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# Geodesic of Two Places on the Surface of the Earth and the Direction from One Place Towards the Other

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**Abstract**

With the help of some standard results in Geometry, Differential geometry, Spherical Trigonometry and the latitude and longitude of two places on the surface of the earth one can measure the Geodesics of the path joining those places and the direction between two places with respect to the geographical position. The study of geodesics on a sphere aroses in connection with geodesy specially with the solution of triangulation networks. The figure of the earth is well approximated by an oblate ellipsoid, a slightly flattened sphere. A geodesic is the shortest path between two points on a curved surface, analogous to a straight line on a plane surface. Instead of finding networks as a two dimensional problem, these problems are now solved by three dimensional methods. The main objectives of this paper is to find the distances and directions of the sum of different renowned places named Dhaka to Mecca, London to Mecca, NewYork to Mecca and Delhi to Mecca with the help of the knowledge of differential geometry.

***Keywords:*** *Geodesics, Sphere, Latitude, Longitude, Direction, Trigonometry, North and South pole*

**Subject Classification, 2010:**32Q15, 53B05, 53C55, 53D05.

## I. Introduction

Earth is an approximate representation of a spherical in real life. However, the earth has a large area and is not a perfect sphere. The term "oblate spheroid" refers to the minor flattening of the earth at the ends of its axis of rotation. For a variety of reasons, humans must determine the distance between two points on Earth, and this is not always an easy task. The shortest distance between two coordinates on the surface of the earth can be found using an approach called the Haversine algorithm. Everywhere in the world, latitude and longitude are commonplace. These understand a coordinate system that may identify or locate geographic locations on planets like as Earth. Latitude is defined in relation to the equatorial reference plane, which contains the great circle that represents the equator and travels through the center of the planet. Meridians, which are half-circles that extend from pole to pole, can be used to define longitude. A reference meridian known as the prime meridian defines longitudes. By using those and spherical geometry, which also estimates the value for travel the shortest distance, the Haversine algorithm is utilized to determine the shortest distance. With the advancement of technology, data and information in cyberspace are becoming easier to access through common mobile devices, such as smartphones. Knowing those two locations on the earth is enough to calculate the distance and with the use of specialized calculators, determine how to get there. The distance between two points on the round and flat planes can be calculated using a variety of formulas. However, trigonometry must be used when dealing with spherically curved planes, such as the globe, which makes these kinds of issues complex. One such method for tackling these kinds of issues is this formula. An intelligent route search is necessary to find the optimal path to the destination. Another issue with route searching is choosing the right branches. The shortest route to the destination can be found with the aid of this one algorithm. Considering the earth as a perfect sphere and the longitudes as the arcs of great circles, the author with the help of the following results describes a mathematical method to find out the distances between two places and the direction from one place to the others. Geodesics has vital importance in several areas for example calculating distances and areas in geographic information systems, in navigation system.

## II. Determination of North pole and South Pole Approximately

To determine the two poles of the earth we have to go an open field and consider a fixed point on this field as a center and with this center taking any radius we have to draw a big circle on this field as we see in fig. 1. Then we have to keep a rod with some length on the center point which is perpendicular on the South surface. When the shade of this rod cuts at a point on the circumference of the circle, we denote this point.

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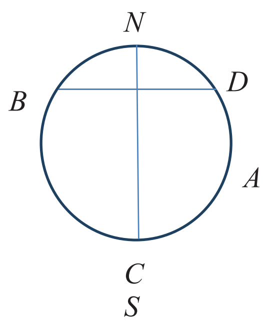


**Fig. 1.** Pole Approximation

Another time of the day when the shade of this rod cuts at point $B$ on the circumference of the circle, we denote this point $B$. Now add $A$ and $B$ and bisect $AB$. We get $CD$. Thus we get $\vec{DC}$ which is the direction of North pole and $\vec{CD}$ which is the direction of South pole.

### III. Approximate Determination of Longitude [2]

We have to find out North and South direction of the earth of any place. We have to stand up on the center point of the circle. When my shade will be coincide with the North and South direction, then note the local time by watch. At this moment we have to know Grenitch time. Also we have to calculate the time difference of this two times say time difference is $x$ minutes. We know that

$$360° = 24 * 60 \text{ minutes}$$

$$\text{so } 1° = 4 \text{ minutes}$$

$$\text{Now } 4 \text{ minutes} = 1° \text{ longitude}$$

$$\text{so } x \text{ minutes} = \frac{x}{4} \text{ longitude.}$$

### IV. Approximate Determination of Latitude [2]

We know that pole star always stay in the sky to the north direction.

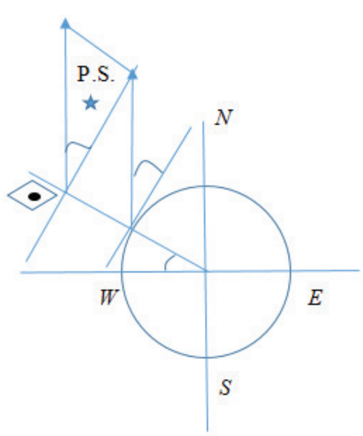


**Fig. 2.** Determination of Latitude

We have to see pole star of any place by standing on the surface of the earth with a telescope as like as fig. 2. Consider a straight line above our head which is parallel to earth surface. Then we can find the angle between telescope and line which goes above our head. Thus this angle is called the latitude of this place.

### V. Calculation of the Distance [2]

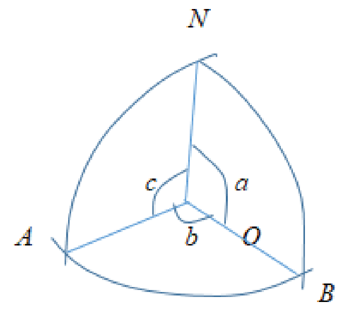


**Fig. 3.** $N$ be the North pole, $A$ and $B$ are two places, $O$ be the center of world

Let $\angle NAB$ be a spherical triangle. We have

$$\cos n = \cos a \cos b + \sin a \sin b \cos N$$
$$a = \angle BON = 90° - x_2^{\circ}$$
$$b = \angle AON = 90° - x_1^{\circ}$$

$N =$ angle between the tangents at $N$ on the arcs $NB$ and $NA = y_2^{\circ} - y_1^{\circ}$, where $y_2^{\circ} > y_1^{\circ}$.
$B =$ angle between the tangents at $B$ on the arcs $BN$ and $BA$, $n = \angle AOB$, $AO = BO = NO = R$ which is the radius of the earth.

$$\cos n = \cos(90° - x_2^{\circ})\cos(90° - x_1^{\circ}) + \sin(90° - x_2^{\circ}) \sin(90° - x_1^{\circ})\cos(y_2^{\circ} - y_1^{\circ})$$

$$\text{or, } \cos n = \sin x_1^{\circ} \sin x_2^{\circ} + \cos x_1^{\circ} \cos x_2^{\circ} \cos(y_2^{\circ} - y_1^{\circ})$$

$$\text{or, } \quad n = \cos^{-1}\left\{\sin x_1^{\circ} \sin x_2^{\circ} + \cos x_1^{\circ} \cos x_2^{\circ} \cos(y_2^{\circ} - y_1^{\circ})\right\}$$

. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . (1)

So the geodesic between $A$ and $B$ on surface of earth length of the arc of the great circle which passes through $A$ and $B$ is $d = Rn$. Where $n$ should be measured in radians.

### VI. Calculation of the Direction [5]

For the spherical triangle $\angle NAB$ (Fig. 3) we have the sine formula

$$\frac{\sin N}{\sin n} = \frac{\sin B}{\sin b}$$

$$\text{or, } \frac{\sin(y_2^{\circ} - y_1^{\circ})}{\sin n} = \frac{\sin B}{\sin(90° - x_1^{\circ})}$$

$$\text{or, } \sin B = \frac{\sin(y_2^\circ - y_1^\circ)\cos x_1^\circ}{\sin n}$$

$$\text{or, } \sin B = \frac{\sin(y_2^\circ - y_1^\circ)\cos x_1^\circ}{\sqrt{1-\cos^2 n}}$$

$$\text{or, } \sin B = \frac{\sin(y_2^\circ - y_1^\circ)\cos x_1^\circ}{\sqrt{1-\left\{\sin x_1^\circ \sin x_2^\circ + \cos x_1^\circ \cos x_2^\circ \cos(y_2^\circ - y_1^\circ)\right\}^2}}$$

$$\text{or, } B = \sin^{-1}\left[\frac{\sin(y_2^\circ - y_1^\circ)\cos x_1^\circ}{\sqrt{1-\left\{\sin x_1^\circ \sin x_2^\circ + \cos x_1^\circ \cos x_2^\circ \cos(y_2^\circ - y_1^\circ)\right\}^2}}\right]$$

$$\text{Similarly, } A = \sin^{-1}\left[\frac{\sin(y_2^\circ - y_1^\circ)\cos x_2^\circ}{\sqrt{1-\left\{\sin x_1^\circ \sin x_2^\circ + \cos x_1^\circ \cos x_2^\circ \cos(y_2^\circ - y_1^\circ)\right\}^2}}\right]$$

......................................(2)

The angle $A, B$ will give us the directions.

### VII. Some Important Latitude and Longitude [1]

$R =$ Radius of the earth = 6378.388 km

$M_c =$ Mecca $(21.26^\circ N, 39.49^\circ E)$

$D_k =$ Dhaka $(23.42^\circ N, 90.22^\circ E)$

$D_l =$ Delhi $(28.7041^\circ N, 77.1025^\circ E)$

$L_n =$ London $(51.5072^\circ N, 0.1276^\circ W)$

$N_y =$ New York $(40.712772^\circ N, 74.006058^\circ W)$

**Example-1** The latitude and longitude of Dhaka are $23.42^\circ N$ and $90.22^\circ E$ respectively. Also the latitude and longitude of Mecca are $21.26^\circ N$ and $39.49^\circ E$ respectively. The approximate radius of the earth is 6378.388 km.

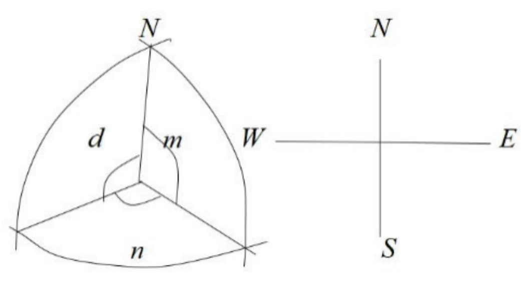


**Fig. 4.** Latitude and Longitude

Here in figure 4., let

$M_c =$ Mecca

$D_k =$ Dhaka

$d =$ Shortest distance between $M_c$ and $D_k$

$R =$ Radius of the earth

$\angle M_c =$ The Heading of Mecca in perspective to Dhaka towards North pole

$\angle D_k =$ The Heading of Dhaka in perspective to Mecca towards North pole

Putting the values of latitudes and longitudes in equation (1) we get

$$n = \cos^{-1}\left\{\sin x_1^\circ \sin x_2^\circ + \cos x_1^\circ \cos x_2^\circ \cos(y_2^\circ - y_1^\circ)\right\}$$

$$\text{or, } n = \cos^{-1}\left\{\begin{matrix}\sin 21.26^\circ \sin 23.42^\circ + \\ \cos 21.26^\circ \cos 23.42^\circ \cos 50.73^\circ\end{matrix}\right\}$$

$$\text{or, } n = \cos^{-1}\left\{0.6854221^\circ\right\}$$

$$\text{or, } n = 46.731188^\circ$$

$$\text{or, } n = 46.731188^\circ \times \frac{\pi}{180}\text{radian}$$

$$\text{or, } n = 0.815613\,\text{radian}$$

So the shortest distance between Mecca and Dhaka is

$d = R \times n = 6378.388 \times 0.815613$

=5202.297 km

Now putting the values of latitudes and longitudes in equation (2) and we will get directions as

$$\angle M_c = \sin^{-1}\left[\frac{\sin 50.73^\circ \cos 23.42^\circ}{\sqrt{1-\left\{\begin{matrix}\sin 21.26^\circ \sin 23.42^\circ + \\ \cos 21.26^\circ \cos 23.42^\circ \cos 50.73^\circ\end{matrix}\right\}^2}}\right]$$

$= 77.322^\circ$

and

$$\angle D_k = \sin^{-1}\left[\frac{\sin 50.73^\circ \cos 21.26^\circ}{\sqrt{1-\left\{\begin{matrix}\sin 21.26^\circ \sin 23.42^\circ + \\ \cos 21.26^\circ \cos 23.42^\circ \cos 50.73^\circ\end{matrix}\right\}^2}}\right]$$

$= 82.244^\circ$

So the heading of Mecca in perspective to Dhaka towards North pole is $\angle M_c = 77.322^\circ$.

So the heading of Dhaka in perspective to Mecca towards North pole is $\angle D_k = 82.244^\circ$

**Example-2** The latitude and longitude of Delhi are $28.7041^\circ N$ and $77.1025^\circ E$ respectively. Also the latitude and longitude of Mecca are $21.26^\circ N$ and $39.49^\circ E$ respectively . The approximate radius of the earth is $6378.388$ km.

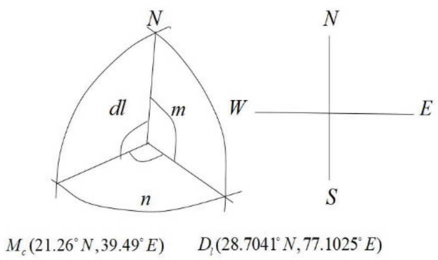


**Fig. 5.** Latitude and Longitude

Here in Figure 6. let

$M_c$ = Mecca
$D_l$ = Delhi
$d$ = Shortest distance between $M_c$ and $D_l$
$R$ = Radius of the earth
$\angle M_c$ = The Heading of Mecca in perspective to Delhi towards North pole
$\angle D_l$ = The Heading of Delhi in perspective to Mecca towards North pole

Putting the values of latitudes and longitudes in equation (1) we get

$$n = \cos^{-1}\left\{\sin x_1^\circ \sin x_2^\circ + \cos x_1^\circ \cos x_2^\circ \cos(y_2^\circ - y_1^\circ)\right\}$$

$$\text{or}, n = \cos^{-1}\left\{\begin{matrix}\sin 28.7041^\circ \sin 21.26^\circ + \\ \cos 28.7041^\circ \cos 21.26^\circ \cos 37.6125^\circ\end{matrix}\right\}$$

$$\text{or}, n = \cos^{-1}\left\{0.821676394^\circ\right\}$$

$$\text{or}, n = 34.74703898^\circ$$

$$\text{or}, n = 34.74703898^\circ \times \frac{\pi}{180} \text{radians}$$

$$\text{or}, n = 0.606451653 \text{ radian}$$

So the shortest distance between Mecca and Delhi is

$$d = R \times n = 6378.388 \times 0.606451653$$
$$=3868.1839 \text{ km}$$

Now putting the values of latitudes and longitudes in equation (2) and we will get directions as

$$\angle M_c = \sin^{-1}\left[\frac{\sin 37.6125^\circ \cos 28.7041^\circ}{\sqrt{1-\left\{\begin{matrix}\sin 28.7041^\circ \sin 21.26^\circ + \\ \cos 28.7041^\circ \cos 21.26^\circ \cos 37.6125^\circ\end{matrix}\right\}^2}}\right]$$
$$= 86.39^\circ$$

and

$$\angle D_l = \sin^{-1}\left[\frac{\sin 37.6125^\circ \cos 21.26^\circ}{\sqrt{1-\left\{\begin{matrix}\sin 28.7041^\circ \sin 21.26^\circ + \\ \cos 28.7041^\circ \cos 21.26^\circ \cos 37.6125^\circ\end{matrix}\right\}^2}}\right]$$
$$= 69.9334^\circ$$

So the heading of Mecca in perspective to Delhi towards North pole is $\angle M_c = 86.39^\circ$

So the heading of Delhi in perspective to Mecca towards North pole is $\angle D_l = 69.9334^\circ$.

**Example-3** The latitude and longitude of London are $51.5072^\circ$ $N$ and $0.1276^\circ$ $W$ respectively. Also the latitude and longitude of Mecca are $21.26^\circ N$ and $39.49^\circ E$ respectively . The approximate radius of the earth is $6378.388$ km.

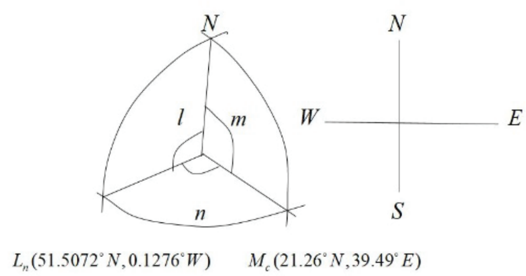


**Fig. 6.** Latitude and Longitude

Here in Figure 6, let

$M_c \equiv$ Mecca
$L_n \equiv$ London
$d$ = Shortest distance between $M_c$ and $L_n$
$R$ = Radius of the earth
$\angle M_c$ = The Heading of Mecca in perspective to London towards n- pole
$\angle L_n$ = The Heading of London in perspective to Mecca towards n- pole

Putting the values of latitudes and longitudes in equation (1) we get

$$n=\cos^{-1}\left\{\sin x_1^{\circ}\sin x_2^{\circ}+\cos x_1^{\circ}\cos x_2^{\circ}\cos(y_2^{\circ}-y_1^{\circ})\right\}$$

$$\text{or}, n=\cos^{-1}\begin{Bmatrix}\sin 51.5072^{\circ}\sin 21.26^{\circ}+\\ \cos 51.5072^{\circ}\cos 21.26^{\circ}\cos 39.6176^{\circ}\end{Bmatrix}$$

$$\text{or}, n=\cos^{-1}\left\{0.730631011^{\circ}\right\}$$

$$\text{or}, n=43.0606799^{\circ}$$

$$\text{or}, n=43.0606799^{\circ}\times\frac{\pi}{180}\text{radian}$$

$$\text{or}, n=0.751552399\,\text{radian}$$

So the shortest distance between Mecca and London is

$$d=R\times n=6378.388\times 0.751552399$$
$$=4793.692 \text{ km}$$

Now putting the values of latitudes and longitudes in equation (2) and we will get directions as

$$\angle M_c=\sin^{-1}\left[\frac{\sin 39.6176^{\circ}\cos 51.5072^{\circ}}{\sqrt{1-\begin{Bmatrix}\sin 51.5072^{\circ}\sin 21.26^{\circ}+\\ \cos 51.5072^{\circ}\cos 21.26^{\circ}\cos 39.6176^{\circ}\end{Bmatrix}^2}}\right]$$
$$=60.501607^{\circ}$$

and

$$\angle L_n=\sin^{-1}\left[\frac{\sin 39.6176^{\circ}\cos 21.26^{\circ}}{\sqrt{1-\begin{Bmatrix}\sin 51.5072^{\circ}\sin 21.26^{\circ}+\\ \cos 51.5072^{\circ}\cos 21.26^{\circ}\cos 39.6176^{\circ}\end{Bmatrix}^2}}\right]$$
$$=35.5414^{\circ}$$

So the heading of Mecca in perspective to London towards North pole is $\angle M_c=60.501607^{\circ}$

So the heading of London in perspective to Mecca towards North pole is $\angle L_n=35.5414^{\circ}$.

**Example-4** The latitude and longitude of New York are $40.712772^{\circ}\,N$ and $74.006058^{\circ}\,W$ respectively. Also the latitude and longitude of Mecca are $21.26^{\circ}N$ and $39.49^{\circ}E$ respectively. The approximate radius of the earth is 6378.388 km.

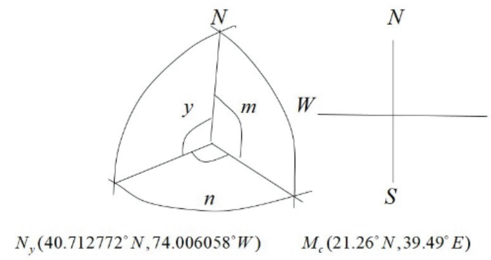


**Fig. 7.** Latitude and Longitude

Here in figure 7, let

$M_c$ = Mecca
$N_y$ = NewYork
$d$ = Shortest distance between $M_c$ and $N_y$
$R$ = Radius of the earth
$\angle M_c$ = The Heading of Mecca in perspective to New York towards North pole
$\angle N_y$ = The Heading of New York in perspective to Mecca towards North pole

Putting the values of latitudes and longitudes in equation (1) we get

$$n=\cos^{-1}\left\{\sin x_1^{\circ}\sin x_2^{\circ}+\cos x_1^{\circ}\cos x_2^{\circ}\cos(y_2^{\circ}-y_1^{\circ})\right\}$$

$$\text{or}, n=\cos^{-1}\begin{Bmatrix}\sin 40.712127^{\circ}\sin 21.26^{\circ}+\\ \cos 40.712127^{\circ}\cos 21.26^{\circ}\cos 113.496058^{\circ}\end{Bmatrix}$$

$$\text{or}, n=\cos^{-1}\left\{-0.045126464^{\circ}\right\}$$

$$\text{or}, n=92.58643428^{\circ}$$

$$\text{or}, n=92.58643428^{\circ}\times\frac{\pi}{180}\text{radian}$$

$$\text{or}, n=1.6159419\,\text{radian}$$

So the shortest distance between Mecca and New York is

$$d=R\times n=6378.388\times 1.6159419$$
$$=10307.10442 \text{ km}$$

Now putting the values of latitudes and longitudes in equation (2) and we will get directions as

$$\angle M_c=\sin^{-1}\left[\frac{\sin 113.496058^{\circ}\cos 40.71212772^{\circ}}{\sqrt{1-\begin{Bmatrix}\sin 40.712127^{\circ}\sin 21.26^{\circ}+\\ \cos 40.712127^{\circ}\cos 21.26^{\circ}\cos 113.496058^{\circ}\end{Bmatrix}^2}}\right]$$
$$=44.0956^{\circ}$$

and

$$\angle N_y = \sin^{-1}\left[\frac{\sin 113.496058^\circ \cos 21.26^\circ}{\sqrt{1-\begin{Bmatrix}\sin 40.712127^\circ \sin 21.26^\circ + \\ \cos 40.712127^\circ \cos 21.26^\circ \cos 113.496058^\circ\end{Bmatrix}^2}}\right]$$

$$= 58.82^\circ$$

So the heading of Mecca in perspective to New York towards North pole is $\angle M_c = 44.0956^\circ$

So the heading of New York in perspective to Mecca towards North pole is $\angle L_n = 58.82^\circ$.

## VIII. Conclusion

The earth is not a perfect sphere. For this reason the radius $R$ of the earth varies from place to place. So $6378.388$ kilometers is taken as the average radius. The distance and direction are approximately found here.

The earth rotates about its axis and the hot liquid of its core is in motion. Due to this motion of the hot liquid, electric current is produced and for this electric magnetism is produced. This magnetism has polarities. It is observed by the Royal Observatory Greewhich that the north pole of the magnetism of the earth rotates on $73^\circ N$ latitude and one complete rotation will take place in $960$ years approximately.

The earth's magnetic north pole was at ($73^\circ N$, $0^\circ$ longitude) in the year $1659$. This pole moves towards west $\left(\frac{3}{8}\right)^\circ$ per year. Due to this secular variation the declaration formula for the place $A(x^\circ N, y^\circ E)$ will be

$$\delta_n = \left[\frac{\cos 73^\circ \sin\left\{y^\circ + \frac{3}{8}(n-1659)^\circ\right\}}{\sqrt{1-\left[\sin x^\circ \sin 73^\circ + \cos x^\circ \cos 73^\circ \cos\left\{y^\circ + \frac{3}{8}(n-1659)^\circ\right\}\right]^2}}\right].$$

Where $n$ represents the year and it starts from $1659$. With the help of magnetic compass we can find the direction of magnetic pole. If $B$ be the angle with the direction of true north of the earth then the angle with the direction of the earth's magnetic pole will be $B-\delta_n$ in the year represented by $n$.

Several kind of variations of the earth's magnetism are observed such as secular variations, annual variation, daily variation, eleven year period variation, variation due to magnetic stromes, etc. Apart from these there are local deposits of magnetic material. For all these magnetic compass does not give regular direction. If we could know the nature of the motion of hot liquid of the core of the earth and we could use a compass to know the direction. But till today the nature of the motion of the hot liquid of the core is not known.

There are methods to know the direction of the geographical north pole. One of the method is by the shadow of a vertical rod on a horizontal plane. We may find it by pole star. Actually in this paper we have tried to find out the direction and distance of Mecca from different places so that the muslims may use the direction for saying prayers towards Mecca (KIBLA). We may want to elaborate our works in geometry of manifolds in future.

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