \in (s \cup t)$ *such that* $\mu_N(x_2) > 0$, $\nu_N(x_2) = 0$, $\mu_N(x_1) = 0$, $\nu_N(x_1) \geq \alpha$

Theorem 2.1: *The features IBF-*R_0*(i), IBF-*R_0*(ii), IBF-*R_0*(iii) and IBF-*R_0*(iv) are all autonomous.*

Proof: In order to demonstrate that there are no consequences among these features, we have a look at the instances below.

Example 2.1: Let $X = \{x_1, x_2\}$ and $(s \cup t)$ be the intuitionistic fuzzy bitopology on X generated by $\{M, N\}$ where $M = \{(x_1, 1, 0)\ (x_2, 0, 0.5)\}$ and $N = \{(x_1, 0.3, 0.2)\ (x_2, 0.1, 0.4)\}$. We see that the IFBTS $(X, s \cup t)$ *is* $IFB - R_0(i)$ but it is neither $IFB - R_0(ii)$ nor $IFB - R_0(iv)$.

Example 2.2: Let $X = \{x_1, x_2\}$ and $(s \cup t)$ be the intuitionistic fuzzy bitopology on X generated by $\{M, N\}$ where $M = \{(x_1, 0.6, 0)\ (x_2, 0, 1)\}$ and $N = \{(x_1, 0.2, 0.7)\ (x_2, 0.6, 0.3)\}$. We see that the IFBTS $(X, s \cup t)$ is $IFB - R_0(i)$ but it is not $IFB - R_0(iii)$.

Example 2.3: Let $X = \{x_1, x_2\}$ and $(s \cup t)$ be the intuitionistic fuzzy bitopology on X generated by $\{M, N\}$ where $M = \{(x_1, 0.5, 0)\ (x_2, 0, 1)\}$ and $N = \{(x_1, 0.2, 0.4)\ (x_2, 0.1, 0.6)\}$. We see that the IFBTS $(X, s \cup t)$ is $IFB - R_0(ii)$ but it is neither $IFB - R_0(iii)$ nor $IFB - R_0(iv)$.

Example 2.4: Let $X = \{x_1, x_2\}$ and $(s \cup t)$ be the intuitionistic fuzzy bitopology on X generated by $\{M, N\}$ where $M = \{(x_1, 1, 0)\ (x_2, 0, 1)\}$ and $N = \{(x_1, 0, 0.2)\ (x_2, 1, 0)\}$. We see that the IFBTS $(X, s \cup t)$ is IFB-R_0(iii) but it is not IFB-R_0(i).

Example 2.5: Let $X = \{x_1, x_2\}$ and $(s \cup t)$ be the intuitionistic fuzzy bitopology on X generated by $\{M, N\}$ where $M = \{(x_1, 0.7, 0)\ (x_2, 0, 0.4)\}$ and $N = \{(x_1, 0.3, 0.2)\ (x_2, 0.5, 0.2)\}$. We see that the IFBTS $(X, s \cup t)$ is IFB-R_0(iii) but it is not IFB-R_0(iv)

Example 2.6: Let $X = \{x_1, x_2\}$ and $(s \cup t)$ be the intuitionistic fuzzy bitopology on X generated by $\{M, N\}$ where $M = \{(x_1, 1, 0)\ (x_2, 0, 1)\}$ and $N = \{(x_1, 0, 1)\ (x_2, 0.5, 0)\}$. We see that the IFBTS $(X, s \cup t)$ is IFB-R_0(iii) but it is not IFB-R_0(i).

Example 2.7: Let $X = \{x_1, x_2\}$ and $(s \cup t)$ be the intuitionistic fuzzy bitopology on X generated by $\{M, N\}$ where $M = \{(x_1, 0.2, 0)\ (x_2, 0, 1)\}$ and $N = \{(x_1, 0, 0.5)\ (x_2, 0.7, 0)\}$. We see that the IFBTS $(X, s \cup t)$ is IFB-R_0(iv) but it is not IFB-R_0(iii).

Theorem 2.2: The properties α −IBF-R_0(i), $\alpha -$ IBF-R_0(ii), and α −IBF-R_0(iii) are all independent.

Proof: To prove the non- implications among these properties, we consider the following examples.

Example 2.8: Let X={ x_1, x_2} and $(s \cup t)$ be the intuitionistic fuzzy bitopology on X generated by $\{M, N\}$ where M = { (x_1, 0.6, 0) (x_2, 0, 0.7)} and N = {(x_1, 0.4, 0.3) (x_2, 0.5, 0.2)}. For $\alpha = 0.2$. We see that the IFBTS (X, s∪ t) is $\alpha -$ IFB-R_0(i) but it is neither $\alpha -$ IFB-R_0(ii) nor $\alpha -$ IFB-R_0(iii).

Example 2.9: Let X={ x_1, x_2} and $(s \cup t)$ be the intuitionistic fuzzy bitopology on X generated by $\{M, N\}$ where M = {(x_1, 1, 0) (x_2, 0, 0.7)} and N = {(x_1, 0, 0.6) (x_2, 0.5, 0)}. For $\alpha = 0.3$. We see that the IFBTS $(X, s \cup t)$ is $\alpha -$ IFB-R_0(ii) but it is not $\alpha -$ IFB-R_0(i).

Example 2.10: Let $X = \{ x , y\}$ and $(s \cup t)$ be the intuitionistic fuzzy bitopology on X generated by $\{M, N\}$ where M = {(x, 0.2, 0) (y, 0, 0.6)} and N= {(x, 0, 0.2) (y, 0.3, 0)}. For $\alpha = 0.5$.We see that the IFBTS (X, s∪ t) is $\alpha -$ IFB-R_0(ii) but it is not $\alpha -$ IFB-R_0(iii).

Example 2.11: Let $X = \{ x_1, x_2\}$ and $(s \cup t)$ be the intuitionistic fuzzy bitopology on X generated by $\{M, N\}$ where M = {(x_1, 1, 0) (x_2, 0, 0.7)} and N = {(x_1, 0, 0.3) (x_2, 0.2, 0)}. For $\alpha = 0.3$. We see that the IFBTS $(X, s \cup t)$ is $\alpha -$ IFB-R_0(iii) but it is neither $\alpha -$ IFB-R_0(i) nor $\alpha -$ IFB-R_0(ii).

3. Hereditary property

Theorem 3.1: Let (X, s, t) be an intuitionistic fuzzy bitopological space, $W \subseteq X$ and $s_W = \{ M/W : M \in s\}$ and $t_W = \{ N/W : N \in t\}$ and $\alpha \in$ (0,1) then

1. *(X, s, t) is IFB-R_0(i)⟹$(W, s_W, t_W,)$ is IFB-R_0(i)*
2. *(X, s, t) is IFB-R_0(ii)⟹(W, s_W, t_W) is IFB-R_0(ii)*
3. *(X, s, t) is IFB-R_0(iii)⟹$(W, s_W, t_W,)$ is IFB-R_0(iii)*
4. *(X, s, t) is IFB-R_0(iv)⟹(W, s_W, t_W) is IFB-R_0(iv)*
5. *(X, s, t) is $\alpha -$ IFB-R_0(i)⟹(W, s_W, t_W) is $\alpha -$ IFB-R_0(i)*
6. *(X, s, t) is $\alpha -$ IFB-R_0(ii)⟹(W, s_W, t_W) is $\alpha -$ IFB-R_0(ii)*
7. *(X, s, t) is $\alpha -$ IFB-R_0(iii)⟹(W, s_W, t_W) is $\alpha -$ IFB-R_0(iii).*

The proof (1), (2), (3), (4), (5), (6), (7) are similar. As an example, we proved (1).

Proof (1): Suppose (X, s, t) is the fuzzy bitopological space and is also IFB-R_0(i). We shall prove that (W, s_W, t_W) is IFB -R_0(i). Let $x_1, x_2 \in W$, $x_1 \neq x_2$ with M_W= $(\mu_{M_W}, \nu_{M_W}) \in (s_W \cup t_W)$ such that $\mu_{M_W}(x_1) = 1, \nu_{M_W}(x_1) = 0$ and $\mu_{M_W}(x_2) = 0, \nu_{M_W}(x_2) = 1$.

Suppose $M = (\mu_M , \nu_M) \in (s \cup t)$ is the extension IFBS of M_W on X, $\mu_M(x_1) = 1$, $\nu_M(x_1)$= 0 and $\mu_M(x_2) = 0$, $\nu_M(x_2) = 1$. Since $x_1, x_2 \in W \subseteq X$. That is $x_1, x_2 \in X$. Again since (X, s, t) $IFB - R_0$(i) then there exists $N = (\mu_N , \nu_N) \in (s \cup t)$ such that $\mu_N(x_2)$= 1, $\nu_N(x_2)$= 0 and $\mu_N(x_1)$= 0, $\nu_N(x_1)$= 1

$\Rightarrow (\mu_N / W) (x_2) = 1, (\nu_N / W)(x_2) = 0$ and $(\mu_N / W)(x_1) = 0, (\nu_N / W)(x_1) = 1$.

Hence $(\mu_N / W, \nu_N / W)) \in (s_W \cup t_W)$.

Therefore (U, s_W, t_W) is $IFB - R_0$(i).

Definition 3.1: *An intuitionistic bitopological space (X ,τ_1 , τ_2) is called intuitionistic PR_0-space (I -PR_0 space) if for all $x_1, x_2 \in X$, $x_1 \neq x_2$ whenever $\exists$ $M = (M_1 , M_2) \in \tau_1 \cup \tau_2$ with $x_1 \in M_1, x_1 \notin M_2$ and $x_2 \notin M_1 , x_2 \in M_2$ then $\exists$ $N = (N_1 , N_2) \in (\tau_1 \cup \tau_2)$ such that $x_2 \in N_1 ,x_2 \notin N_2$ and $x_1 \notin N_1, x_1 \notin N_2$.*

4. Good Extension Property

We demonstrate in this part that our conceptions adhere to the good extension property.

Theorem 4.1: *Let (X, τ_1, τ_2) be an intuitionistic bitopological space and let (X, t_1, t_2) be an intuitionistic fuzzy bitopological space. Then we have the following implication*

a. (X, τ_1, τ_2) *is* $I - PR_0 \Leftrightarrow (X, t_1, t_2)$ *is* $IF - PR_0$ *(i)*.

b. (X, τ_1, τ_2) *is* $I - PR_0 \Rightarrow (X, t_1, t_2)$ *is* $IF - PR_0$ *(ii)*.

c. (X, τ_1, τ_2) *is* $I - PR_0 \Rightarrow (X, t_1, t_2)$ *is* $IF - PR_0$ *(iii)*.

d. (X, τ_1, τ_2) *is* $I - PR_0 \Rightarrow (X, t_1, t_2)$ *is* $IF - PR_0$ *(iv)*.

Proof (a): Let (X , τ_1 , τ_2) be $I - PR_0$ space we shall show that (X , t_1 , t_2) *is* $IF - PR_0$(i). Suppose (X, τ_1 , τ_2) *is* $I - PR_0$. Let $x_1, x_2 \in X$ $x_1 \neq x_2$ with $(1M_1 ,1M_2) \in (t_1 \cup t_2)$ such that $1M_1(x_1) = 1, 1M_2(x_1) = 0, 1M_1(x_2) = 0, 1M_2(x_2) = 1$. $\Rightarrow x_1 \in M_1, x_1 \notin M_2 , x_2 \notin M_1 , x_2 \in M_2$. Hence $(M_1, M_2) \in (\tau_1 \cup \tau_2)$. Since (X, τ_1 , τ_2)*is* $I - PR_0$ then there exist $(N_1, N_2) \in (\tau_1 \cup \tau_2)$ such that $x_2 \in N_1, x_2 \notin N_2$ and $x_1 \notin N_1 , x_1 \in N_2$.$\Rightarrow 1N_1(x_2) = 1, 1N_2(x_2) = 0$ and $1N_1(x_1) = 0 ,1N_2(x_1) = 1$.$\Rightarrow (1N_1, 1N_2) \in (t_1 \cup t_2)$.

Hence (X, t_1, t_2) is IF-PR$_0$(i). Therefore $I - PR_0 \Rightarrow IF - PR_0$ (i).

Conversely let (X, t_1, t_2) be $IF - PR_0(i)$, we shall show that (X, τ_1 , τ_2) is $I - PR_0$. Suppose (X , t_1 , t_2) is $IF -$

PR_0 (i). Let $x_1, x_2 \in X, x_1 \neq x_2$ with $M = (M_1, M_2) \in (\tau_1 \cup \tau_2)$ such that $x_1 \in M_1, x_1 \notin M_2$ and $x_2 \notin M_1, x_2 \in M_2. \Rightarrow 1M_1(x_1) = 1, 1M_2(x_1) = 0$ and $1M_1(x_2) = 0, 1M_2(x_2) = 1$. Hence $(1M_1, 1M_2) \in (t_1 \cup t_2)$. Since $(X, t_1, t_2) is\ IF - PR_0(\ i\ )$ then there exist $(1N_1, 1N_2) \in (t_1 \cup t_2)$ such that $1N_1(y) = 1, 1N_2(y) = 0$ and $1N_1(x_1) = 0, 1N_2(x_1) = 1 \Rightarrow x_2 \in N_1, x_2 \notin N_2$ and $x_1 \notin N_1, x_1 \in N_2 \Rightarrow (N_1, N_2) \in (\tau_1 \cup \tau_2)$. Hence $(X, \tau_1, \tau_2)\ is\ I - PR_0$.Hence $IF - PR_0(\ i\ ) \Rightarrow I - PR_0$.
Therefore $I - PR_0 \Leftrightarrow IF - PR_0(\ i\ )$.
Furthermore, it can verify that $I - PR_0 \Longrightarrow IF - PR_0(\ ii),\ I - PR_0(iii)\ and\ I - PR_0(iv)$.

Theorem 4.2: *Let $(X,\ \tau_1,\ \tau_2)$ be an intuitionistic bitopological space and let $(X,\ t_1,\ t_2)$ be an intuitionistic fuzzy bitopological space. Then we have the following implication*
a. $(X, \tau_1,\ \tau_2)\ is\ I - PR_0 \Rightarrow (X,\ t_1,\ t_2\ )\ is\ \alpha - IF - PR_0\ (i)$
b. $(X,\ \tau_1,\ \tau_2)\ is\ I - PR_0 \Rightarrow (X,\ t_1,\ t_2\ )\ is\ \alpha -\ IF - PR_0\ (ii)$
c. $(X,\ \tau_1,\ \tau_2)\ is\ I - PR_0 \Rightarrow (X,\ t_1,\ t_2\ )\ is\ \alpha -\ IF - PR_0\ (iii)$.

Proof (b): Let $(X,\ \tau_1,\ \tau_2)$ be $I - PR_0$ space we shall show that $(X,\ t_1,\ t_2)$ is $\alpha -\ IF - PR_0$(ii). Let $\alpha \in (0, 1)$. Suppose $(X, \tau_1,\ \tau_2)$ is $I - PR_0$. Let $x_1,\ x_2 \in X, x_1 \neq x_2$ with $(1M_1, 1M_2) \in (t_1 \cup t_2)$ such that $1M_1(x_1) \geq \alpha, 1M_2(x_1) = 0, 1M_1(x_2) = 0, 1M_2(x_2) \geq \alpha. \Rightarrow 1M_1(x_1) = 1, 1M_2(x_1) = 0, 1M_1(x_2) = 0, 1M_2(x_2) = 1$ for any $\alpha \in (0,1). \Rightarrow x_1 \in M_1, x_1 \notin M_2, x_2 \notin M_1, x_2 \in M_2$. Hence $(M_1, M_2) \in (\tau_1 \cup \tau_2)$. Since $(X, \tau_1, \tau_2) is\ I - PR_0$ then there exist $(N_1, N_2) \in (\tau_1 \cup \tau_2)$ such that $x_2 \in N_1, x_2 \notin N_2$ and $x_1 \notin N_1, x_1 \in N_2. \Rightarrow 1N_1(x_2) = 1, 1N_2(x_2) = 0$ and $1N_1(x_1) = 0, 1N_2(x_1) = 1. \Rightarrow 1N_1(x_2) \geq \alpha, 1N_2(x_2) = 0$ and $1N_1(x_1) = 0, 1N_2(x_1) \geq \alpha$ for $\alpha \in (0,1) \Rightarrow (1N_1, 1N_2) \in (t_1 \cup t_2)$.
Hence (X, t_1, t_2) is $\alpha -$ IF-P$R_0(\ ii)$.Therefore $I - PR_0 \Rightarrow \alpha - IF - PR_0\ (\ ii)$.
Furthermore, it can be verified that $I - PR_0 \Longrightarrow\ \alpha - IF - PR_0(\ i\ )$ and $I - PR_0 \Longrightarrow\ \alpha - PR_0(iii)$.

5. Mappings in Fuzzy Intuitionistic R_0 Bitopological Space

In this part, we explore order of preserving attribute of the idea IFP-R_0(j), where $j =\ i, ii, iii, iv$ under one- one, onto, fuzzy open and fuzzy continuous mappings.

Theorem 5.1: *Let $(X, s_1,\ s_2)$ and $(Y, t_1,\ t_2)$ be two intuitionistic fuzzy bitopological space and if $X \to Y$ be one –one onto continuous open mapping, then*

a. $(X, s_1, s_2)\ is\ IF - PR_0(\ i\ ) \Leftrightarrow (Y, t_1, t_2)\ is\ IF - PR_0(\ i\ )$.
b. $(X, s_1, s_2)\ is\ IF - PR_0(\ ii) \Leftrightarrow (Y, t_1, t_2)\ is\ IF - PR_0(\ ii)$.
c. $(X, s_1, s_2)\ is\ IF - PR_0(iii) \Leftrightarrow (Y, t_1, t_2)\ is\ IF - PR_0(iii)$.
d. $(X, s_1, s_2)\ is\ IF - PR_0(iv) \Leftrightarrow (Y, t_1, t_2)\ is\ IF - PR_0(iv)$.
e. $(X, s_1, s_2) is\ \alpha - IF - PR_0(\ i\ ) \Leftrightarrow (Y, t_1, t_2)\ is\ \alpha -\ IF - PR_0(\ i\ )$.
f. $(X, s_1, s_2)\ is\ \alpha -\ IF - PR_0(\ ii) \Leftrightarrow (Y, t_1, t_2)\ is\ \alpha -\ IF - PR_0(\ ii)$.
g. $(X, s_1, s_2)\ is\ \alpha - IF - PR_0(iii) \Leftrightarrow (Y, t_1, t_2)\ is\ \alpha -\ IF - PR_0(iii)$.

The proofs (a), (b), (c), (d), (e), (f) and (g) are similar. As an example, we proved (e).

Proof (e): Assume that the intuitionistic fuzzy bitopological space (X, s_1, s_2) is $\alpha -\ IF - PR_0(i)$,we shall prove that the intuitionistic fuzzy bitopological space (Y, t_1, t_2) is $\alpha - IF - PR_0(i)$. Let $\alpha = (0,1)$.Let $y_1, y_2 \in Y, y_1 \neq y_2$ with $M = (\mu_M, \nu_M) \in\ (s_1 \cup s_2)$, such that $\mu_M(y_1) = 1, \nu_M(y_2) \geq \alpha$. since f is onto, then $\exists\ x_1, x_2 \in X$ such that $x_1 = f^{-1}(y_1)$ and $x_2 = f^{-1}(y_2)$. Since $y_1 \neq y_2$, then $f^{-1}(y_1) \neq f^{-1}(y_2)$. Hence $x_1 \neq x_2$.We have $\left(f^{-1}(\mu_M), f^{-1}(\nu_M)\right) \in (s_1 \cup s_2)$, as f is $IF -$continuous.
Now if $\left(f^{-1}(\mu_M)\right)(x_1) = \mu_M f(x_1) = \mu_M(y_1) = 1$ and $(f^{-1}(\nu_M))(x_2) = \nu_M(y_2) = 1.$ Therefore since (X, s_1, s_2) is $\alpha - IF - PR_0(\ i\ )$ then there exits $N = (\mu_N, \nu_N) \in (s_1 \cup s_2)$, such that $\mu_N(x_2) = 1, \nu_N(x_1) \geq \alpha$ and $(f(\nu_N))(y_1) = \nu_N(f^{-1}(y_1))$=$\nu_N(x_1) = 1$ as f is one - one and onto. Hence $(f(\mu_N), f(\nu_N)) \in (t_1 \cup t_2)$. Therefore (Y, t_1, t_2) is $IF - PR_0(\ i\ )$.
In contrast assume the intuitionistic fuzzy bitopological space (Y, t_1, t_2) is $\alpha - IF - PR_0(\ i\ )$. We shall prove that the intuitionistic fuzzy bitopological space $(X, s_1, s_2)$is $IF - PR_0(\ i\ )$. Let $x_1, x_2 \in X, x_1 \neq x_2$ with $M = (\mu_M, \nu_N) \in (s_1 \cup s_2)$ such that $y_i = fx_i,\ i = 1,2$. Hence $f(x_1) \neq f(x_2)$ implies $y_1 \neq y_2$ as f is one one. We have $\left(f(\mu_M), f(\nu_M)\right) \in (t_1 \cup t_2)$ as f is continuous.
Now,$(f(\mu_M))(y_1) = (f(\mu_{M)})(f(x_1) = \mu_M(f^{-1}\left(f(x_1)\right) = \mu_M(x_1) = 1$ and
$f(\nu_M))(y_2) = (f(\nu_{M)})(f(x_2) = \nu_M(f^{-1}\left(f(x_2)\right) = \nu_M(x_2) = 1.$
Therefore since $(Y, t_1, t_2)\ is\ \alpha - IF - PR_0(\ i\ )$ then there exists $N = (\mu_N, \nu_N) \in (t_1 \cup t_2)$ such that $\mu_N(y_2) =$

$1, \nu_N(y_1) \geq \alpha$. Now $\left(f^{-1}(\mu_N)\right)(x_2) = \mu_N\left(f(x_2)\right) = \mu_N(y_2) = 1$ and $\left(f^{-1}(\nu_N)\right)(x_1) = \nu_N\left(f(x_1)\right) = \nu_N(y_1) = 1$. As f is one-one and onto. Hence, $(f^{-1}(\mu_N), f^{-1}(\nu_N) \in (s_1 \cup s_2)$. Therefore (X, s_1, s_2) is $\alpha - IF - PR_0(\ i\ )$.

6. Conclusions

This paper's key accomplishment is the formulation of an innovative notion of an intuitionistic fuzzy bitopological R_0 space. We go over a few aspects of those notions. Additionally, we have found that these notions hold true for both fuzzy open and fuzzy continuous mapping. We anticipate that the outcomes of these studies will contribute to illuminate the subject of contemporary mathematics.

Conflict of Interests

The authors declare no conflict of interest.

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Appendix A: Basic Notions

Definition A.1: [1] A fuzzy set u from X into the unit interval I is called a fuzzy set in X. For every $x \in X, u(x) \in I$ is called the grade of membership of x in u. Some authors say that u is a fuzzy subset of X instead of saying that u is a fuzzy set in X. The class of all fuzzy sets from X into the closed unit interval I will be denoted by I^X.
Definition A.2: [17] A fuzzy set u in X is called a fuzzy singleton if and only if u(x) =r , 0$< r \leq 1$, for a certain $x \in X$ $and\ u(y) = 0$ for all points y of X except x . The fuzzy singleton is denoted by x_r and x is its support. The class of all fuzzy singletons in X will be denoted by $S(X)$ .If u $\in I^x$ and $x_r \in S(X)$, then we say that $x_r \in$ u if and only if $r \leq u(x)$.
Definition A.3: [18] A fuzzy set u in X is called a fuzzy point if and only if $u(x) = r$, $0 < r < 1$ for a certain $x \in X$ and u(y) = 0 for all points y of X without x. The point is denoted by x_r and x is its support.
Definition A.4: [2] Let f be a mapping from a set X into a set Y and v be a fuzzy subset of Y. Then, the inverse of v written as $f^{-1}(v)$ is a fuzzy subset of X defined by $f^{-1}(v)(x) = v(f(x))$, for $x \in X$.
Definition A.5: [19] The function $f : (X, t) \to (Y, s)$ is called fuzzy continuous if and only if for every $v \in s, f^{-1}(v) \in t$, the function f is called fuzzy homeomorphic if and only if f is bijective and both f and f^{-1} are fuzzy continuous.
Definition A.6: [20] The function $f : (X, t) \to (Y, s)$ is called fuzzy open if and only if for every open fuzzy set u in (X, t) , $f(u)$ is open fuzzy set in (Y, s).
Definition A.7: [21] An intuitionistic set A is an object having the form A= (x, A_1, A_2), where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \emptyset$. The set A_1 is called the set the member of A while A_2 is called the set of non –member of A.
Definition A.8: [12] An intuitionistic topology on a set X is a family τ of intuitionistic sets in X satisfying the following axioms:

(1) $\phi_\sim, X_\sim \in \tau$.
(2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$.
(3) $\cup G_i \in \tau$ for any arbitrary family $G_i \in \tau$

In this case, the pair (X, τ) is called an intuitionistic topological space (ITS, in short) and any intuitionistic set in τ in known as an intuitionistic open set in X.
Definition A.9: [11] Let X be a non-empty set and I be the interval [0,1]. An intuitionistic fuzzy set A in X is an object having the form A=$\{(x, \mu_A(x), \nu_A(x)), x \in X\}$, where $\mu_A: X \to I$ and $\nu_A: X \to I$ degree of membership and degree of non-membership respectively, and $\mu_A(x) + \nu_A(x) \leq 1$. Let I(X) denote the set of all intuitionistic fuzzy sets in X. Obviously every fuzzy set μ_A in X is an intuitionistic fuzzy set of the form $(\mu_A(x),\ 1 - \mu_A(x))$.