ON SOME VALUES OF THE SANDOR-SMARANDACHE FUNCTION

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ABSTRACT
Sandor [1] posed a new function, denoted by SS(n), and is defined as follows:

$$SS(n) = \max \left\{ m : 1 \leq m \leq n-1, n \text{ divides } \binom{n}{m}, n \geq 2 \right\}$$

being the binomial coefficients. This paper finds SS(n) for some particular cases of n.

Keywords: Sandor-Smarandache function, Binomial coefficient, Diophantine equation

1. Introduction

Let $C(n, m)$ be the binomial coefficient, defined as follows:

$$C(n, m) = \binom{n}{m} = \frac{n!}{m!(n-m)!}, \quad 0 \leq m \leq n.$$

Then, the Sandor-Smarandache function, denoted by SS(n), is defined as follows:

$$SS(n) = \max \left\{ m : 1 \leq m \leq n-1, n \text{ divides } \binom{n}{m}, n \geq 3 \right\},$$

with

$$SS(1) = 1, SS(2) = 1, SS(3) = 1, SS(4) = 1, SS(6) = 1.$$

Throughout this paper, we use the following formula for $C(n, m)$:

$$C(n, m) = \frac{n(n-1)(n-2) \ldots (n-m+1)}{m!}, \quad 0 \leq m \leq n.$$

Sandor [1] proved the result below.

Lemma 1.1: SS(n) = n − 2 for any odd integer n (≥ 3).

Corollary 1.1: For any prime $p \geq 3$, SS(p) = p − 2, and in general,

$$SS(p_1, p_2, \ldots, p_t) = p_1p_2 \ldots p_t − 2$$

for any odd primes $p_1, p_2, \ldots, p_t$. 

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As has been pointed out by Sandor [1], to find \( SS(n) \), the case of even \( n \) is more involving. This paper considers this case in the next section. We first derive \( SS(n) \) when \( n = p + 1 \), \( p \) being an odd prime. Next, we confine our attention to the prime \( p \) of the four forms, \( p = 5v + 1, p = 5v + 2, p = 5v + 3 \) and \( p = 5v + 4 \) (for some integer \( v \geq 1 \)).

2. Main Results

In a recent book, Majumdar [2] derived the expressions of \( SS(n) \) for some particular cases. In this section, we derive more to supplement the results found in [2]. We concentrate on the functions \( SS(p + 1) \), where \( p \) is an odd prime of the forms \( p = 5v + 1, 5v + 2, 5v + 3, 5v + 4 \).

We first prove the following simple result.

**Lemma 2.1**: Let \( p (\geq 5) \) be an odd prime. Then, \( SS(p + 1) = p - 2 \) if and only if \( p + 1 \) is not a multiple of 3.

**Proof**: We consider

\[
\frac{(p + 1)p(p - 1)}{3!}.
\]

If 3 does not divide \( p + 1 \), then 3 must divide \( p - 1 \), and hence, 6 divides \( p - 1 \).

Conversely, if \( SS(p + 1) = p - 2 \), then 3 must divide \( p - 1 \), and consequently, \( p + 1 \) is not divisible by 3.

Applying Lemma 2.1, we get the following expressions:

\[
SS(8) = 5, SS(14) = 11, SS(20) = 17, SS(32) = 29, SS(38) = 35, SS(44) = 41.
\]

**Corollary 2.1**: Let \( p (\geq 5) \) be an odd prime such that 3 divides \( p + 1 \). Then,

\[
SS(p + 1) \geq p - 3.
\]

**Proof**: follows immediately by virtue of Lemma 2.1.

Lemma 2.2 – Lemma 2.7 below deal with the case when \( p \) is a prime of the form \( p = 5v + 1 \).

**Lemma 2.2**: Let \( p \) be a prime of the form \( p = 5v + 1 \) for some integer \( v \geq 1 \). Then,

\[
SS(p + 1) = p - 3,
\]

if \( v = 8(3s + 1) \) for some integer \( s \geq 0 \).

**Proof**: With \( p = 5v + 1 \), the following expression

\[
\frac{(p + 1) p(p - 1)(p - 2)}{4!}
\]

takes the form
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\[ 5(p + 1)p \frac{v(5v - 1)}{2x3x4}. \]

We consider the case when 8 divides \( v \) and 3 divides \( 5v - 1 \), so that
\[ v = 8x, 5v = 3y + 1 \]
for some integers \( x \geq 1, y \geq 1 \).

The solution of the second Diophantine equation is \( v = 3a + 2, a \geq 0 \). This, when combined with the first equation, gives
\[ 8x = 3a + 2, \]
whose solution is \( x = 3s + 1, s \geq 0 \). Hence, finally
\[ v = 8x = 8(3s + 1). \]

Observe that, in Lemma 2.2, \( p = 5v + 1 = 3(8s + 3) \) is a multiple of 3, a result consistent with Corollary 2.1.

Using Lemma 2.2, we get the functions below:
\[
SS(42) = 38, SS(282) = 278, SS(402) = 398, SS(522) = 518, SS(642) = 638.
\]

**Lemma 2.3**: Let \( p \) be a prime of the form \( p = 5v + 1 \) for some integer \( v \geq 1 \). Then,
\[ SS(p + 1) = p - 4, \]
if \( v = 2(6s + 1) \) for some integer \( s \geq 0 \).

**Proof**: With \( p = 5v + 1 \), we have
\[
(p + 1)p \frac{(p - 1)(p - 2)(p - 3)}{5!} = (p + 1)p \frac{v(5v - 1)(5v - 2)}{2x3x4}.
\]

Now, we consider the case when 3 divides \( 5v - 1 \) and 4 divides \( 5v - 2 \). Then,
\[ 5v = 3x + 1, 5v = 4y + 2 \]
for some integers \( x \geq 1, y \geq 1 \).

The solutions of these two Diophantine equations are
\[ v = 3a + 2 = 4b + 2; a \geq 0, b \geq 0 \]
being any integers.

This shows that \( a = 4s, s \geq 1 \). Therefore,
\[ v = 3a + 2 = 2(6s + 1), \]
which is the desired condition.

Lemma 2.3 gives the following functions:
\[
SS(12) = 7, SS(72) = 65, SS(132) = 125, SS(192) = 185, SS(252) = 247.
\]

The first example shows that Lemma 2.3 is valid for \( s = 0 \) as well.

From the proof of Lemma 2.1, it may be deduced that, if \( p = 5v + 1 \), then \( SS(p + 1) = p - 2 \) if and
only if \( v = 6s \) \((s \geq 1)\). However, in other cases, there might be more than one solution, as the two lemmas below illustrate.

**Lemma 2.4:** Let \( p \) be a prime of the form \( p = 5v + 1 \) with \( v = 4(3s + 2) \), \( s \geq 0 \). Then,

\[
SS(p+1) = \begin{cases} 
  p - 3, & \text{if } s \text{ is even} \\
  p - 4, & \text{if } s \text{ is odd}
\end{cases}
\]

**Proof:** Consider the expression

\[
(p+1)p\frac{(p-1)(p-2)(p-3)}{5!} = (p+1)p\frac{v(5v-1)(5v-2)}{2\times3\times4},
\]

Now, let \( s \) be even, say, \( s = 2r \) for some integer \( r \geq 1 \). Then, \( v = 4(6r + 2) = 8(3r + 1) \), \( 5v - 1 = 3(40r + 13) \).

Then, clearly \((p-1)(p-2) = 5v(5v-1)\) is divisible by \( 4! \), and hence, \( SS(p+1) = p - 3 \).

Next, let \( s \) be odd of the form \( s = 2t + 1 \) for some integer \( t \geq 1 \). Then, \( v = 2(12t + 7) \), \( 5v - 1 = 3(40t + 13) \), \( 5v - 2 = 4(30t + 17) \).

In this case, \((p-1)(p-2)\) is not divisible by \( 4! \), but \((p-1)(p-2)(p-3)\) is divisible by \( 4! \).

From Lemma 2.4, corresponding to \( s = 0 \), we get the prime \( p = 41 \) with \( SS(42) = 38 \); when \( s = 1 \), we get the prime \( p = 101 \) which gives \( SS(102) = 97 \). Again, with \( s = 2 \), we get the prime \( p = 161 \) with \( SS(162) = 158 \). The next prime in the sequence is \( p = 281 \) (corresponding to \( s = 4 \)) with \( SS(282) = 278 \).

**Lemma 2.5:** Let \( p \) be a prime of the form \( p = 5v + 1 \) with \( v = 2(3s + 1) \), \( s \geq 0 \). Then,

\[
SS(p+1) = \begin{cases} 
  p - 3, & \text{if } s = 4t + 1, \ t \geq 0 \\
  p - 4, & \text{otherwise}
\end{cases}
\]

**Proof:** We start with

\[
(p+1)p\frac{(p-1)(p-2)(p-3)}{5!} = (p+1)p\frac{v(5v-1)(5v-2)}{2\times3\times4},
\]

With \( v = 2(3s + 1) \), \((p-1)(p-2) = 30(3s + 1)(10s + 3)\).

When \( s = 4t + 1 \), then \( 3s + 1 = 4(3t + 1) \), so that \( 4! \) divides \((p-1)(p-2)\). Thus, in this case,

\( SS(p+1) = p - 3 \).

Otherwise, \( 4! \) does not divide \((p-1)(p-2)\), but \((p-1)(p-2)(p-3) = 60(3s + 1)(10s + 3) (15s + 4) \) is divisible by \( 5! \), so that \( SS(p+1) = p - 4 \).

From Lemma 2.5, corresponding to \( s = 0 \), we get the prime \( p = 11 \) with \( SS(12) = 7 \); \( s = 1 \) gives the prime \( p = 41 \), which is of the form \( 4t + 1 \), so that \( SS(42) = 38 \). Corresponding to \( s = 3 \), we get the prime \( p = 101 \) with \( SS(102) = 97 \). Continuing, we get successively the functions \( SS(132) = 125 \).
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SS(192) = 187, SS(252) = 247 and SS(282) = 278.

Lemma 2.6: Let $p$ be a prime of the form $p = 5v + 1$ with $v = 2(9s + 10)$, $s \geq 0$. Then,

$$SS(p + 1) = \begin{cases} p - 3, & \text{if } s = 4t + 2, \ t \geq 0 \\ p - 4, & \text{otherwise} \end{cases}$$

Proof: Consider the expression below:

$$(p + 1)p (p - 1)(p - 2)(p - 3) = (p + 1)p \frac{(5v - 1)(5v - 2)}{2 \times 3 \times 4 \times 5!}.$$  

If $s = 4t + 2$, then

$$9s + 10 = 4(9t + 7).$$

Therefore,

$$(p - 1)(p - 2) = 90(9s + 10)(10s + 11),$$

which is divisible by $4!$. Thus, in this case,

$$SS(p + 1) = p - 3.$$  

Otherwise, $(p - 1)(p - 2)$ is not divisible by $4!$, but $5!$ divides

$$(p - 1)(p - 2)(p - 3) = 180(9s + 10)(10s + 11)(45s + 49)$$

so that $SS(p + 1) = p - 4$.

Some functions, obtained from Lemma 2.6, are listed below:

<table>
<thead>
<tr>
<th>Value</th>
<th>SS(102)</th>
<th>SS(192)</th>
<th>SS(252)</th>
<th>SS(282)</th>
<th>SS(462)</th>
<th>SS(642)</th>
<th>SS(822)</th>
<th>SS(912)</th>
<th>SS(1092)</th>
<th>SS(1182)</th>
<th>SS(1362)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>97</td>
<td>187</td>
<td>247</td>
<td>278</td>
<td>457</td>
<td>638</td>
<td>817</td>
<td>907</td>
<td>1087</td>
<td>1177</td>
<td>1358</td>
</tr>
</tbody>
</table>

Lemma 2.7: Let $p$ be a prime of the form $p = 5v + 1$ for some integer $v \geq 1$. Then,

$$p - 2 \leq SS(p + 1) \leq p - 4.$$  

Proof: We prove the lemma by showing that $SS(p + 1) \neq p - 5$. So, we consider the expression below:

$$(p + 1)p (p - 1)(p - 2)(p - 3)(p - 4) = (p + 1)p \frac{(5v - 1)(5v - 2)(5v - 3)}{2 \times 3 \times 4 \times 6}.$$  

Now, we find the condition under which the term inside the square bracket on the right is an integer. To do so, first note that, by Corollary 2.1, $(since 3$ divides $p + 1)$, 9 must divide $5v - 1$. Then, since $5v - 3$ must be odd, we see that it remains dormant, and consequently, in such a case, we must have $SS(p + 1) \leq p - 4$.

The next three lemmas deal with the case when $p$ is a prime of the form $p = 5v + 2$.

Lemma 2.8: Let $p$ be a prime of the form $p = 5v + 2$ for some integer $v \geq 1$. Then,
SS(p + 1) = p – 3,

if \( v = 3(8s + 1) \) for some integer \( s \geq 0 \).

**Proof**: With \( p = 5v + 2 \), we get

\[
(p + 1)p \frac{(p - 1)(p - 2)}{4!} = 5(p + 1)p \frac{v(5v + 1)}{2 \times 3 \times 4}.
\]

Now, consider the case when 8 divides \( 5v + 1 \) while 3 divides \( v \), so that

\[ 5v = 8x - 1, \ v = 3y \text{ for some integers } x \geq 1, y \geq 1. \]

The solution of the first equation is

\[ v = 8a + 3, \ a \geq 0. \]

We are then lead to the equation

\[ 8a = 3y - 3, \]

with the solution \( a = 3(s + 1), \ s \geq 0 \). Plugging in this expression in \( v = 8a + 3 \), we get the desired condition.

The following functions are obtained from Lemma 2.8:

\[ SS(18) = 14, SS(138) = 134, SS(258) = 254, SS(618) = 614, SS(858) = 854. \]

**Lemma 2.9**: Let \( p \) be a prime of the form \( p = 5v + 2 \) for some integer \( v \geq 1 \). Then,

\[ SS(p + 1) = p – 4, \]

if \( v = 3(4s + 3), \ s \geq 0 \).

**Proof**: We start with

\[
(p + 1)p \frac{(p - 1)(p - 2)(p - 3)}{5!} = (p + 1)p \frac{v(5v + 1)(5v - 1)}{2 \times 3 \times 4}.
\]

Now, we consider the case when 4 divides \( 5v - 1 \) and 3 divides \( v \). Then,

\[ 5v = 4x + 1, \ v = 3y \text{ for some integers } x \geq 1, y \geq 1. \]

The first equation has the solution

\[ v = 4a + 1, \ a \geq 0. \]

We are then faced with the Diophantine equation

\[ v = 3y = 4a + 1, \]

whose solution is

\[ a = 3s + 2, \ s \geq 0. \]
After simplification, we get the condition desired.

From Lemma 2.9, we get the functions below.

\[ SS(48) = 43, SS(108) = 103, SS(158) = 153, SS(228) = 223, SS(348) = 343. \]

**Lemma 2.10**: Let \( p \) be a prime of the form \( p = 5v + 2 \) with \( v = 3(2s + 1) \), \( s \geq 0 \). Then,

\[ SS(p + 1) = \begin{cases} p - 3, & \text{if } s \neq 4t, \ t \geq 0 \\ p - 4, & \text{otherwise} \end{cases} \]

**Proof**: Consider the expression below:

\[
( p + 1)p \frac{(p - 1)(p - 2)(p - 3)}{5^t} = ( p + 1)p \frac{5v + 1(5v - 1)}{2 \times 3 \times 4}.
\]

Now, we consider the case when \( 2 \) divides \( 5v + 1 \) and \( 3 \) divides \( v \). Then,

\[ 5v = 2x - 1, \ v = 3y \text{ for some integers } x \geq 1, \ y \geq 1. \]

The first Diophantine equation has the solution

\[ v = 2a + 1, \ a \geq 0, \]

which, combined with the second equation, leads to

\[ 3y = 2a + 1, \]

which gives

\[ y = 2s + 1, \ s \geq 0. \]

And finally, we get \( v = 3(2s + 1) \).

Now, since

\[ 5v + 1 = 2(15s + 8), \]

it follows that

\[ SS(p + 1) = p - 4, \text{ if } 4 \text{ divides } 15s + 8; \]

\[ \text{otherwise, } SS(p + 1) = p - 5. \]

Now, noting that \( 4 \) divides \( 15s + 8 \) if and only if \( s = 4t, \ t \geq 1 \), the lemma is established.

Some functions, obtained from Lemma 2.10, are

\[ SS(18) = 14, SS(48) = 43, SS(108) = 103, SS(138) = 134, SS(168) = 163, \]

\[ SS(198) = 193, SS(228) = 223, SS(258) = 254. \]

**Lemma 2.11** – Lemma 2.14 consider the case when the prime \( p \) is of the form \( p = 5v + 3 \).

**Lemma 2.11**: Let \( p \) be a prime of the form \( p = 5v + 3 \), where \( v = 2(12s + 11) \), \( s \geq 0 \). Then,
\[ SS(p + 1) = p - 3. \]

**Proof**: Letting \( p = 5v + 3 \) in
\[
(p + 1)p \frac{(p - 1)(p - 2)}{4!},
\]
we get
\[
(p + 1)p \left[ \frac{(5v + 2)(5v + 1)}{2x3x4} \right].
\]

Now, in order that the above number is an integer, 8 must divide \( 5v + 2 \), and 3 must divide \( 5v + 1 \). This leads to the following two Diophantine equations
\[
5v = 8x - 2, \quad 5v = 3y - 1 \quad \text{for some integers } x (\geq 1) \text{ and } y (\geq 1),
\]
with solutions
\[
v = 8a + 6, \quad v = 3b + 1 \quad (a \geq 1 \text{ and } b \geq 1 \text{ being integers}),
\]
respectively. Now, combining together, the resulting equation is
\[
8a = 3b - 5,
\]
whose solution is
\[
a = 3s + 2, \quad s \geq 0.
\]

Hence,
\[
v = 8(3s + 2) + 6 = 2(12s + 11),
\]
which is the desired expression we were looking for.

Applying Lemma 2.11, we get the expressions below:
\[
SS(114) = 110, \quad SS(234) = 230, \quad SS(594) = 590.
\]

**Lemma 2.12**: Let \( p \) be a prime of the form \( p = 5v + 3 \), \( v = 4(3s + 1) \) for some integer \( s \geq 0 \). Then,
\[
SS(p + 1) = p - 4.
\]

**Proof**: We start with
\[
(p + 1)p \frac{(p - 1)(p - 2)(p - 3)}{4!} = (p + 1)p \left[ \frac{(5v + 2)(5v + 1)v}{2x3x4} \right].
\]

We consider the case when 3 divides \( 5v + 1 \), 2 divides \( 5v + 2 \) and \( v \) itself is a multiple of 4. Then, we have
\[
5v = 3x - 1, \quad 5v = 2y - 2, \quad v = 4z \quad \text{for some integers } x (\geq 1), y (\geq 1) \text{ and } z (\geq 1).
\]
The first Diophantine equation gives the solution
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\[ v = 3a + 1, \ a \geq 1. \]

This, together with the condition \( v = 4z \), requires that

\[ 3a = 4z - 1, \]

whose solution is

\[ a = 4s + 1, \ s \geq 1. \]

Therefore,

\[ v = 3a + 1 = 3(4s + 1) + 1 = 4(3s + 1). \]

Applying Lemma 2.12, we get the functions below:

SS(24) = 19, SS(84) = 79, SS(264) = 259, SS(384) = 379.

**Lemma 2.13**: Let \( p \) be a prime of the form \( p = 5v + 3, \ v = 8(3s + 2) \) for some integer \( s \geq 1 \). Then,

\[ \text{SS}(p + 1) = p - 4. \]

**Proof**: We start with the following simplified form:

\[
(p + 1)p \left( \frac{(p - 1)(p - 2)(p - 3)}{5!} \right) = (p + 1)p \left\{ \sum_{u=0}^{\infty} \frac{(5u + 2)(5u + 1)u}{2 \times 3 \times 4} \right\}.
\]

Now, we consider the case when \( 3 \) divides \( 5v + 1 \) and \( 8 \) divides \( v \). Then,

\[ 5v = 3x - 1, \ v = 8y \text{ for some integers } x (\geq 1) \text{ and } y (\geq 1). \]

The first equation has the solution \( v = 3a + 1, \ a (\geq 1) \), and hence, we have to consider

\[ 3a = 8y - 1, \]

whose solution is \( a = 8s + 5, \ s \geq 1 \).

Therefore,

\[ v = 3(8s + 5) + 1 = 8(3s + 2), \]

which we intended to establish.

Application of Lemma 2.13 gives the functions:

SS(84) = 79, SS(444) = 439.

**Lemma 2.14**: Let \( p \) be a prime of the form \( p = 5v + 3, \ v = 2(3s + 2) \) for some integer \( s \geq 1 \) such that \( s \neq 4t + 3 \) for any \( t \geq 0 \). Then,

\[ \text{SS}(p + 1) = p - 4. \]

**Proof**: We start with
\( (p + 1)p \frac{(p - 1)(p - 2)(p - 3)}{5!} = (p + 1)p \left\lceil \frac{(5v + 2)(5v + 1)v}{2x3x4} \right\rceil. \)

We consider the case when \(3\) divides \(5v + 1\) and \(2\) divides \(v\). Then,
\[ 5v = 3x - 1, \quad v = 2y \]
for some integers \(x \geq 1\) and \(y \geq 1\).

The solution of the first equation is
\[ v = 3a + 1, \quad a \geq 1. \]

Then, the combined Diophantine equation is
\[ 2y = 3a + 1, \]
with the solution
\[ y = 3s + 2, \quad s \geq 0. \]

Therefore,
\[ v = 3(8s + 5) + 1 = 8(3s + 2). \]

Note that, \(5v + 2 = 2(15s + 11)\), and hence, to complete the proof, we have to guarantee that \(4\) does not divide \(15s + 11\). To do so, we consider the equation \(15s = 4b - 11\), which has the solution
\[ s = 4t + 3, \quad t \geq 0. \]

Thus, if \(s \neq 4t + 3\) for any \(t \geq 0\), then \(SS(p + 1) = p - 4\).

Hence, the proof of the lemma is complete.

Lemma 2.14 gives the following functions
\[ SS(24) = 19, \quad SS(54) = 49, \quad SS(84) = 79, \quad SS(174) = 169, \quad SS(264) = 259. \]

It may be mentioned here that, if \(s = 4t + 3 (t \geq 0)\) in Lemma 2.14, then by Lemma 2.11,
\[ SS(p + 1) = p - 3. \]

Finally, we have the following lemma, dealing with the case when \(p\) is a prime of the form
\[ p = 5v + 4. \]

**Lemma 2.15:** Let \(p\) be a prime of the form \(p = 5v + 4, v = 24s + 17\) for some \(s \geq 0\). Then,
\[ SS(p + 1) = p - 3. \]

**Proof:** With \(p = 5v + 4\), we get
\[ (p + 1)p \frac{(p - 1)(p - 2)}{4!} = (p + 1)p \left\lceil \frac{(5v + 3)(5v + 2)}{2x3x4} \right\rceil. \]

We consider the case when \(8\) divides \(5v + 3\) and \(3\) divides \(5v + 2\). Thus,
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\[ 5v + 3 = 8x, \quad 5v + 2 = 3y \text{ for some integers } x (\geq 1) \text{ and } y (\geq 1), \quad (1) \]

that is, \( 5v = 8x - 3, \quad 5v = 3y - 2. \)

The solutions of the above two equations are

\[ v = 8a + 1, \quad v = 3b + 2; \quad a \geq 1 \text{ and } b \geq 1 \text{ being integers.} \]

Now, combining together the above two Diophantine equations, we get the equation

\[ 8a = 3b + 1, \]

with the solution

\[ a = 3s + 2, \quad s \geq 0. \]

Thus,

\[ v = 8(3s + 2) + 1 = 24s + 17, \]

which is the condition desired.

Some of the functions found from Lemma 2.15 are given below.

\[ SS(90) = 86, \quad SS(450) = 446, \quad SS(570) = 566, \quad SS(810) = 806. \]

Remark 2.1: In the proof of Lemma 2.15, writing the two equations in (1) in the form

\[ 5v + 3 = 8x = 3y + 1, \]

we get the solution

\[ x = 3t + 2, \quad t \geq 0. \]

Then,

\[ p = 5v + 4 = 8x + 1 = 8(3t + 2) + 1 = 24t + 17. \]

Having the solution in the above form, we get the functions below:

\[ SS(18) = 14, \quad SS(42) = 38, \quad SS(90) = 86, \quad SS(114) = 110, \quad SS(138) = 134, \]

\[ SS(234) = 230, \quad SS(258) = 254, \quad SS(282) = 278, \quad SS(354) = 350. \]

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