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A Stochastic Capacity Expansion Model for an Electricity Generation Company

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Abstract

The business organizations in Bangladesh are operating their business by intuition based planning. They are not using scientific techniques for making decisions. But the scientific models can play an important role to maximize their profit and to save idle times, wastage of raw materials and finished products. As a result, the business organizations in our country fail achieve their goals.This is the case for electricity generation companies too. This research paper will focus on developing a stochastic capacity expansion model to help an electricity generation company. Capacity expansion problems are concerned with the timing of facility expansions and levels of investments to meet increasing demand. Since demand is difficult to forecast and expansion plans may need to change over time, stochastic programming offers a convenient way to solve these problems. In this research paper, we will develop a multi-stage stochastic programming model to formulate the real world problem which will satisfy the future demand of an electricity generation company. The model will help the company to find optimal investment in different power plants by allocating the limited resources and capacities to different power terminals.To observe whether the company can maximize the utilization of resources and minimize cost, and to demonstrate the applicability of our model, we will analyze a two-stage problem derived from this model. We will use mathematical programming language LINDO and LINGO for solving resulting SPs. We will also use MATHEMATICA to draw the continuous function.

Key word: Stochastic programming, recourse problem, capacity expansion, electricity generation

I. Introduction

To become a developed country, Bangladesh must develop its economic sector because it is one of the most important areas for the development of the country. To achieve the Sustainable Development Goals (SDGs), Bangladesh needs strong national economy. Total economy of this country depends on the development of the different business organizations. On the other hand, the development of the business organizations depends on the implementation of the mathematical and scientific models which gives the manager a perfect insight about the demand, supply, production and inventory.

Business organizations must do more with their limited resources to match their priorities with the needs of citizens. But almost all of the business organizations in Bangladesh are still operating their business through intuition based planning. They are not using scientific forecast techniques. In order to be sustainable, the companies should plan the future not the present. The organizations must take necessary measures to manage the cost of the inventory, labor, and financial resources in the best way to avoid wastage. This will result into cost minimization and profit maximization. We will first analyze various problems related to electricity generation. To satisfy the future demand of electricity, an investment problem will be considered. It will help the company to decide the amount of investment in different power plants will be appropriate. Capacity expansion problems are concerned with the timing of facility expansions and levels of investment to meet increasing demand. Since demand is difficult to forecast and expansion plans may need to change over time, stochastic programming offers a convenient way to solve these problems. The model will help the company by optimal allocation of limited capacity to different power terminals.We will develop a multistage stochastic programming model so that the electricity company will be able to determine optimal levels of investment to meet their future demands. To observe whether the company can maximize the utilization of resources and maximize profit, and to demonstrate the applicability of our model, we will analyze a two-stage problem derived from this model. We will use programming language LINDO or LINGO for solving resulting SPs. We will also use MATHEMATICA to draw the continuous function.

Literature Review

We will first review various relevant research articles to get an overview of the current research in the electricity generation sector. Dimitrios and Giorgia[1] develop a stochastic model for evaluating the optimal timing and capacity of investments in flexible combined heat and power generation for energy-intensive industries. Salvador and Morales[2] proposed a capacity expansion model of stochastic power generation under two-stage electricity markets. Hasanand Chakroborty[3] discussed about the stochastic programming to solve a real life oriented farmer's problem. Louveaux and Smeers[4,5] presented the stochastic models for optimal investment in electricity generation. Hashnayne[6] presented a case study of stochastic programming problems modeling applications. Mainly he tried to develop a study of farmer's problem based on two-stage SP with simple recourse. Zhengyangand Guiping[7] presented two-stage SP framework for scheduling

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problem. Murfy and Weiss[8] have developed a capacity expansion modelfor electric utility. Lara et. al[9] presented a stochastic infrastructure planning model for electric power. Rajagopalan et. al[10] presented a capacity replacement model. Sainee[11] developed a multi-period capacity expansion model for telecommunications. Grinold[12]developed a multistage convex programming model for the correction of end effect. Goel. Grossmann[13,14] analyzed a class of stochastic programs with decision dependent uncertainty.

The knowledge and information gathered from the above literature review will help us to develop capacity expansion stochastic program model for the electricity generation company. This will also help us to develop a multi-stage stochastic program for using decomposition technique.

II. Basics of Stochastic Program

In mathematical planning, an optimization problem finds the best solution among the set of all alternatives. It is classified as deterministic and stochastic depending on the basis of uncertainties on parameters and variables[15,16]. In this section, we discuss about Stochastic Programming (SP), importance and applications in several fields of real life.

The mathematical form of a SP is as follows[13,14].
Minimize or Maximize $z = cx$

subject to $Ax \geq b, x \geq 0$

There may be uncertainty in demand **b**, in input prices **c** and in the technical coefficient matrix[17,18]:

Classes of SP

The SP problems can be classified in Figure 2.1.

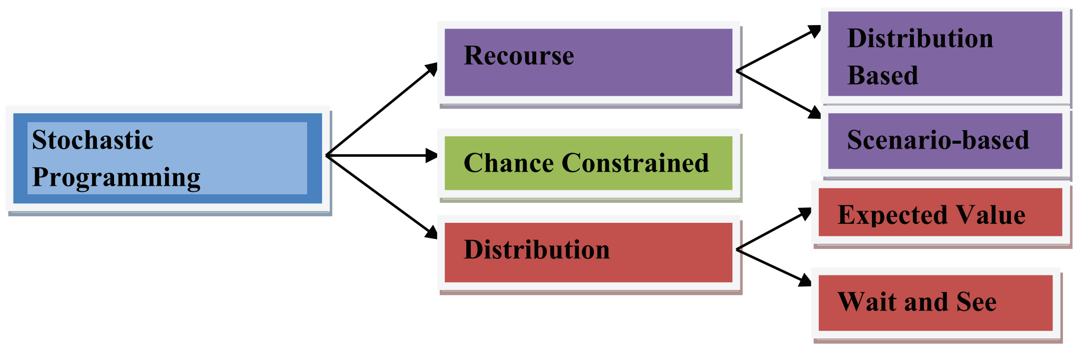


Fig. 2.1. Classification of SP

III. Stochastic Program with Simple Recourse

In this Section, we will discuss about multi stage recourse problems and scenario generation[21]. We will first present a two-stage model. Then, we will generalize this to multistage model.

Two-Stage SP Problem

In two-stage recourse problem, we first make a decision now which is termed as first stage decision. Then nature makes a random decision called after observing the random realization. Finally, we make a second stage decision that attempts to repair the uncertainty called two-stage recourse problem. The mathematical form of a two-stage recourse model is as follows[19,20.]

First stage

$$Min\ c_x x + c_z z + E[h(z,\xi)]$$

subject to

$$A_x x + A_z z = b$$
$$0 \leq x \leq u_x,\ \ 0 \leq z \leq u_z$$

Second stage

$$h\,(z,\xi) = Min(fy)$$

subject to

$$Tz + Wy = d$$
$$0 \leq y \leq u_y,$$
$$\xi = vec(f, u_y, T, W, d)$$

The first stage decisions are the vectors *x* and *z*, where the decisions *x* appear in the first stage but *z* connects the first and second stage. The first stage decisions contribute directly to the objective value through the first stage cost. These decisions are constrained by linear constraints and simple upper bounds. The random variables affecting the second stage are described by the vectors ξ. In most general case the vector may affect all the parameters of the second stage problem. The cost of the second stage, $h\,(z,\xi)$, depends on the realizations and on z. This cost is a random variable. *y* is the second stage decision variable which depends on the 1st stage decision *x*.

Multi-Stage SP Problem

Initially, we first take a decision known as initial decision. After the realization from the random parameters, we make decisions for the 2nd stage. In a similar manner, we take decision for the succeeding stages are called recourse decisions.

Consider a multistage SP problem where c_1, A_1 and b_1 are certain, but some or all the entries of the cost vectors c_t, coefficient matrices B_t and A_t, and the right hand side vectors b_t, $t = 2, \ldots, T$, are uncertain. Consider the random vector ξ_t becomes known at time period t. Let the decision vector for stage t is x_t and depends on the realization available at time t and is defined as $\xi_{[t]}$. The initial decision is x_1. Then the realization from the random data is $\xi_2 := (c_2, B_2, A_2, b_2)$. After this we make a new decision in the second stage as x_2. In a similar way we take the decision for T^{th} stage as x_T after observing the realization from $\xi_T := (c_T, B_T, A_T, b_T)$. This decision process minimizes the total cost allowing the decisions to be made at every stage $t = 1, \ldots, T$. Figure 3.1 shows a flowchart of the stage wise decision.

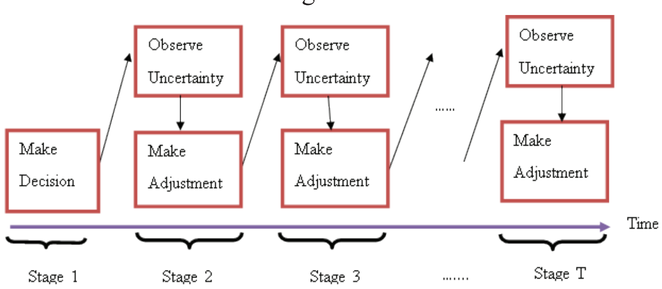


Fig. 3.1. Multi-stage decision procedure

The problem can be formulated as follows:

$$\min_{x_1 \in X_1} f_1(x_1) + E\Big[\min_{x_2 \in X_2(x_1,\xi_2)} f_2(x_2,\xi_2) + E\Big[\ldots + E\Big[\min_{x_T \in X_T(x_{T-1},\xi_T)} f_T(x_T,\xi_T)\Big]\Big]\Big]$$

where

$$f_t(x_t,\xi_t) = c_t^T x_t$$

$$X_1 = \{x_1 : A_1 x_1 = b_1, x_1 \geq 0\}$$

$$X_T = \{x_t : B_t x_{t-1} + A_t x_t = b_t, x_t \geq 0\}, t = 2, \ldots, T$$

Here $\xi_t, t = 1, \ldots, T$, is a sequence of random variables and X_t is the set of decision variables and depends upon $\xi_{[t]}$ where, $\xi_{[t]} = (\xi_1, \xi_2, \ldots, \xi_t)$ denote the random observations up to period t.

Scenario based problem

The idea is to construct or sample possible futures and solve the corresponding problems for these values. After obtaining a number of decisions in this way, we either pick the best of them, or we try to make a good combinations of the decisions.

Scenario Tree

In a scenario tree each scenario creates a path from the root of the tree to one of its leaves. The visited nodes are assigned with a random value[21,22,23]. As an example, here we discuss the construction of a scenario tree for well-known news vendor problem which maximize the total expected profit at the end a planning horizon.

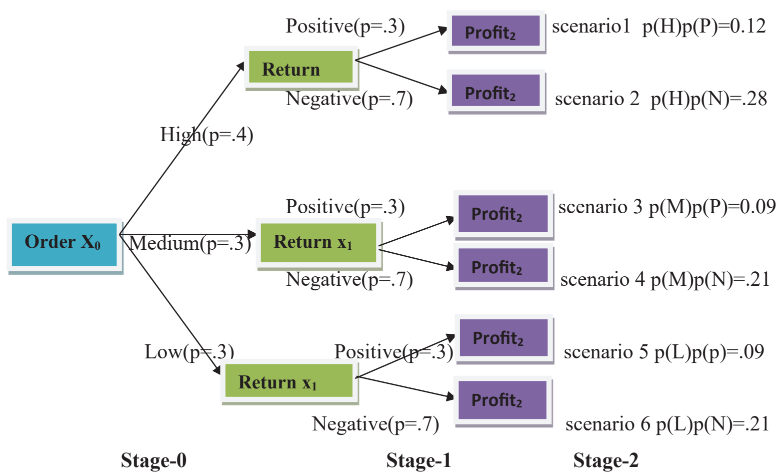


Fig. 3.2. Scenario tree of news vendor problem

In this problem, a news vendor first decides the amount to order and then he realize the amount unsold amount to be returned to the company with a reduced price. Initially decide an order quantity. After observing the random parameter the demand is revealed. At the end of 1st stage, he decides how much of the products to be returned to the source company at a reduced price. This price may be either positive or negative. In Figure 3.2, each root-leaf path defines a scenario.

IV. Capacity Expansion Model

Capacity expansion problems are concerned with the timing of facility expansions and levels of investment to meet increasing demand. We first study the optimal investments problem for an electricity generation company. Analyzing different existing problems related to electricity generation, we will develop a multi-stage stochastic model which will be able to handle the uncertainties faced by the company. The model will obtain optimal amount of investment in different power plants to meet future demands. This model concerns the optimal allocation of limited capacity to different power terminals.We will discuss the justification of the requirement of a multistage stochastic model. We will then demonstrate the proposed model by considering a two-stage test-problem.

The Electricity Generation Problem

In this section, we first present a deterministic analysis of the electricity generation problem. Currently the company is operating its business on the basis of deterministic decisions. The company has i number of power plants. The costs involve are investment cost $c_{i,}$ operating cost q_i and the availability factor a_i which is a percentage of time the power plant can effectively be operated. The demand for electricity of the company is uncertain. The electricity company represents the demand by a ”load-curve” as shown in Figure 4.1. It indicates that the demand is decreasing over time.

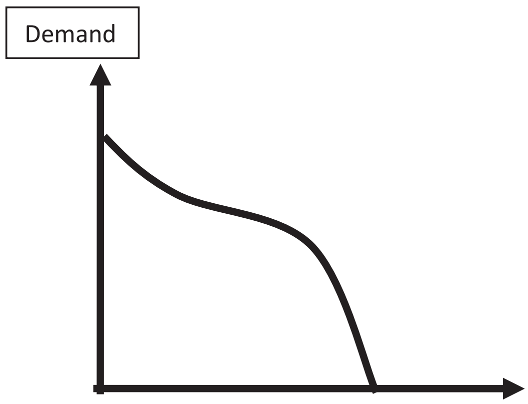


Fig. 4.1. The load duration curve

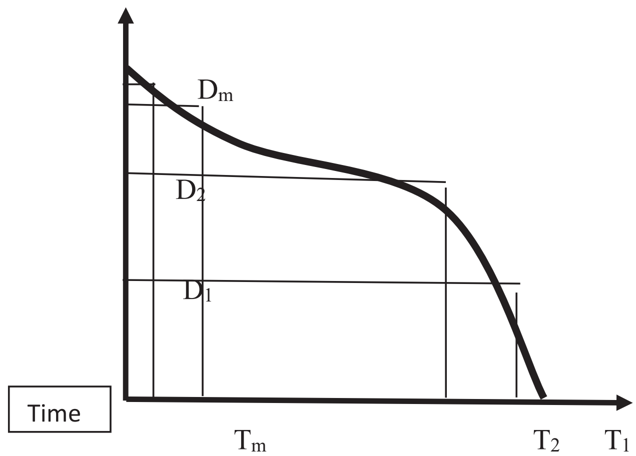


Fig. 4.2.A piecewise constant curve with *m*segments

Figure 4.2 shows the piece wise distribution of load which is known as load distribution curve. This curve has m segments. Let $d_1 = D_1$, and $d_j = D_j - D_j - 1$, $j = 2, \ldots, m$ is the additional demand of power in ‘mode j’ for a duration T_j. In the deterministic situation, the problem consists of finding the optimal investment for each mode j. It is given by

$$i(j) = argmin_{i=1,\ldots,n} \{\frac{c_i+q_iT_j}{a_i}\} \ldots\ldots\ldots\ldots\ldots (1)$$

where *argmin* represents the index i for which the minimum value is achieved and n is the number of available technologies[5].

Deterministic Model Formulation

We consider equipment costs, load curve, new technologies and the currently available equipment[24].The equipment costs depends on technological progress and fuel costs.

The indices for the proposed multi-stage model are $t = 1, \ldots, M$ for the periods or stages, $i= 1, \ldots, n$ for the available technologies and $j = 1, \ldots, m$ the operating modes in the load duration curve. The parameter for proposed multi-stage model a_i stands for availability factor of i, L_i stands for lifetime of i, g_i^t indicates existing capacity of i at time t, decided before $t= 1$, c_i^t is the investment cost per unit for $i$at time t, q_i^t is the production cost per unit for $i$at time t, d_j^t is the demanded of power in mode j at time t and T_j^t is the duration of mode j at time t. The decision variables for the proposed multi-stage model are x_i^t which is the available capacity made for technology i at time t, w_i^t is the total available capacity of i at time t and y_{ij}^t is the capacity of i effectively used at time t in mode j.

Then the deterministic multi-stage electricity generation model takes the following form.

$$\boldsymbol{min \sum_{t=1}^{M} (\sum_{i=1}^{n} c_i^t . w_i^t + \sum_{i=1}^{n}\sum_{j=1}^{m} q_i^t . T_j^t . y_{ij}^t)} \ldots\ldots\ldots (4.2)$$

$\boldsymbol{s.t}$

$$\boldsymbol{w_i^t = w_i^{t-1} + x_i^t - x_i^{t-L_i}, i = 1, \ldots, n, t = 1, \ldots, M} \ldots\ldots\ldots (4.3)$$

$$\boldsymbol{\sum_{i=1}^{n} y_{ij}^t = d_j^t , j = 1, \ldots., m, t = 1, \ldots, M} \ldots\ldots\ldots (4.4)$$

$$\boldsymbol{\sum_{j=1}^{m} y_{ij}^t \leq a_i(g_i^t + w_i^t), \quad i = 1, \ldots, n, t = 1, \ldots, M} \ldots\ldots\ldots (4.5)$$

$$\boldsymbol{w, x, y \geq 0}$$

The constraint set (4.3) states that an equipment becomes obsolete after its lifetime. It also states that the newly decided capacities increase the total capacity w_i^t. Constraint set (4.4) is the match between the demand and capacity in all modes. Constraint set (4.5) is a relationship between capacity and capacities g_i^t decided before $t = 1$, newly decided capacities x_i^t, and the availability factor. The objective function (4.2) is the sum of the investment cost, maintenance costs, and operating costs.

A Real Life Deterministic Problem

In this section, we consider a real life example taking secondary data from a company of Dhaka. The example is a deterministic model with 3 operating modes namely North, Middle and South of Dhaka. The problem has $n = 4$ technologies such as gas, coal, nuclear and wind. One period construction delay for all technologies, full availabilities, $a = (1,1,1,1)$ and there is no equipment available. As a result $g = (0,0,0,0)$. Consider $d_1=5$, $d_2= 3$, $d_3= 2$. Moreover, $T_2= .6T_1$ and $T_3= .1\ T_1$ and let $T_1 = 10$. Available budget is 120. The investment costs for the equipment are (10, 7, 16, 6) and production costs per unit are q = (4, 4.5, 3.2, 5.5) respectively. The company needs to find the optimal allocation of limited capacity of gas, coal, nuclear and wind to different power terminals at the location North, Middle and South.

The decision variables and parameters are considered as follows.

x_1 = Unit of new capacity made available in Gas Plant

x_2 = Unit of new capacity made available in Coal Plant

x_3 = Unit of new capacity made available in Nuclear Plant

x_4 = Unit of new capacity made available in Wind Plant

y_{11} = Unit of allocated capacity of Gas in North Mode

y_{21} = Unit of allocated capacity of Coal effectively used in North Mode

y_{31} = Unit of allocated capacity of Nuclear effectively used in North Mode

y_{41} = Unit of allocated capacity of Wind effectively used in North Mode

y_{12} = Unit of allocated capacity of Gas effectively used in Middle Mode

y_{22} = Unit of allocated capacity of Coal effectively used in Middle Mode

y_{32} = Unit of allocated capacity of Nuclear effectively used in Middle Mode

y_{42} = Unit of allocated capacity of Wind effectively used in Middle Mode

y_{13} = Unit of allocated capacity of Gas effectively used in South Mode

y_{23} = Unit of allocated capacity of Coal effectively used in South Mode

y_{33} = Unit of allocated capacity of Nuclear effectively used in South Mode

y_{43} = Unit of allocated capacity of Wind effectively used in South Mode

Table 4.1. Install and operating cost per unit

Technology (x_i)	Cost/unit to install (c_i)	Cost/unit to operate (q_i)
Gas (x_1)	10	4
Coal (x_2)	7	4.5
Nuclear (x_3)	16	3.2
Wind (x_4)	6	5.5

Table 4.2. Demands

Modes	North	Middle	South
Demands (d_j)	5	3	2

Table 4.3. Combined cost

Technology(x_i)	North	Middle	South
Gas	40	24	4
Coal	45	27	4.5
Nuclear	32	19.2	3.2
Wind	55	33	5.5

The resulting deterministic problem is:

$$z = \min 10x_1 + 7x_2 + 16x_3 + 6x_4 + 40y_{11} + 45y_{21} + 32y_{31} + 55y_{41} + 24y_{12} + 27y_{22} + 19.2y_{32} + 33y_{42} + 4y_{13} + 4.5y_{23} + 3.2y_{33} + 5.5y_{43}$$

subject to

$$10x_1 + 7x_2 + 16\,x_3 + 6\,x_4 \le 120$$
$$y_{11} + y_{12} + y_{13} - x_1 \le 0$$
$$y_{21} + y_{22} + y_{23} - x_2 \le 0$$
$$y_{31} + y_{32} + y_{33} - x_3 \le 0$$
$$y_{41} + y_{42} + y_{43} - x_4 \le 0$$
$$y_{11} + y_{21} + y_{31} + y_{41} \ge 5$$
$$y_{12} + y_{22} + y_{32} + y_{42} \ge 3$$
$$y_{13} + y_{23} + y_{33} + y_{43} \ge 2$$
$$x_i \ge 0, y_{ij} \ge 0, i=1,\dots,4 \text{ and } j=1,\dots,3.$$

This is a simple linear optimization problem which can be solved by LINDO software as follows:

Output of the Deterministic LP problem

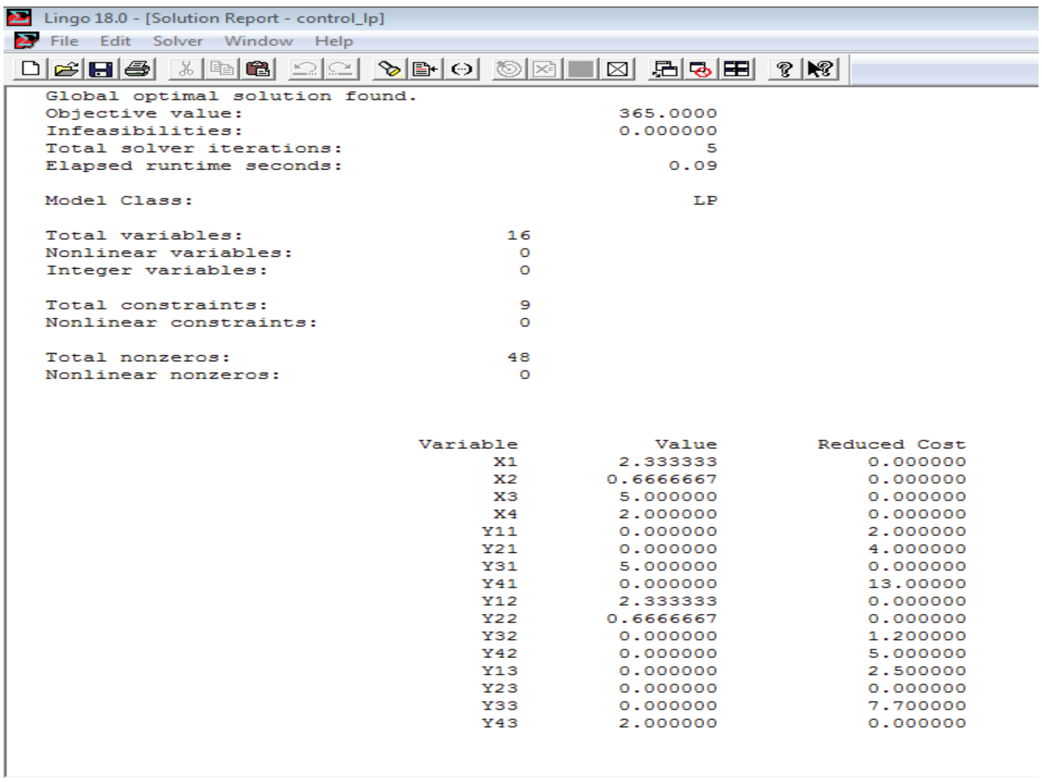


Fig. 4.3. Solution of deterministic LP problem

Table 4.4. Solution obtained by Lindo

Technology	North	Middle	South	Capacity installed
Gas	0	2.3333	0	2.3333
Coal	0	0.6667	0	.6667
Nuclear	5	0	0	5
Wind	0	0	2	2

Solution obtained by solving **Lindo** with description has given below table: where x_1=2.33,x_2=.67, x_3=5, x_4=2. Therefore, the objective value is

$$z = (10 * 2.3333 + 7 * .6667 + 16 * 5 + 6 * 2 + 40 * 0 + 45 * 0 + 32 * 5 + 55 * 0 + 24 * 2.3333 + 27 * 0.6667 + 19.2 * 0 + 33 * 0 + 4 * 0 + 4.5 * 0 + 3.2 * 0 + 5.5 * 2) = 365$$

Stochastic Capacity Expansion model

In the deterministic model discussed in the section 4.3, we considered all the variables and parameters as certain. But in real life it is not true for all. So in this section, we develop a multi-stage stochastic model to deal with these uncertainties. In this model, we will consider that demand and costs are uncertain. In the current model, x_i^t will represent the new capacity of i at time t, as x_i^{t+hi}, where hi is the construction delay for equipment i. Because of the uncertainty, the choice of investment decisions may yield an infeasible operations policy. To overcome this problem, let there exists a technology with high operating costs and zero construction delay. Let ξ be the realization of random event for any period t on the basis of technology an investment is made. Let x_i^t be the new capacity at time t for equipment i where, $i = 1, \ldots, n$, w_i^t be the total capacity of i available in order at time t, n be a technology such that $h_n = 0$ and ξ_t is the random variable at time t. Then the stochastic model takes the following form.

$$\min E_\xi \sum_{i=1}^{M} \Big(\sum_{i=1}^{n} c_i^t . w_i^t + \sum_{i=1}^{n}\sum_{i=1}^{m} q_i^t . T_j^t . y_{ij}^t\Big) \ldots\ldots\ldots (4.6)$$

$$s.t$$

$$w_i^t = w_i^{t-1} + x_i^t - x_i^{t-L_i}, i = 1, \ldots, n, t = 1, \ldots, M \ldots\ldots\ldots (4.7)$$

$$\sum_{i=1}^{n} y_{ij}^t = d_j^t, j = 1, \ldots, m, t = 1, \ldots, M \ldots\ldots\ldots (4.8)$$

$$\sum_{j=1}^{m} y_{ij}^t \le a_i\big(g_i^t + w_i^{t-hi}\big), \quad i = 1, \ldots, n, t = 1, \ldots, M \ldots\ldots\ldots (4.9)$$

$$w, x, y \ge 0$$

where, $\xi = (\xi^{2,\ldots,}\xi^H)$ is a random vector. And $\xi^t=(d_1^t,\ldots,d_k^t)$ are the random demands, and the cost vector is (c^t, q^t). Now we will demonstrate the model on the basis of scenarios as follows.

$$\min \quad \sum_{i=1}^{4} c_i x_i + \sum_{i=1}^{4}\sum_{j=1}^{3} p_j\, y_j$$

$$Budget \;\; constraints \quad \sum_{j=1}^{4} a_j x_j \le b$$

$$city \;\; constraints \quad \sum_{i=1}^{4}\sum_{j=1}^{3} y_j \le x_i,$$

$$and \;\; constraints \quad \sum_{i=1}^{4} y_j \le \xi, \quad j = 1$$

$$\sum_{i=1}^{4}\sum_{j=2}^{3} y_j \le b_j, \quad j = 2, \quad 3$$

$$x_i \ge 0, \quad y_j \ge 0,$$

Similar models can be formulated for Scenario 2 and 3.

A Real Life Stochastic Problem

Considering the stages M= 2, we analyze a two-stage, stochastic linear model of (4.6)-(4.9). In this real life problem, we assume that the available resources are stochastic.The deterministic decisions are the capacities assigned (X_i) to each power plants, subject to certain resource limitations. The stochastic decisions (2nd stage) are the amount of power is to be supplied from each plant to each demand location (Y_{ij}). The objective of the company is to minimize the total expected cost. At the beginning of stage-1, demands at three locations are revealed, and at the end of stage-1, the demand is satisfied at minimum cost by solving the resulting problem. The problem has 3 operating modes, n = 4 technologies, h_i =1, i.e, one period construction delay for all technologies, full availabilities, a = (1,1,1,1) and no equipment available, so g = (0,0,0,0). We also assume d_3= 2, d_2= 3 and the only random variable is $d_1 = \boldsymbol{\xi}$,where $\boldsymbol{\xi}$ can take the value 3for low demand, 5for moderate demand and 7for highest demand with probability 0.3, 0.4 and 0.3 respectively. Moreover, T_2= .6$T_1$and T_3= .1T_1 and we assume T_1 = 10. The maximum available budget is 120. The investment costs for the four equipment's are 10,7, 16, 6 respectively. Assuming,T_1 = 10, the operating costs/production costs in mode 1 are 40,45,32,55. Then, if T_2= 6 and T_3= 1, the resulting two-period stochastic program is given as follow.

Table 4.5. Different costs and demand

Technology (x_i)	Cost/unitto install (c_i)	Cost/unitto operate (q_i)	North (y_{i1})	Middle (y_{i2})	South (y_{i2})
Gas (x_1)	10	4	40	24	4
Coal (x_2)	7	4.5	45	27	4.5
Nuclear (x_3)	16	3.2	32	19.2	3.2
Wind (x_4)	6	5.5	55	33	5.5
Demands(d_j)			5	3	2
Transport rate(T_j)			10	6	1

Table 4.6. Scenario based demand

North Mode	Scenario 1 "Low Demand"	Scenario 2 "Moderate Demand"	Scenario 3 "Highest Demand"
Probability	0.3	0.4	0.3
Demands (d_1)	3	5	7

Optimal solution based on Scenario-1:

$$z1 = \min 10x_1 + 7x_2 + 16x_3 + 6x_4 + 40y_{11} + 45y_{21} + 32y_{31} + 55y_{41} + 24y_{12} + 27y_{22} + 19.2y_{32} + 33y_{42} + 4y_{13} + 4.5y_{23} + 3.2y_{33} + 5.5y_{43}$$

subject to

$$10x_1 + 7x_2 + 16\,x_3 + 6\,x_4 \le 120$$
$$y_{11} + y_{12} + y_{13} - x_1 \le 0$$
$$y_{21} + y_{22} + y_{23} - x_2 \le 0$$
$$y_{31} + y_{32} + y_{33} - x_3 \le 0$$
$$y_{41} + y_{42} + y_{43} - x_4 \le 0$$
$$y_{11} + y_{21} + y_{31} + y_{41} \ge 3$$
$$y_{12} + y_{22} + y_{32} + y_{42} \ge 3$$
$$y_{13} + y_{23} + y_{33} + y_{43} \ge 2$$
$$x_i \ge 0, y_{ij} \ge 0, \; i = 1,\dots,4 \text{ and } j = 1,\dots,3.$$

Table 4.7. Solution obtained by solving Lindo for 3 scenarios

	Scenario 1			Scenario 2			Scenario 3		
Technology	North	Middle	South	North	Middle	South	North	Middle	South
Gas	0	3	0	0	2.3333	0	4.1667	0	0
Coal	0	0	2	0	0.6667	0	0	3	0
Nuclear	3	0	0	5	0	0	2.8333	0	0
Wind	0	0	0	0	0	2	0	0	2
Optimal costs ($)		269			365			459	

Solution obtained by solving **Lindo** with description has been given in Table 4.7

Considering three scenarios the long run average cost is $\frac{(269 + 365 + 469.33)}{3}$ = $367.78. This is the Expected Value of WS problem where one needs to wait and see the outcomes random variables before making final decisions. For minimization type problem WS <= EV. This is the situation under perfect information i.e assuming that the power plants have advance knowledge on the demands in the North mode and can base the decision upon that knowledge.

We suppose that the demand is cyclical. A year with 'low demand in North mode' is always followed by a year with 'moderate demand in North mode' and then a year with 'high demand in North mode'. The optimal solutions of the power plants for three different scenarios are shown in Table 4.9, Table 4.10 and Table 4.11 respectively. The optimal cost of the company is $269 for the first year, $365 for the second year and $469.33 for the third year. The average expected cost over the three years is $367.78 per year.

From the above discussion, we observe that if the Power plants get the information on the demand before investment, they will be able to invest optimally to the different modes. As a result they will be gain an optimal expected cost **$367.78** per year.

Generalized stochastic programming formulation

Without using any scientific model, the power plants are unable to make a perfect decision that would be best in all circumstances. To get a reasonable solution it is required to formulate an expected general stochastic programming Problem as follows.

$$z = min \sum_{i=1}^{4} c_i x_i + E_\xi \left[\sum_{i=1}^{4}\sum_{j=1}^{3} p_{ij}\, y_{ij} + \sum_{i=1}^{4}\sum_{j=1}^{3} q_{ij}\, w_{ij} + \sum_{i=1}^{4}\sum_{j=1}^{3} s_{ij}\, z_{ij} \right]$$

$$s.t. \quad Budget \quad constra\,int\,s \quad \sum_{j=1}^{4} a_j x_j \le b$$

$$Capacity \quad constra\,int\,s \quad \sum_{i=1}^{4}\sum_{j=1}^{3} y_{ij} \le x_i\,,$$

$$\sum_{i=1}^{4}\sum_{j=1}^{3} w_{ij} \le x_i\,,$$

$$\sum_{i=1}^{4}\sum_{j=1}^{3} z_{ij} \le x_i\,,$$

Demand constraints

$$\sum_{i=1}^{4} y_{ij} \leq \xi, \quad j = 1$$
$$\sum_{i=1}^{4}\sum_{j=2}^{3} y_{ij} \leq b_j, \quad j = 2, \quad 3$$
$$\sum_{i=1}^{4} w_{ij} \leq \xi, \quad j = 1$$
$$\sum_{i=1}^{4}\sum_{j=2}^{3} w_{ij} \leq m_j, \quad j = 2, \quad 3$$
$$\sum_{i=1}^{4} z_{ij} \leq \xi, \quad j = 1$$
$$\sum_{i=1}^{4}\sum_{j=2}^{3} z_{ij} \leq n_j, \quad j = 2, \quad 3$$

$$x_i \geq 0, \quad y_{ij} \geq 0, \quad w_{ij} \geq 0, \quad z_{ij} \geq 0$$

Here, we assume an equal probability of happening the three scenarios. The detailed model for the electricity company is presented as follows.

$$z = \min 10x_1 + 7x_2 + 16x_3 + 6x_4 + E\text{-}_\xi \{\min (40y_{11} + 45y_{21} + 32y_{31} + 55y_{41} + 24y_{12} + 27y_{22} + 19.2y_{32} + 33y_{42} + 4y_{13} + 4.5y_{23} + 3.2y_{33} + 5.5y_{43}) + \quad 40w\text{-}_{11} + 45w_{21} + 32w_{31} + 55w_{41} + 24w_{12} + 27w_{22} + 19.2w_{32} + 33w_{42} + 4w_{13} + 4.5w_{23} + 3.2w_{33} + 5.5w_{43} \quad () + \quad (40z\text{-}_{11} + 45z_{21} + 32z_{31} + 55z_{41} + 24z_{12} + 27z_{22} + 19.2z_{32} + 33z_{42} + 4z_{13} + 4.5z_{23} + 3.2z_{33} + 5.5z_{43})\}$$

subject to

Budget Constraints:

$$10x_1 + 7x_2 + 16\,x_3 + 6\,x_4 \leq 120$$

Capacity Constraints:

$$y_{11} + y_{12} + y_{13} \leq x_1$$
$$y_{21} + y_{22} + y_{23} \leq x_2$$
$$y_{31} + y_{32} + y_{33} \leq x_3$$
$$y_{41} + y_{42} + y_{43} \leq x_4 \qquad w_{11} + w_{12} + w_{13} \leq x_1$$
$$w_{21} + w_{22} + w_{23} \leq x_2$$
$$w_{31} + w_{32} + w_{33} \leq x_3$$
$$w_{41} + w_{42} + w_{43} \leq x_4$$
$$z_{11} + z_{12} + z_{13} \leq x_1$$
$$z_{21} + z_{22} + z_{23} \leq x_2$$
$$z_{31} + z_{32} + z_{33} \leq x_3$$
$$z_{41} + z_{42} + z_{43} \leq x_4$$

Demand Constraints:

$$y_{11} + y_{21} + y_{31} + y_{41} \geq \xi$$
$$y_{12} + y_{22} + y_{32} + y_{42} \geq 3$$
$$y_{13} + y_{23} + y_{33} + y_{43} \geq 2$$
$$w_{11} + w_{21} + w_{31} + w_{41} \geq \xi$$
$$w_{12} + w_{22} + w_{32} + w_{42} \geq 3$$
$$w_{13} + w_{23} + w_{33} + w_{43} \geq 2$$
$$z_{11} + z_{21} + z_{31} + z_{41} \geq \xi$$
$$z_{12} + z_{22} + z_{32} + z_{42} \geq 3$$
$$z_{13} + z_{23} + z_{33} + z_{43} \geq 2$$

$$x_i \geq 0, y_{ij} \geq 0, w_{ij} \geq 0, z_{ij} \geq 0$$

where, ξ can take the value 3, 5 or 7 with probability 0.3, 0.4 and 0.3 respectively. The output of the model is presented as follows.

Table 4.8. Solution of the generalized SP problem

Scenarios	Technology	North	Middle	South	Total Costs
"Low Demand in North mode"	Gas	0	2.667	0	
	Coal	0	0	2	
	Nuclear	3	0.333	0	295.400
	Wind	0	0	0	
"Moderate demand in North mode"	Gas	1.667	1	0	
	Coal	0	2	0	
	Nuclear	3.333	0	0	380.333
	Wind	0	0	2	
"Highest demand in North mode"	Gas	2.667	0	0	
	Coal	1	3	0	470.333
	Nuclear	3.333	0	0	
	Wind	0	0	2	

From Table 4.8, we observe that, the optimal solution HN extensive form is given by x_1 = 2.6667, x_2= 4, x_3= 3.3333, x_4= 2 and the objective value z = 381.853. The values of x_1, x_2, x_3, x_4 give the new capacities made available in Gas, Coal, Nuclear and wind respectively, which must be determined before realizing the level of demands in the North mode of the 1st stage. The table describe the allocated capacities of all technologies and total costs in 3 modes in the three scenarios as second stage solution. The value of z is the overall expected cost.

Comparison between results in different scenarios

In this section, we present a comparison between the costs obtained for each scenarios. We see from the above discussions that total cost changes depending on an uncertain parameter demand in North mode. From Table-5, we see that minimum cost is 295.4000 units which is obtained for low demand in North mode and maximum cost is 470.3333 units which is obtained for highest demand in North mode. We present the comparison between profits for each scenario in Figure 4.7.

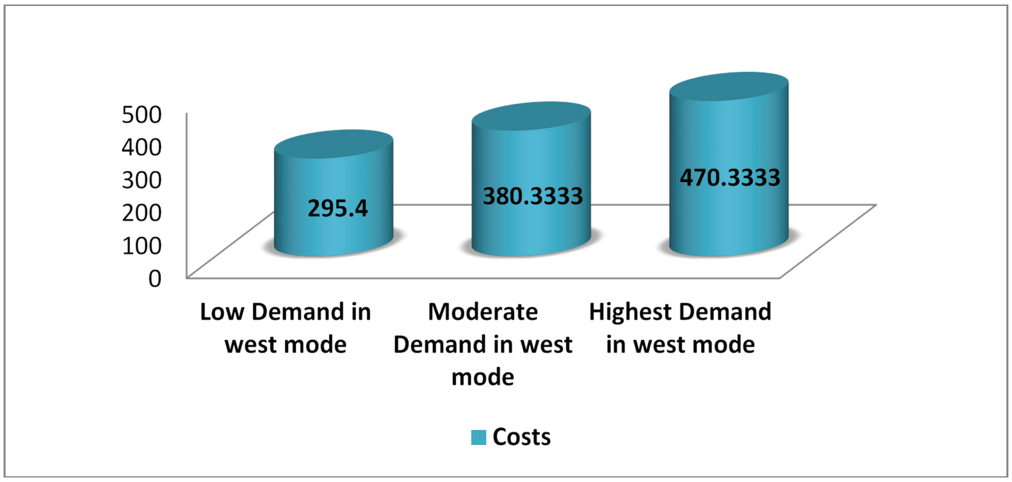


Fig. 4.5. Scenario based comparison between costs

From Figure 4.5, we observe that the column diagram shows that as the demand increase in North mode the total costs for installing new technologies increases. It happens because transportation costs are highest for each technology in North mode. Here, the expected value (EV) obtained from different scenarios is 381.853. Therefore, the Expected Value of Perfect Information (EVPI) = abs (EV- WS) = abs (381.853-367.777) = 14.076. This indicates that the company will be able to reduce the cost by 14.074 if they make a decision from the realization of random data.

Two Stage Recourse Model

In this section, considering M = 2, we develop a 2-stage stochastic recourse model for the electricity generation company as follows.

Table 4.9. Stage wise description of parameters and variables

Stage-1 Variables (to be taken without full information of future demands)	**Stage-2 Variables**
x_1, x_2, x_3, x_4	$y_{11}, y_{12}, y_{13}, y_{14}, y_{21}, y_{22}, y_{23}, y_{24}, y_{31}, y_{32}, y_{33}, y_{34},$
	$w_{11}, w_{12}, w_{13}, w_{14}, w_{21}, w_{22}, w_{23}, w_{24}, w_{31}, w_{32}, w_{33}, w_{34},$
	$z_{11}, z_{12}, z_{13}, z_{14}, z_{21}, z_{22}, z_{23}, z_{24}, z_{31}, z_{32}, z_{33}, z_{34}$

Stage-1 Model:

$$z = min \sum_{i=1}^{4} c_i x_i + \sum_{j=1}^{3} E_{\xi}\ Q_j(x_j,\ \xi)$$

$$s.t.\quad Budget\ \ constraints\quad \sum_{j=1}^{4} a_j x_j \le b$$

$$x_i \ge 0$$

Stage-2 Model:

Scenario 1

$$z = min \quad \sum_{i=1}^{4}\sum_{j=1}^{3} p_{ij}\, y_{ij}$$

$$Capacity\ \ constraints\quad \sum_{i=1}^{4}\sum_{j=1}^{3} y_{ij} \le x_i\ ,$$

Demand constraints

$$\sum_{i=1}^{4}\sum_{j=2}^{3} y_{ij} \le b_j\ ,$$

$$x_i \ge 0,\quad y_{ij} \ge 0$$

Similar models can be defined for Scenario 2 and Scenario 3.

$$z = \min 10x_1 + 7x_2 + 16x_3 + 6x_4 + E_{\xi} \min Q(x, \xi)$$

St

$$10x_1 + 7x_2 + 16\, x_3 + 6\, x_4 \le 120$$

Stage-2:

Scenario-1:

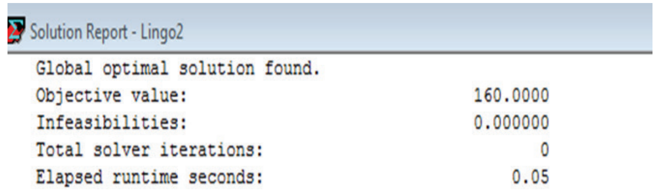
$$Q_1 = \min 40y_{11} + 45y_{21} + 32y_{31} + 55y_{41} + 24y_{12} + 27y_{22} + 19.2y_{32} + 33y_{42} + 4y_{13} + 4.5y_{23} + 3.2y_{33} + 5.5y_{43}$$

subject to

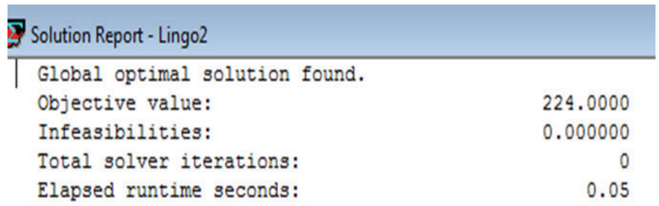
$$y_{11} + y_{12} + y_{13} - x_1 \le 0$$
$$y_{21} + y_{22} + y_{23} - x_2 \le 0$$
$$y_{31} + y_{32} + y_{33} - x_3 \le 0$$
$$y_{41} + y_{42} + y_{43} - x_4 \le 0$$
$$y_{11} + y_{21} + y_{31} + y_{41} \ge 3$$
$$y_{12} + y_{22} + y_{32} + y_{42} \ge 3$$
$$y_{13} + y_{23} + y_{33} + y_{43} \ge 2$$
$$x_i \ge, y_{ij} \ge 0$$

Detailed 2-stage SP model for the company is discussed as follows.

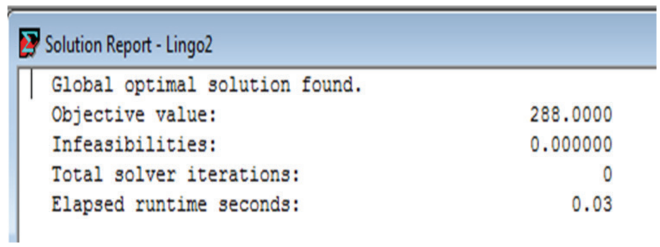
Solution by LINGO: Scenario 1



Solution by LINGO: Scenario 2



Scenario-3



1st stage cost for the Medium demand, x_1=2.33, x_2= 67, x_3 = 5, x_4 = 2. $1st\ Stage\ Cost = (10 * 2.3333 + 7 * .6667 + 16 * 5 + 6 * 2)$ = 120, 2nd stage Cost = .3*160+.4*224+.3*288 = 224. Total cost = 120+224 = 344 is the Expected Value Model Solution.

Value of stochastic solution (VSS**)** is the possible gain from the difference between the solution of the deterministic equivalent of the SP problem and the solution from the Expected Value Model. Therefore, VSS. The above results shows that VSS is not equal to EVPI, and, may in fact be larger than the EVPI.

We developed a stochastic model or the power generation problem handling the uncertainty in the electricity demand. The problem aims at making the most effective investment decisions based on different scenarios. In the end, a visual comparison between EVPI and VSS has been shown. EVPI measures the value of knowing the future with certainty but VSS assesses the value of knowing and using distributions on future outcomes.

V. Conclusion

In this research paper, we have developed a multistage stochastic recourse model expanding capacity of an electricity generation company. The model was concerned with the timing of facility expansions and levels of investment to meet increasing demand. Since demand is difficult to forecast and expansion plans may need to change over time, our stochastic programming model offers a convenient way to solve these problems. Our multi stage stochastic capacity expansion model formulated the real world problem of the company so that it can satisfy the future demand of the company. We also observed that our model is an efficient tool to obtain optimal levels of investment in various types of power plants by allocating the limited resources and capacities to different power terminals. To observe whether the company can maximize the utilization of resources and maximize profit, and to demonstrate the applicability of our model, we analyzed a two-stage problem derived from this model. We used a mathematical programming language LINDO and LINGO for solving resulting SPs. We also used MATHEMATICA to draw the continuous function. In our stochastic model, we assumed that the demand and cost are stochastic. We hope that if our model is implemented, the electricity generation company will be able to minimize their cost and maximize the utilization of the limited resources and maximize their profit. In future research, we will try to generalize the model considering lifetime the delay factors and the availability factors as stochastic too.

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