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Effect of Magnetic Field on Unsteady Free Convection Flow Over a Vertical Wavy Surface

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Abstract

Unsteady free convection flow over a vertical wavy surface in the presence of magnetic field is presented. The governing equations are reduced to a system of dimensionless equations by using appropriate transforms. Resulting equations are then solved numerically employing implicit finite difference method. Effects of the amplitude-wavelength ratio and the magnetic field parameter are discussed in terms of the local skin-friction coefficient, the local Nusselt number and the local Sherwood number.

I. Introduction

In engineering applications free convection flow problems are important. These applications include the chemical distillatory processes, design of heat exchangers, formation and dispersion of fog, channel type solar energy collectors, and thermo-protection systems. The study of effects of magnetic field on free convection flow is often found importance in liquid metals, electrolytes and ionized gases. The ancillary of magnetohydrodynamics (MHD) has allured the attention of a large number of scholars due to its diverse applications in several problems of technological importance. The interaction of ionized gas or plasma can be made with the magnetic field.

The notion of natural convection along a vertical wavy surface was first studied by Yao[1] using an extension of Prandtl's transposition theorem and a finite difference scheme. He proposed a simple transformation to study the natural convection heat transfer from isothermal vertical wavy surfaces, such as the sinusoidal surface. Elbashbeshy[2] studied the heat and mass transfer along a vertical plate with variable surface temperature and concentration in the presence of the magnetic field. Alam et al.[3] have studied the problem of free convection from a wavy vertical surface in the presence of a transverse magnetic field. Al-Odat [4] investigated magnetic field effect on heat and fluid flow over a wavy surface. Heating effect on MHD natural convection flow along a vertical wavy surface with viscosity dependent on temperature was investigated by Parveen and Alim[5]. Besides, Jang and Yan[6] investigated transient analysis of heat and mass transfer by natural convection over a vertical wavy surface. Elgazery and Elazem[7] studied the effects of variable properties on MHD unsteady natural convection heat and mass transfer over a vertical wavy surface. Yang et al.[8] discussed about the natural convection of non-Newtonian fluids along a wavy vertical plate including the magnetic field effect. MHD boundary layer flow and heat transfer on a continuous moving wavy surface was studied by Hossain and Pop[9]. Also Kumari et al.[10] investigated free convection boundary layer flow of a non-Newtonian fluid along a vertical wavy surface. The extension of mixed-convection boundary-layer flow along a symmetric wedge with variable surface temperature embedded in a saturated porous median was explored by Al-Harbi and Ibrahim[11]. The magneto hydrodynamics mixed convection flow over a permeable non-isothermal wedge was analyzed by Prasad et al[12] in the presence of variable thermal conductivity.

The objective of the present work is to analyze the unsteady natural convection flow over a vertical wavy surface with magnetic field. The governing equations are first transformed into a non-dimensional form by using appropriate non-dimensional variables. These equations are then transformed into a system of non linear partial differential equations and finally the equations are solved numerically by using implicit finite difference method. Results are presented in terms of the local skin-friction coefficient, the local Nusselt number and the local Sherwood number.

II. Mathematical Formulation

The unsteady, two-dimensional, laminar boundary layer flow of a viscous incompressible fluid over a vertical wavy surface is considered. The geometry of this problem is schematically shown in Fig. 1. It is assumed that the surface temperature of the wavy plate T_w is greater than quiescent ambient fluid temperature, T_∞. The thermo-physical properties are assumed to be constant except the buoyancy term which obeys Boussinesq approximation and thus varies only when multiplied by gravity in the x momentum equation.

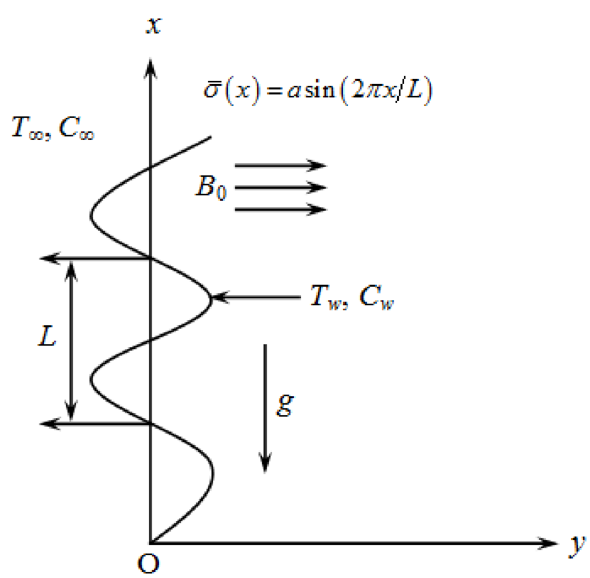

Fig. 1. Schematic diagram of the physical system

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The wavy surface of the plate is described in the function below

$$y=\bar{\sigma}(x)=a\sin(2\pi x/L) \tag{1}$$

where *a* is the dimensional amplitude of wavy surface, *L* the characteristic length scale associated with the waves and the origin of the coordinate system is placed at the leading edge of the vertical surface. The governing equations of such flow along a vertical wavy surface under the usual Boussinesq approximations can be written in a dimensional form as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{3}$$

$$+g\beta_T(T-T_\infty)+g\beta_C(c-c_\infty)-\frac{\sigma_c B_0^2 u}{\rho}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{4}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_T\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{5}$$

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\left(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}\right) \tag{6}$$

where u and v are the velocity components in the x and y directions respectively, T is the temperature of the fluid, p is the pressure, σ_c is the electrical conductivity, D is the diffusivity, ρ is the density of the fluid, g is the acceleration due to gravity, β_T and β_c are the thermal expansion coefficient and concentration expansion coefficient, B_0 is the applied magnetic field, α_T, ν, c_w and c_∞ are the thermal diffusivity, kinematic viscosity of the fluid, the average value of the fluid concentration at the plate and the concentration of the fluid far away from the plate, respectively.

Now we introduce the following dimensionless variables and parameters

$$\begin{gathered}
x^*=\frac{x}{L},\quad y^*=\frac{y-\bar{\sigma}}{L}Gr^{1/4},\quad p^*=\frac{\rho L^2}{\mu^2 Gr}p,\\
v^*=\frac{\rho L}{\mu Gr^{1/4}}(v-\sigma' u),\ Sc=\frac{\mu}{\rho D},\ \Pr=\frac{\nu}{\alpha_T},\\
\tau=\frac{\mu Gr^{1/2}t}{\rho L^2},\ C=\frac{c-c_\infty}{c_w-c_\infty},\ \theta=\frac{T-T_\infty}{T_w-T_\infty},\\
\sigma=\frac{\bar{\sigma}}{L},\ N=\frac{\beta_C(c_w-c_\infty)}{\beta_T(T_w-T_\infty)},u^*=\frac{\rho L}{\mu Gr^{1/2}}u,
\end{gathered} \tag{7}$$

Substituting the dimensionless variables into equations (2)-(6) and ignoring the small terms in *Gr*, the governing equations then become

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \tag{8}$$

$$\frac{\partial u^*}{\partial \tau}+u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{\partial p^*}{\partial x^*}+\theta+NC \tag{9}$$

$$+\left(1+\sigma'^2\right)\frac{\partial^2 p^*}{\partial y^{*2}}-Mu^*+\sigma'\frac{\partial p^*}{\partial y^*}Gr^{\frac{1}{4}}$$

$$u^{*2}\sigma''+\sigma'\left(\frac{\partial u^*}{\partial \tau}+u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}\right) \tag{10}$$

$$=-Gr^{1/4}\frac{\partial p^*}{\partial y^*}+\sigma'(1+\sigma')\frac{\partial^2 u^*}{\partial y^{*2}}$$

$$\frac{\partial \theta^*}{\partial \tau}+u^*\frac{\partial \theta^*}{\partial x^*}+v^*\frac{\partial \theta^*}{\partial y^*}=\frac{1+\sigma'^2}{\Pr}\frac{\partial^2\theta^*}{\partial y^{*2}} \tag{11}$$

$$\frac{\partial C^*}{\partial \tau}+u^*\frac{\partial C^*}{\partial x^*}+v^*\frac{\partial C^*}{\partial y^*}=\frac{1+\sigma'^2}{Sc}\frac{\partial^2 C^*}{\partial y^{*2}} \tag{12}$$

where $M=\sigma_C B_0^2 L Gr^{-1/2}/\rho^2$, which is called dimensionless magnetic parameter. It is noticeable that equation (9) indicates that $\partial p/\partial y$ is of order $Gr^{1/4}$, which implies that the lower order pressure gradient along the x axis is determined from the inviscid solution. However, for the present problem this gives $\partial p/\partial x=0$. Further, we multiply equation (10) by σ' and the resulting equation is added to equation (9) in order to eliminate the term $Gr^{1/4}\frac{\partial p}{\partial y}$ from equations (9) and (10). After some manipulation, we get

$$\frac{\partial u^*}{\partial \tau}+u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{\theta^*+NC^*}{1+\sigma'^2} -\frac{u^{*2}\sigma'\sigma''}{1+\sigma'^2}+(1+\sigma'^2)\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{Mu^*}{1+\sigma'^2} \tag{13}$$

This problem does not have a similarity solution and to obtain a solution for all $x \geq 0$ the governing equations (11), (12) and (13) have to be solved numerically. To obtain such a numerical solution we use the following transformation

$$X=x^*,\ Y=\frac{y^*\left(1-e^{-\zeta}\right)^{-\frac{1}{4}}}{\left(4x^*\right)^{1/4}},\ \zeta=\frac{\tau}{\left(4x^*\right)^{1/2}}, \tag{14}$$

$$\psi=\left(4x^*\right)^{3/4}\left(1-e^{-\zeta}\right)^{1/2} f\left(X,Y,\zeta\right),$$

$$\theta^*=\theta\left(X,Y,\zeta\right),\quad C^*=C\left(X,Y,\zeta\right)$$

where ψ is the stream function which is defined in the usual way $\left(u,v\right)=\left(\partial\psi/\partial y,-\partial\psi/\partial x\right)$. The above mentioned equation then become

$$\left(1-e^{-\zeta}\right)\frac{\partial f'}{\partial \zeta}+4X\left(1-e^{-\zeta}\right)f'\frac{\partial f'}{\partial X} -\left\{\left(1-e^{-\zeta}\right)\left(3f+4X\frac{\partial f}{\partial X}\right)+\frac{e^{-\zeta}}{2}Y\right\}\frac{\partial f'}{\partial Y} +\left(1-e^{-\zeta}\right)\left(2+\frac{4X\sigma'\sigma''}{1+\sigma'^2}\right)f'^2 =\frac{1-e^{-\zeta}}{1+\sigma'^2}\left(\theta+NC\right)+\left(1+\sigma'^2\right)f'''-\frac{1-e^{-\zeta}}{1+\sigma'^2}Mf' \tag{15}$$

$$\left(1-e^{-\zeta}\right)\frac{\partial \theta}{\partial \zeta}+4X\left(1-e^{-\zeta}\right)f'\frac{\partial \theta}{\partial X} -\left\{\left(1-e^{-\zeta}\right)\left(3f+4X\frac{\partial f}{\partial X}\right)+\frac{e^{-\zeta}}{2}Y\right\}\frac{\partial \theta}{\partial Y} =\frac{\left(1+\sigma'^2\right)}{\Pr}\frac{\partial^2\theta}{\partial Y^2} \tag{16}$$

$$\left(1-e^{-\zeta}\right)\frac{\partial C}{\partial \zeta}+4X\left(1-e^{-\zeta}\right)f'\frac{\partial C}{\partial X} -\left\{\left(1-e^{-\zeta}\right)\left(3f+4X\frac{\partial f}{\partial X}\right)+\frac{e^{-\zeta}}{2}Y\right\}\frac{\partial C}{\partial Y} =\frac{\left(1+\sigma'^2\right)}{Sc}\frac{\partial^2 C}{\partial Y^2} \tag{17}$$

At time t = 0, the temperature and the concentration of the wavy surface are suddenly changed to new levels. The appropriate initial conditions can be written as

$$T=T_\infty,\ c=c_\infty \text{ and } u=v=0 \text{ for } t<0 \tag{18}$$

For $t \geq 0$,

$$u=0,\ v=0,\ T=T_w,\ c=c_w \text{ at } y=0 \tag{19}$$

$$u=0, v=0, T=T_\infty, c=c_\infty \text{ as } y\to\infty \tag{20}$$

Using (9) and (14) into the equations (18)–(20), the corresponding initial and boundary conditions are

$$\zeta<0:\ f'=f=\theta=C=0 \tag{21}$$

$$\zeta\geq 0: f'=f=0, \theta=1 \text{ and } C=1 \text{ at } Y=0 \tag{22}$$

$$f'\to 0,\ \theta\to 0 \text{ and } C\to 0 \text{ as } Y\to\infty \tag{23}$$

The local skin-friction coefficient, C_f, the local Nusselt number, Nu, and the local Sherwood number, Sh, are defined respectively as

$$C_f=\frac{2\tau_w}{\rho\tilde{U}^2},\ Nu=\frac{hL}{k},\ Sh=\frac{h_D L}{D} \tag{24}$$

where $\tilde{U}=\mu Gr^{1/2}/\left(\rho L\right)$ is a characteristic velocity and

$$\tau_w=\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]_{y=0}, \tag{25}$$

$$h_x=-\frac{k\left(\partial T/\partial n\right)}{T_w-T_\infty},\ h_D=-\frac{D\left(\partial c/\partial n\right)}{T_w-T_\infty}$$

where

$$\frac{\partial c}{\partial n}=\sqrt{\left(\frac{\partial c}{\partial x}\right)^2+\left(\frac{\partial c}{\partial y}\right)^2}, \quad \frac{\partial T}{\partial n}=\sqrt{\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2} \tag{26}$$

Substituting equations (8) and (15) into equation (24), we have

$$C_{fx}=2\left(1-\sigma'^2\right)\left(1-e^{-\zeta}\right)^{-1/2}\left(\frac{\partial U}{\partial Y}\right)_{Y=0} \tag{27}$$

$$Nu_x=-\left(1+\sigma'^2\right)^{1/2}\left(1-e^{-\zeta}\right)^{-1/2}\left(\frac{\partial \theta}{\partial Y}\right)_{Y=0} \tag{28}$$

$$Sh_x=-\left(1+\sigma'^2\right)^{1/2}\left(1-e^{-\zeta}\right)^{-1/2}\left(\frac{\partial C}{\partial Y}\right)_{Y=0} \tag{29}$$

where

$$C_{fx} = \left(\frac{Gr}{4X}\right)^{1/4} C_f, \quad Nu_X = \left(\frac{Gr}{4X}\right)^{-1/4} Nu \quad \text{and}$$

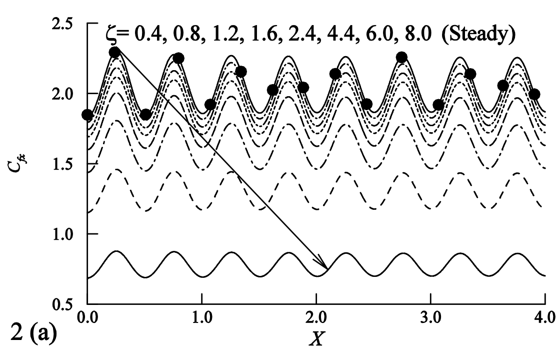
$$Sh_X = \left(\frac{Gr}{4X}\right)^{-1/4} Sh$$

III. Results and Discussion

In this paper, we have analyzed the problem of unsteady free convection over a vertical wavy surface in the presence of magnetic field. The governing equations have been solved using implicit finite difference method[13]. The local skin-friction coefficient, the local Nusselt number and the local Sherwood number are discussed with the variation of different time, ζ, the amplitude-wavelength ratio, α, and the magnetic field parameter, M.

The local skin-friction coefficient, the local Nusselt number and the local Sherwood number for different time are shown in Figs. 2(a), (b) and (c) respectively. From Figs. 2(a) and 2(c) we observe that the local skin-friction coefficient and the local Sherwood number gradually increases as time elapses and become steady state value after a large value of time $(\zeta = 8.0)$. Fig. 2(b) shows that the local Nusselt number decreases as time increases and reaches a steady state value at a large value of time.

The effects of amplitude-wavelength ratio, α, on the local skin-friction coefficient, the local Nusselt number and the local Sherwood number are depicted in Figs. 3(a), (b) and (c) respectively. Fig. 3(a) indicates that with the increase of α, the skin friction coefficient fluctuates with greater amplitude. But the maximum values of skin-friction coefficient remain almost the same. Figs. 3(b) and 3(c) demonstrate that the amplitudes of local Nusselt number and the local Sherwood number are reduced with increasing the amplitude wavelength ratio. The reasons for such a behavior are that when the amplitude-wavelength ratio is higher the flow field encounters stronger drag force so that the minimum values of the skin friction coefficient and the Sherwood number show smaller values.



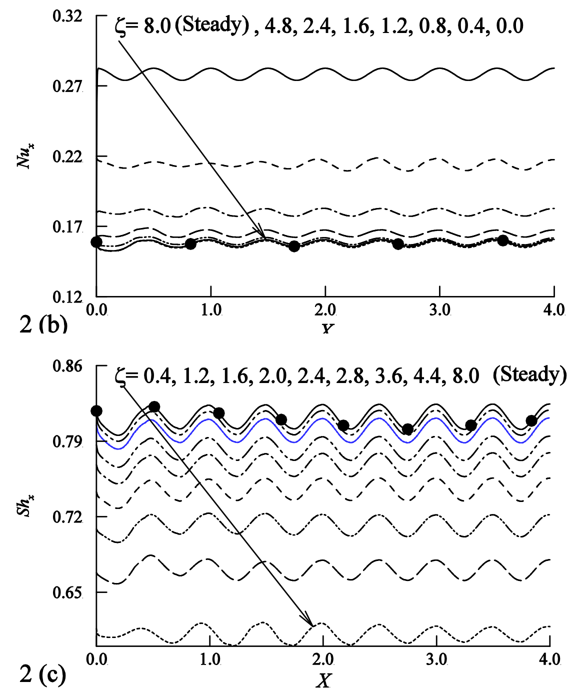
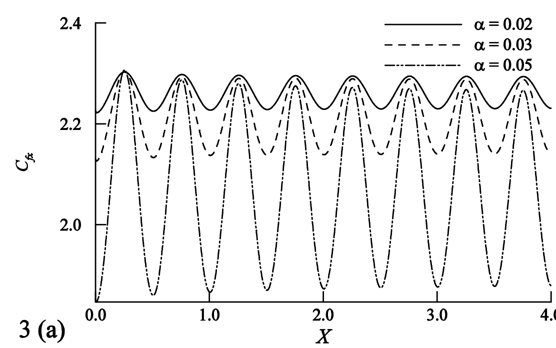


Fig. 2. Numerical values of (a) local skin-friction coefficient, (b) the local Nusselt number and (c) the local Sherwood number for different values of ζ when $\Pr = 0.1$, $Sc = 1.3$, $N = 1.0$ and α = 0.05.

The effects of varying the dimensionless magnetic parameter, M, on the local skin-friction coefficient, the local Nusselt number and the local Sherwood number are illustrated in Figs. 4(a), (b) and (c) respectively. In Fig. 4(a), we can see that with the increase of the dimensionless magnetic parameter, M, the skin-friction coefficient decreases. In Fig. 4(b) and (c) show that the local Nusselt number and the local Sherwood number also decrease owing to the increase of the dimensionless magnetic parameter, M. From the definition of the magnetic parameter, it is found that magnetic parameter increases with the increase of the strength of magnetic field. Hence the decrease of the amplitude of the local skin-friction coefficient is the result of this change of the fluid property.



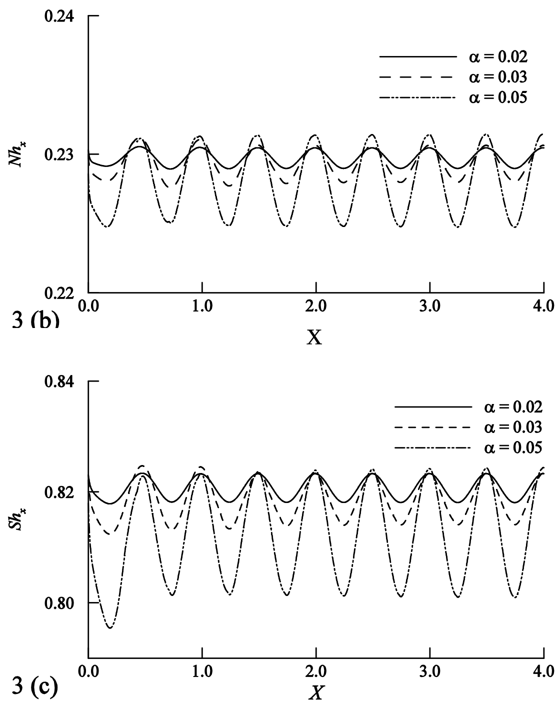
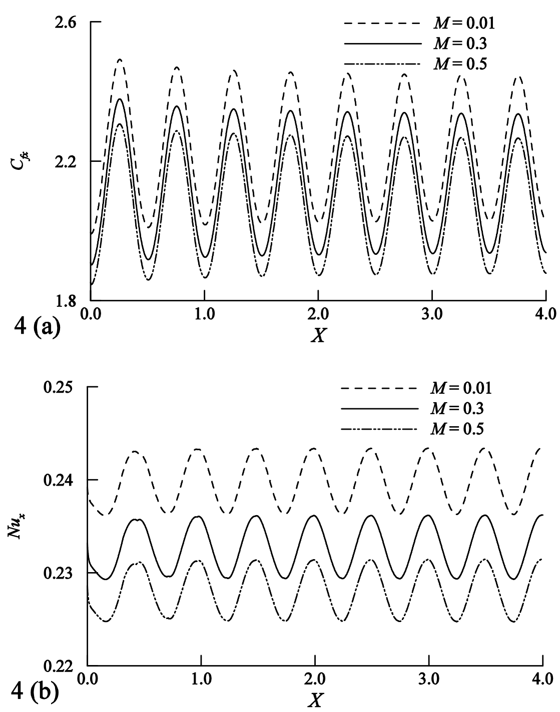


Fig. 3. Numerical values of steady state (a) local skin-friction coefficient, (b) the local Nusselt number and (c) the local Sherwood number for different values of amplitude wavelength ratio, α, while Pr = 0.1, Sc = 1.3, N = 1.0.



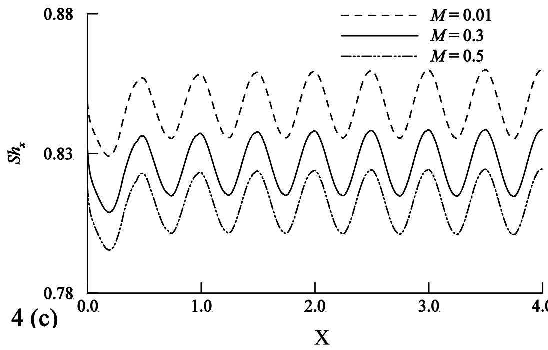


Fig. 4. Numerical values of steady state (a) local skin-friction coefficient, (b) the local Nusselt number and (c) the local Sherwood number for different values of dimensionless magnetic parameter, M, while Pr = 0.1, Sc = 1.3, N = 1.0, Q = 0.5 and α = 0.05.

IV. Conclusions

We have investigated the problem of natural convection boundary layer flow over a vertical wavy surface in the presence of magnetic field. The non-dimensional governing equations have been solved numerically by using finite difference method. With the increasing values of magnetic field parameter, the skin-friction coefficient, the local Nusselt number and the local Sherwood number are found to decrease. When the amplitude wave-length ratio is increased, the local skin-friction coefficient flactuates with greater amplitudes.

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