

A Review on the Proposed Quasi-Concave Quadratic Programming Problems with Bounded Variable: by Counter-example, by Computer Algebra

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I. Introduction

Quasi concave quadratic programming (QP) bounded variable problems in which the objective function involve the product of two factorized linear functions and constraints functions are in the form of linear inequalities and the variables are bounded. The purpose of this research is to series of study on nonlinear programming (NLP) problems, QP Problems, linear fractional programming (LFP) problems. In this series study, firstly H. K. Das, and Hasan developed a technique for solving LFP bounded variable problems⁸ in 2012 by following LP bounded variable Problems and in the same time M. B. Hasan developed a technique for solving special type QP Problems². Latter in 2013, H.K. Das and Hasan³ developed a generalized technique for solving unconstrained NLP problems. Again in 2013 H. K. Das and Hasan⁷ improved decomposition approach and its computer technique for solving primal dual LP and LFP problems. In 2014, H.K Das , T. Saha and Hasan⁵ studied on 1-D Simplex Search and its numerical experiments through computer algebra. In 2015 H. K. Das and Hasan⁴ studied on the algorithmic technique for solving NLP and QP problems. Finally, in 2016 H. K. Das⁹ developed a decomposition procedure for solving NLP and QP problems based on Lagrange and Sander's Method. However an algorithm for solving quasi-concave QP bounded variable problems proposed by M. Asadujjaman and Hasan¹ in 2015. But unfortunately, this proposed method¹ arises a big question to solve the quasi-concave QPBV problems. So, it becomes very important to study on the quasi-concave QPBV problems.

Therefore, this paper is concerned with the analysis of the proposed algorithm¹ of paper's concerning of the quasi-concave QP bounded variables problems, failed to solve in this type of problems. A counter example is given to verify the proposed algorithm¹ and compare the result obtained from the counter example for the proposed algorithm¹ with the build in command in Mathematica and developed code in Mathematica. Finally, it is suggested that the counter example might be adequate to justify the proposed algorithm¹.

II. Experimental

Numerical Experiment

This Section is designed for the justification of proposed algorithm¹.

Counter Example 1

This example is taken from Asadujjaman and Hasan¹.

$$\text{Max } z = (5x_1 + x_2 + 10)(4x_1 + 2x_2 + 12)$$

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subject to:

$$5x_1 + x_2 + x_3 = 20$$

$$4x_1 - x_3 + x_4 = 14$$

$$2 \leq x_1 \leq 5, 4 \leq x_2 \leq 12, 0 \leq x_3 \leq 25, 0 \leq x_4 \leq 18$$

Solution Using Proposed Algorithm¹

Since x_1 and x_2 has positive lower bound so we substituted at its lower bound. Let, $x_1 = 2 + y_1$, then $0 \leq y_1 \leq 3$ and $x_2 = 4 + y_2$, then $0 \leq y_2 \leq 8$. Substitute these into the above problem we get following

$$\text{Max } z = (5y_1 + y_2 + 24)(4y_1 + 2y_2 + 28)$$

subject to:

$$5y_1 + y_2 + x_3 = 6$$

$$4y_1 - x_3 + x_4 = 6$$

$$0 \leq y_1 \leq 3, 0 \leq y_2 \leq 8, 0 \leq x_3 \leq 25, 0 \leq x_4 \leq 18$$

Table 1. Initial Table

c_B	d_B	$c_j \rightarrow$	5	1	0	0
\downarrow	\downarrow	$d_j \rightarrow$	4	2	0	0
		$x_{Bi} \rightarrow$	y_1	y_2	x_3	x_4
1	2	$y_2 = 6$	Ⓢ	1	1	0
0	0	$x_4 = 6$	4	0	-1	1
$z_1 =$	$z^2 =$	$z =$				
30	40	1200				
		$c_j - z_j^1$	0	0	-1	0
		$d_j - z_j^2$	-6	0	-2	0
		$\Delta_j \rightarrow$	180	0	30	0

$$\theta_1 = \min\left\{\frac{6}{5}, \frac{6}{4}\right\} = 6/5, \text{ since } (\alpha^j_i > 0, \theta_2 = \infty,$$

$$U_1 = \text{upper bound of } y_1 = 3.$$

$$\theta = \min\{\theta_1, \theta_2, U_j\} = 6/5 = \theta_1. \text{ So the entering}$$

variable is y_1 in replace of y_2 .

In the following table, all $\Delta_j \leq 0$ so this table is given the

optimal solution. Here, the optimal solution is as follows:

$$x_1 = 2 + 6/5 = \frac{16}{5}, x_2 = 4 + 0 = 4, x_3 = 0, x_4 = \frac{6}{5}$$

and the optimal value is 984.

Table 2. Optimal table

c_B ↓	d_B ↓	$c_j \rightarrow$	5	1	0	0
		$d_j \rightarrow$	4	2	0	0
		x_{Bi}	y_1	y_2	x_3	x_4
5	4	$y_1 = 6/5$	1	1/5	1/5	0
0	0	$x_4 = 6/5$	0	-4/5	-9/5	1
$z_1 =$ 30	$z^2 =$ $\frac{164}{5}$	$z =$ 984				
		$c_j - z_j^1$	0	0	-1	0
		$d_j - z_j^2$	0	6/5	-4/5	0
		$\Delta_j \rightarrow$	0	-36	-44/5	0

III. Results and Discussion

Following table observes that the coding output, build in command output and exact results are identical, but the proposed algorithm¹ is not identical with the exact solution.

Table 3. Result comparison

Method	Results
Original Problem	Optimal value: 1200 Optimal solution: {2,10,0,6} Comments: Exact solution
Proposed algorithm ¹	Optimal value: 978 Optimal solution: $\{\frac{16}{5}, 4, 0, \frac{6}{5}\}$ Comments: Not identical with exact solution
Mathematica Code	Optimal value: 1200 Optimal solution: {2,10,0,6} Comments: Identical with exact solution
Build in Command	Optimal value: 1200 Optimal solution: {2,10,0,6} Comments: Identical with exact solution

In page 114, example 2, addressed as counter example-1 in the current paper is failed. In page 116, reference¹ Mathematica "Output for Numerical Example 2" is not optimal solution for the quasi-concave QP problem. The possible all basic solution is given in the following table.

Table 4. All basic solution for numerical example-2 ref¹

All Possible basic solution	Obj. function value
{2,4,6,12,0,3,0,8,19,6}	672
$\{\frac{16}{5}, 4, 0, \frac{16}{5}, \frac{6}{5}, \frac{9}{5}, 0, 8, 25, \frac{84}{5}\}$	984
{2,10,0,6,0,3,6,2,25,12}	1200

Therefore the optimal solution is {2,10,0,6,0,3,6,2,25,12} and the optimal value is 1200. However, in reference¹, on page 116, "Output for numerical example 2", the optimal value the quasi-concave QP is 984 and optimal solution is $\{\frac{16}{5}, 4, 0, \frac{16}{5}, \frac{6}{5}, \frac{9}{5}, 0, 8, 25, \frac{84}{5}\}$.

IV. Conclusion

The aim of the research was to study a series of study on NLP problems, QP Problems, LFP problems and quasi-concave QPBV problems. We then found that proposed quasi-concave QPBV algorithm¹ was failed to solve in this type of problem. A counter example was given to demonstrate this argument. A computer technique also introduced by using programming language Mathematica for quasi-concave QPBV problems. We therefore, hope that that the counter example might be adequate to justify the proposed algorithm¹.

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